Cosmic 21 cm Delensing of Microwave Background Polarization and the Minimum Detectable Energy Scale of Inflation

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We propose a new method for removing gravitational lensing from maps of cosmic microwave background (CMB) polarization anisotropies. Using observations of anisotropies or structures in the cosmic 21 cm radiation, emitted or absorbed by neutral hydrogen atoms at redshifts 10 to 200, the CMB can be delensed. We find this method could allow CMB experiments to have increased sensitivity to a background of inflationary gravitational waves (IGWs) compared to methods relying on the CMB alone and may constrain models of inflation which were heretofore considered to have undetectable IGW amplitudes.

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Introduction.—Cosmological inflation [1] provides an explanation for both the predominant isotropy and homogeneity of the Universe and also the tiny, a few parts in 105, anisotropies and inhomogeneities observed in the cosmic microwave background (CMB) radiation. These tiny perturbations grew via gravitational instability to form the large-scale structure and galaxies we see today. While the details of a theory of inflation remain a mystery, a generic outcome of inflationary models is the generation of a long-wavelength background of gravitational waves [2]. Since the amplitude of inflationary gravitational waves (IGWs) is proportional to \( V \), the value of the inflaton potential \( V(\phi) \) during inflation, a reliable detection of IGWs could establish the energy scale of inflation \( 10^{14} \) [3] and help discriminate between physical models of the early Universe.

A promising technique to detect IGWs is to observe and study the polarization statistics of the CMB in detail. Since gravitational waves have a handedness, they lead to curl-like patterns of polarization (the \( B \) modes) in the two-dimensional CMB polarization field [4]. Density perturbations (which are scalar rather than tensor fluctuations and possess no handedness) cannot produce curl-like patterns to leading order and instead produce gradientlike patterns (the \( E \) modes). The main confusion to the detection of the IGW polarization signal is the transfer of power in the \( E \) modes to the \( B \) modes via gravitational lensing of CMB photons by foreground density fluctuations between our detectors and the CMB surface of last scattering [5].

In this Letter we propose a new method for discriminating between the lensing-induced contamination and the \( B \) modes from the IGW polarization signal. This method, described in detail below, relies on observations of the cosmic 21 cm radiation emitted or absorbed by neutral hydrogen atoms at redshifts 10 to 200. We begin by summarizing how the lensing confusion arises.

Gravitational lensing.—Lensing induces a remapping of the CMB polarization field at the last-scattering surface \( \hat{X}(\hat{n}) \) such that \( \hat{X}(\hat{n}) = \hat{X}[\hat{n} + \nabla \phi(\hat{n})] \) is the observed polarization field, where \( \hat{X} = Q + iU \) are linear combinations of the Stokes parameters \( Q \) and \( U \) and \( \phi(\hat{n}) \) is the lensing deflection angle. Here,

\[
\phi(\hat{n}, z_s) = -2 \int_0^{(z_s)} \frac{r(z)}{r(z_s)} \Phi(\hat{n}, r) \, \, (1)
\]

is the deflection potential, a line-of-sight projection of the gravitational potential \( \Phi \) to redshift \( z_s \). The total lensing potential \( \phi(\hat{n}) = \phi(\hat{n}; z_{\text{CMB}}) \) is this quantity evaluated at \( z_s \rightarrow z_{\text{CMB}} = 1100 \).

Using the flat-sky approximation and the \( E \)-mode or \( B \)-mode decomposition [4], the lensed \( B \)-mode polarization power spectrum, in the relevant limit \( C^{BB}_l \ll C^{EE}_l \), is

\[
\hat{C}^{BB}_l = C^{BB}_l + \int \frac{d^2l'}{(2\pi)^2} [l'' \cdot l']^2 \sin^2(2\theta_l') C^{\phi\phi}_l C^{EE}_l (2)
\]

where \( l'' = l - l' \). The second term in this expression is the lensing confusion in the \( B \)-mode map which must be separated from \( C^{BB}_l \) (the IGW signal). Here, \( C^{\phi\phi}_l \) is the angular power spectrum of the total deflection potential and is simply related to a weighted projection of the matter power spectrum [5]; \( C^{\phi\phi}_l(z_s) \) is the incomplete power spectrum out to source redshift \( z_s < z_{\text{CMB}} \).

Unlike the \( B \) modes generated by IGWs, \( C^{EE}_l \) is dominated by larger amplitude scalar perturbations. The expected few-percent conversion of \( E \) modes to the \( B \)-mode power spectrum [5]. We can relate the amplitude of fluctuations, in terms of the tensor-to-scalar ratio \( T/S \), to the inflationary energy scale via [6]

\[
\mathcal{V}^{1/4} = 3 \times 10^{-3} (T/S)^{1/4} m_{Pl},
\]

For a tensor-to-scalar ratio below \( 2.6 \times 10^{-4} \) or \( \mathcal{V}^{1/4} \) below \( 4.6 \times 10^{13} \) GeV, the IGW signal is completely confused by the lensing contaminant [7]. To bypass this limit one must separate the lensing-induced \( B \) modes from those due to IGWs. Clearly, the lensing confusion could...
be exactly removed if one knew the three-dimensional distribution of mass out to the CMB last-scattering surface. However, as our goal is a measurement of $C_{l}^{BB}$, knowledge of the projected quantity $\phi(\hat{n})$ is sufficient. One way to estimate $\phi(\hat{n}; z_s)$ is by using quadratic estimators or maximum likelihood methods to statistically infer the deflection-angle field given some lensed random field $\chi(\hat{n})$ at source redshift $z_s$. For instance, arcminute resolution CMB temperature and polarization maps could be used to make such an estimate [8–10]. Another way to estimate $\phi(\hat{n}; z_s)$ is by observing the weak-lensing distortions of the shapes of objects of a known average shape at source redshift $z_s$. However, observations of the weak lensing of galaxies, which have $z_s \sim 1$–2, cannot be used to delense CMB maps because an order unity fraction of the lensing contamination arises from the structure at $z > 3$. Higher source redshifts are needed for effective delensing.

**Cosmic 21 cm radiation.**—Neutral atoms kinetically decouple from the thermal bath of CMB photons at $z \sim 200$ and cool adiabatically as $T_s \propto (1 + z)^2$ [11]. Since the spin temperature of hydrogen atoms remains collisionally coupled to $T_s$ these atoms resonantly absorb CMB photons at $\lambda_{21} = 21.1$ cm (the hyperfine transition of the ground state of hydrogen). The cosmic 21 cm radiation is thus first observable in absorption by low-frequency radio telescopes which could detect brightness-temperature fluctuations at wavelength $\lambda = \lambda_{21}(1 + z)$ [12–14]. During reionization, the neutral gas distribution is likely to be complex due to the first luminous sources [15] and cosmic 21 cm signatures shift to emission [16]. Yet, even before reionization, it is possible the 21 cm sky is brightened by emission from neutral hydrogen gas contained in minihalos with masses $\sim 10^{3}–10^{7}$ $M_{\odot}$ [17].

Like the CMB, 21 cm fluctuations are gravitationally lensed by the foreground large-scale mass distribution. Since 21 cm radiation probes an epoch very close to CMB decoupling, CMB and 21 cm radiation are affected, essentially, by the same foreground mass distribution.

**Quadratic estimators.**—Quadratic estimators can be used to extract lensing information from the gravitationally lensed field $\chi(\hat{n})$ of some intrinsic field $\chi(\hat{n})$ at redshift $z_s$. The quadratic form $\nabla \cdot [\chi(\hat{n}) \nabla \chi(\hat{n})]$ provides an estimate of the deflection angle at position $\hat{n}$ on the sky given the $\chi$ anisotropy map. For the CMB, the quantity $\chi$ could be the temperature anisotropies [9], the polarization anisotropies, or some combination of both [10]. The brightness-temperature fluctuations in the 21 cm transition of neutral hydrogen at redshifts 10 to 200 could similarly be used [18].

In Fourier space, the quadratic estimator for the deflection potential is

$$\hat{Q}(\ell; z_s) = \frac{1}{Q_{\ell}(z_s)} \int \frac{d^2l}{(2\pi)^2} \frac{Q_{\ell}(l) \chi(l) \chi(l)}{2T_{\ell}^{XX} T_{\ell}^{XY}}.$$  

(4)

where $C_{l}^{XX}$ is the unlensed power spectrum and $T_{\ell}^{XX} = C_{l}^{XX} + N_{l}^{XX}$ is to total power spectrum, including lensing corrections and a noise power spectrum $N_{l}^{XX}$. The expectation value of the deflection-potential estimator $\langle \hat{Q}(\ell; z_s) \rangle$ (the ensemble average over realizations of the random field $\chi$) is just $\phi(\hat{n}; z_s)$. Here,

$$Q_{\ell}(z_s)^{-1} = \int \frac{d^2l}{(2\pi)^2} \frac{(l \cdot l') C_{l'}^{XX} + l \cdot l'^{*} C_{l'}^{XY})^2}{2T_{l'}^{XX} T_{l'}^{XY}}$$

(5)

is the noise power spectrum associated with a quadratic reconstruction of $C_{l}^{\phi\phi}(z_s)$ using the field $\chi$ [9].

**Partial delensing bias.**—An estimate of $\phi(\hat{n})$ can be used to delense the CMB $B$-mode polarization map. In the limit $z_s \to z_{\mathrm{CMB}}$ (conventional CMB delensing) the extraction of $C_{l}^{BB}$ from the delensed map is limited by the noise introduced during delensing. The residual contamination of the $B$ modes is given by the second term of Eq. (2) with the replacement $C_{l}^{\phi\phi} \to Q_{\ell}$. However if $z_s < z_{\mathrm{CMB}}$ this noise is not necessarily the factor limiting a measurement of the IGW signal. Using $\phi(\hat{n}; z_s)$ as a proxy for $\phi(\hat{n})$ to delense the map leaves a residual lensing contamination not due to noise. Accounting for this partial delensing bias $B_{\ell}(z_s) \equiv C_{l}^{\phi\phi} - C_{l}^{\phi\phi}(z_s)$ (due to the difference in source redshift between the lensed field $\chi(\hat{n})$ and the CMB) the residual contamination of the $B$-mode power spectrum is instead the second term of Eq. (2) with $C_{l}^{\phi\phi} \to B_{\ell}(z_s) + N_{\ell}(z_s)$. This is true whether $\phi(\hat{n}; z_s)$ is estimated using quadratic estimators or by some other method. Here, $N_{\ell}(z_s)$ is the residual noise power spectrum of the deflection potential due to noise associated with the delensing process—for quadratic reconstruction $N_{\ell}(z_s) = Q_{\ell}(z_s)$. If the deflection potential is reconstructed from a line, as is the case for cosmic 21 cm radiation, the source redshift is exactly known and $B_{\ell}(z_s)$ can be reliably estimated.

**Quadratic reconstruction.**—Unlike the CMB anisotropies, which lack power on angular scales below a few arcminutes due to Silk damping, the cosmic 21 cm anisotropies extend to much higher values of $l$ (limited by the Jeans wavelength of the gas) and peak in amplitude at higher values of $l$ [13,14]. Additionally, measurements of cosmic 21 cm anisotropies in different frequency bins provide several estimates of essentially the same deflection field.

As shown in Fig. 1, we estimate a 21 cm experiment centered around $z_s \sim 30$ with a 20 MHz coverage in frequency space capable of observing anisotropies out to $l \sim 5000$ would have an $N_{\ell}(z_s)$ higher than the planned CMBpol mission. In this case residual confusion arises from noise rather than bias. Following the approach used in Ref. [19], using the residual noise level for lensing contamination after a quadratic reconstruction of the deflection field, a CMB polarization experiment could detect $T/S \geq 2.5 \times 10^{-5}$ or $\mathcal{V}^{1/4} > 2.6 \times 10^{15}$ GeV. (Limits
sheared by weak gravitational lensing, so will be the shapes of these minihalos. Ellipticity information obtained from such 21 cm minihalos could be used to reconstruct the projected potential out to high $z$ [22]. Based on the dark-matter halo mass function, we expect roughly a surface density of $10^{11}/sr$ of such minihalos at $z \sim 30$ for a bandwidth of 1 MHz with masses between $10^6$ and $10^7$ $M_\odot$. A typical halo of mass $10^6$ $M_\odot$ has a characteristic projected angular size of $\sim 60$ milliarcseconds. If resolved, then techniques currently applied to measure shear in background galaxies in the low-$z$ Universe could be adapted for this application. In the case of minihalos, one can target a smaller area with 21 cm observations than with quadratic reconstruction to determine the projected mass distribution because ellipticities provide a direct measure of the local projected mass distribution (which can be used to delense CMB maps within the same field).

Bias-limited delensing.—To understand to what extent bias-limited reconstructions, where the residual lensing contamination is dominated by $B_L(z_s)$, would result in the removal of lensing confusion, we have calculated the residual $B$-mode power spectrum after correcting for the modified lensing kernel when $z_s < z_{CMB}$. We have adapted the formalism of Ref. [19] to estimate the smallest detectable background of IGWs, while the residual noise levels are summarized in Fig. 2. While knowing the projected mass distribution out to $z_s = 1$ does not allow the confusion to be reduced significantly, if it is known to $z_s = 10$ the confusion is reduced by an order of magnitude and the minimum detectable energy scale of inflation is reduced below the limit derived using quadratic CMB statistics [19]. A lensing-source redshift $z_s \gtrsim 30$ would be required to improve beyond the practical $1.1 \times 10^{15}$ GeV limit of the more sophisticated maximum likelihood method with high-resolution and sensitivity CMB polarization observations [20]. However, we emphasize here that in the case where a bias-limited reconstruction of the deflection field exists for $z_s \sim 30$, a very high-resolution CMB polarization experiment is not necessary and a lower-resolution (but high sensitivity) CMB polarization experiment could do the same job if paired with high-resolution observations of the cosmic 21 cm radiation.

For a bias-limited reconstruction out to a $z_s \sim 100$, the limit on the tensor-to-scalar ratio is $7.0 \times 10^{-8}$ or $\mathcal{V}^{1/4} > 6.0 \times 10^{14}$ GeV. For $z_s \sim 200$, the maximum redshift where 21 cm fluctuations are expected to be nonzero, one could probe down to $\mathcal{V}^{1/4} > 3 \times 10^{14}$ GeV.

Discussion.—While obtaining bias-limited measurements out to $z_s = 100$ is a daunting task, with many experimental and theoretical obstacles to overcome, there is great interest in detecting the fluctuations in the cosmic 21 cm radiation at $z \sim 10–200$ for their own sake and for a variety of other reasons (e.g., [13,22–25]). The observational study of 21 cm fluctuations, especially during and prior to the era of reionization, is now being pursued by a
assuming redshift $z_l$. Previous estimates of residual confusion arising in CMB-only delensing are shown for quadratic (thin short-dashed line) and maximum likelihood (thin dotted line) methods. The IGW induced $B$-mode signal is shown with (thin solid line) and without (thin long-dashed line) a low-$l$ early reionization bump assuming $T/S = 1.0 \times 10^{-1}$ or, from Eq. (3), $Y_{1/4} = 2.0 \times 10^{16}$ GeV. Limits quoted in the text can be roughly read from this figure by requiring that the lensing confusion be less than peak of the ($\tau = 0.17$) IGW signal—a lower $\tau$ would increase the minimum $Y_{1/4}$ by a factor of $\leq 2$–3. For reference, the noise curves of a high-resolution CMB polarization experiment with 2 arcminute beams and a pixel noise of 0.25 $\mu$K arcminute (thin long-short-dashed line) and a lower-resolution experiment with 30 arcminute beams (thin dot-dashed line) and similar pixel noise are shown. This latter low-resolution experiment could nevertheless detect IGWs when paired with a cosmic 21 cm lensing reconstruction. For very efficient delensing other foregrounds, such as patchy reionization [27], might dominate confusion.

A variety of low-frequency radio interferometers. While certain models of inflation may be related to grand unified theories, some supersymmetric theories of inflation have energy scales of several times $10^{14}$ GeV [26]. New methods, such as the idea of using observations of the cosmic 21 cm radiation to delense the CMB $B$-mode polarization suggested here, are needed to push the minimum detectable energy scale of inflation below $10^{13}$ GeV and discriminate between physical theories at these high energy scales.

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FIG. 2 (color online). Shown (thick line) are the residual contamination of the CMB $B$-mode polarization induced by gravitational lensing for bias-limited delensing with source redshift $z_l$. Previous estimates of residual confusion arising in CMB-only delensing are shown for quadratic (thin short-dashed line) and maximum likelihood (thin dotted line) methods. The IGW induced $B$-mode signal is shown with (thin solid line) and without (thin long-dashed line) a low-$l$ early reionization bump assuming $T/S = 1.0 \times 10^{-1}$ or, from Eq. (3), $Y_{1/4} = 2.0 \times 10^{16}$ GeV. Limits quoted in the text can be roughly read from this figure by requiring that the lensing confusion be less than peak of the ($\tau = 0.17$) IGW signal—a lower $\tau$ would increase the minimum $Y_{1/4}$ by a factor of $\leq 2$–3. For reference, the noise curves of a high-resolution CMB polarization experiment with 2 arcminute beams and a pixel noise of 0.25 $\mu$K arcminute (thin long-short-dashed line) and a lower-resolution experiment with 30 arcminute beams (thin dot-dashed line) and similar pixel noise are shown. This latter low-resolution experiment could nevertheless detect IGWs when paired with a cosmic 21 cm lensing reconstruction. For very efficient delensing other foregrounds, such as patchy reionization [27], might dominate confusion.

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