WHAT CAN WE LEARN FROM THE RISING LIGHTCURVES OF RADIOACTIVELY-POWERED SUPERNOVAE?

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ABSTRACT

The lightcurve of the explosion of a star with a radius \( \lesssim 10 - 100 R_\odot \) is powered mostly by radioactive decay. Observationally such events are dominated by hydrogen deficient progenitors and classified as Type I supernovae (SNe I), i.e., white dwarf thermonuclear explosions (Type Ia), and core collapses of hydrogen-stripped massive stars (Type Ib/c). Current transient surveys are finding SNe I in increasing numbers and at earlier times, allowing their early emission to be studied in unprecedented detail. Motivated by these developments, we summarize the physics that produces their rising lightcurves and discuss ways in which observations can be utilized to study these exploding stars. The early radioactive-powered lightcurves probe the shallowest deposits of \(^{56}\text{Ni}\). If the amount of \(^{56}\text{Ni}\) mixing in the outermost layers of the star can be deduced, then it places important constraints on the progenitor and properties of the explosive burning. In practice, we find that it is difficult to determine the level of mixing because it is hard to disentangle whether the explosion occurred recently and one is seeing radioactive heating near the surface or whether the explosion began in the past and the radioactive heating is deeper in the ejecta. In the latter case there is a “dark phase” between the moment of explosion and the first observed light emitted once the shallowest layers of \(^{56}\text{Ni}\) are exposed. Because of this, simply extrapolating a lightcurve from radioactive heating back in time is not a reliable method for estimating the explosion time. The best solution is to directly identify the moment of explosion, either through observing shock breakout (in X-ray/UV) or the cooling of the shock-heated surface (in UV/optical), so that the depth being probed by the rising lightcurve is known. However, since this is typically not available, we identify and discuss a number of other diagnostics that are helpful for deciphering how recently an explosion occurred. As an example, we apply these arguments to the recent SN Ic PTF 10gyv. We demonstrate that just a single measurement of the photospheric velocity and temperature during the rise places interesting constraints on its explosion time, radius, and level of \(^{56}\text{Ni}\) mixing.

Subject headings: hydrodynamics — shock waves — supernovae: general

1. INTRODUCTION

A typical supernova (SN) lightcurve is powered by a combination of two sources: (1) the energy deposited by the SN shock and (2) the radioactive decay of \(^{56}\text{Ni}\) that was synthesized during the explosion\textsuperscript{3}. The first light from shock breakout and the following early emission (for the first minutes to days) is dominated by the shock-deposited energy. The very late emission, after the radiation can efficiently diffuse through the entire ejecta (\(\gtrsim 100\) d), is dominated by radioactive decay. The relative influence of these power sources during the time in between depends on two main factors: (1) the progenitor radius \(R_\star\) and (2) the amount of \(^{56}\text{Ni}\). The reason for this is that the bolometric luminosity from shock heating increases linearly with the progenitor radius (as we discuss in \S\textsuperscript{2.2}), while the peak of the radioactively-powered luminosity is roughly linear with the total mass of \(^{56}\text{Ni}\) \cite{1979ARNPS...9...79A}.

Core-collapse SNe typically synthesize \(\sim 0.01 - 0.1 M_\odot\) of \(^{56}\text{Ni}\), and the progenitor radii range between \(\sim 1 - 1000 R_\odot\). Therefore, in more extended progenitors the main SN event is dominated by shock heating, while in more compact ones it is dominated by radioactive power. The most common SNe II-P are explosions of red supergiants with \(R_\star \sim 500 R_\odot\). They exhibit an extended plateau phase for about a hundred days, which is powered by the cooling of shock-heated material. The progenitors of the rare 1987A-like SNe II are blue supergiants with \(R_\star \sim 50 R_\odot\) \cite{1988ApJ...326L..55W,Kleiser2011}, and their lightcurves are dominated by radioactive decay starting a week after the explosion. Similarly, SNe Ib with compact progenitors (\(R_\star \lesssim 10 R_\odot\)) like SN 2011dh \cite{2011Natur.477..487A} show evidence of both shock-heated material and a separate radioactive peak. The progenitors of SNe Ib/c are massive stars that were stripped of their hydrogen envelope and are also fairly compact with \(R_\star \lesssim 10 R_\odot\). Indeed SNe Ib/c are dominated by radioactive power starting a few days (or earlier) after the explosion. Finally, for the compact white dwarf progenitors of SNe Ia with \(R_\star \lesssim 0.01 R_\odot\), the shock-heating lightcurves are so dim that they have never been observed, and our knowledge of these events is only possible due to the synthesis of \(\sim 0.5 M_\odot\) of \(^{56}\text{Ni}\).

Since the physics that governs the lightcurves of most SNe with \(R_\star \lesssim 50 R_\odot\) has many similarities, a single theoretical framework should roughly describe their main qualitative features. Their emission during the first few days is especially exciting because it probes the shallowest layers of the progenitor. This can teach us about the exploding star’s radius, and constrain the surface composition and velocity/density gradients that reflect details of the explosive burning. The shock-heating contribution in SNe Ia has been well-studied in the literature, both with semi-analytic models \cite{2010ApJ...715L.147N,2011PhRvD..84f1201R} and with detailed radiative transfer simulations \cite{2011PhRvD..84f1201D}. The rising lightcurve from radioactive heating has been explored for the cases of SNe Ia.
Concurrent with these theoretical studies, transient surveys like with the Katzman Automatic Imaging Telescope (KAIT; Filippenko et al. 2001), and by the Palomar Transient Factory (PTF; Rau et al. 2009, Law et al. 2009) and the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS; Kaiser et al. 2002) are finding increasing numbers of these events, especially at early times. Best known among these is SN 2011fe, the closest SNe Ia in the last 25 years (Nugent et al. 2011, Bloom et al. 2012). Other SNe Ia reported within the last year with early data include SN 2009ig (Foley et al. 2012), SN 2010hn (Hachinger et al. 2012), and SN 2012cg (Silverman et al. 2012). SNe Ib/c have also been increasingly well-studied in the optical at early times, as summarized by Drout et al. (2011). Other particular recent events include SN 2008D (Soderberg et al. 2008, Modjaz et al. 2009) and PTF 12gz (Ben-Ami et al. 2012). Motivated by these exciting developments, we ask: what can and what cannot be learned from the early optical lightcurve of radioactively-powered SNe? Being more abundant, we focus on parameters that are typical to SNe I, although most of our conclusions can also be applied to subclasses of SNe II that are dominated by radioactive heating.

In our study we first consider instances where only a photometric lightcurve is available, as is often the case at early times. One of the main issues is how much can we infer about $^{56}$Ni mixing in the outermost layers of the star with this limited information. Our main conclusion is that it is hard to determine the mixing of $^{56}$Ni based on the lightcurve alone without additional information. The reason is that the lightcurve can provide an estimate for the total mass of $^{56}$Ni that is exposed to the observer at any given time, but it cannot provide a good handle on the fractional mass of $^{56}$Ni with respect to the total exposed mass (which can be determined if the time of explosion is well constrained, e.g., by detection of the shock breakout or the cooling envelope phase). Putting it differently, there is a degeneracy in the lightcurve between a recent explosion with a high fraction of $^{56}$Ni in the outermost layers and an older explosion where $^{56}$Ni resides only in deep material. In the latter case the SN has a “dark phase” that can persist for up to a few days after the explosion. During this time no $^{56}$Ni is exposed and its shock-heated cooling emission is often too faint for detection even when deep observations exist. For this reason, a simple extrapolation of the lightcurve to early times cannot reliably constrain the time of explosion.

We then consider what additional information can be extracted by using limited spectral data that provide the photosphere temperature and velocity. We show that if detailed color evolution, or if one or even better two spectra (possibly of low signal to noise) are available during the rise, then the degeneracy between explosion time and $^{56}$Ni depth can be at least partially alleviated. Key among our results is equation (19), which allows one to estimate a lower limit on the time of explosion using merely a single simultaneous measurement of the bolometric luminosity, temperature, and photospheric velocity. The methods we discuss are complementary to detailed spectral studies that probe element mixing based on their absorption and emission features in the rare cases where early high signal to noise spectra are available (e.g., Sauer et al. 2006, Parrent et al. 2012).

In the following we begin in §2 by discussing each of the ingredients that shape the electromagnetic emission starting less than an hour and up to weeks after explosion. This discussion provides a useful general guide for interpreting early observations of radioactively-dominated SNe. In §3 we investigate a specific SN Ic in some detail (PTF 10vgv) as a test case for applying these arguments and techniques. We conclude in §4 with a discussion of our results and a summary of important conclusions that should help facilitate better constraints from future SNe I observations.

2. EARLY EMISSION FROM TYPE I SUPERNOVAE

When a hydrogen-poor star explodes as a SN, a few important events occur in the moments before and after the first optical emission is seen. In this section we summarize the main properties of each of these events, their observational consequences, and how detections of some or all of these events can be used to put constraints on the properties of the exploding star. To guide the discussion we will be referring to the diagram in Figure 1, which shows the time-dependent luminosity components, photospheric radius, and velocity.

![Figure 1](image-url)
2.1. Shock Breakout

Just prior to emission, a shock is traveling through the envelope. This heats and accelerates the material, unbinding it from the star. A radiation-dominated shock accelerates in the decreasing density of the outer edge of the star with a shock velocity that scales with the density as $v_s \propto \rho^{-\beta}$ where $\beta \approx 0.19$ (Sakurai 1960). The shock continues to shallower regions until the optical depth falls to $\tau \approx c/v_s$, where $c$ is the light speed. At this point the photons are no longer trapped; they stream away and the shock dies. This is what is typically referred to as the “shock breakout” UV/X-ray flash, and it has been frequently studied because of its strong dependence on the radius (which determines both the energy budget of the shock and the timescale of the emission), allowing the progenitor star to be studied from its detection (Colgate 1974; Falk 1978; Klein & Chevalier 1978).

Recently it was realized that the radiation behind the shock falls out of thermal equilibrium for small radii hydrogen-stripped massive stars with high shock velocities, and that the breakout will be in X-rays (Katz et al. 2010; Nakar & Sari 2010). In SNe Ia, the shock achieves relativistic velocity, and its breakout emission is in the MeV range (Nakar & Sari 2012). In any case, although shock breakout is the first indication of the explosion, its impact for SNe I in the optical/UV bands is negligible. For this reason it is not discussed further in this paper and not plotted in Figure 1.

2.2. Shock-Heated Cooling Lightcurve

Immediately after shock breakout, the observed radiation is out of thermal equilibrium until roughly the time when the gas doubles its radius at $t \approx R_*/v_f$ (Nakar & Sari 2010), where $v_f \approx 2v_s$ is the final velocity of the material (Matzner & McKee 1999), which is within minutes or less. The observed photons then gain thermal equilibrium and the expansion enters its spherical homologous phase (Chevalier 1992; Piro et al. 2010; Nakar & Sari 2010; Rabinak & Waxman 2011). As the material that has been heated by the shock expands, a thermal diffusion wave begins backing its way through the ejecta. Above the depth of the diffusion wave, material cools via photon diffusion. Below this depth, material evolves adiabatically. For each fluid element, there is then a competition between adiabatic cooling and diffusive cooling that controls the energy density of that material at the moment its photons begin streaming out of the star. This determines the observational signature of shock-heated cooling, leading to a bolometric luminosity (Nakar & Sari 2010):

$$L \approx 2 \times 10^{44} \frac{E_{51}^{0.91} R_1}{\kappa_{0.2}^{0.23} M_1^{0.73}} \left( \frac{t}{1 \text{ hr}} \right)^{-0.35} \text{ erg s}^{-1}, \quad (1)$$

and an observed (color) temperature of

$$T_c \approx 4 \frac{E_{51}^{0.11} R_1^{0.38}}{\kappa_{0.2}^{0.23} M_1^{0.11}} \left( \frac{t}{1 \text{ hr}} \right)^{-0.61} \text{ eV}, \quad (2)$$

where $E = E_{51} 10^{51}$ erg is the explosion energy, $M = M_1 M_5$, is the ejecta mass, $R_1 = R_*/R_\odot$, and $\kappa$ is the opacity with $\kappa_{0.2} = \kappa / 0.2 \text{ cm}^2 \text{ g}^{-1}$ being the canonical value during the cooling phase for fully ionized hydrogen-free gas. For all the scalings in this paper we assume a polytropic index of $n = 3$, as is relevant for compact, radiative stars forming SNe Ib/c, or relativistic, degenerate WDs exploding as SNe Ia. The key result is that the luminosity during the shock-heated cooling is directly proportional to the progenitor radius.

As long as the opacity is dominated by scattering the observed temperature is determined at the thermalization depth, below the photosphere, where the optical depth is $\tau > 1$. The thermalization optical depth, $\tau_c$, and the absorption optical depth, $\tau_{abs}$, satisfy at the thermalization depth $\tau_c \tau_{abs} \approx 1$. Equation (2) assumes that $\tau_{abs}$ is dominated by free-free and minor correction are expected when bound-free absorption dominates (for a detailed discussion, see [Nakar & Sari 2012]).

In the optical/UV, the luminosity is rising as long it is in the Rayleigh-Jeans tail, resulting in (see Appendix A)

$$L_{\text{opt/UV}} \propto t^{-1.5}. \quad (3)$$

When optical/UV luminosity is observed to rise more steeply than this (as in PTF 12gzk; Ben-Ami et al. 2012), it indicates that the rise from shock-heated cooling is not being seen, and the time of explosion is not confidently constrained. This phase continues for several hours in a core-collapse SN until $T_c$ crosses the UV (at which point the UV flux starts to drop) and reaches the optical band. This is also the point that recombinant starts in cases that radioactive decay does not play an important role. In SNe Ia this phase is terminated after about an hour when the diffusion wave reaches material where the shock was not radiation dominated and the cooling envelope emission drops significantly (Rabinak et al. 2011).

Once $T_c$ drops sufficiently in a core-collapse SN, a recombinant wave begins backing its way in from the surface. The temperature drop becomes more gradual, settling at $\approx 5000 – 8000 \text{ K}$ and the luminosity drop stops or even gently rises (as discussed in Dessart et al. 2011, also see Goldoni, Sari & Nakar, in preparation). As far as optical photometry is considered, both the temperature and the luminosity are roughly constant and the SN enters a “plateau phase.” This plateau phase is similar to the one observed in SN II-P (Popov 1993; Kasen & Woosley 2009), except that for SNe I it is dimmer and that it starts earlier, both due to the smaller progenitor and enhanced adiabatic losses. Setting $T_c \approx 0.6 \text{ eV}$ as found for the plateau in the SN Ib/c models of Dessart et al. (2011), equation (2) implies that the optical plateau starts at

$$t_p \approx 20 \frac{E_{51}^{0.18} R_1^{0.62}}{\kappa_{0.2}^{0.23} M_1^{0.18}} \text{ hrs}, \quad (4)$$

and together with equation (1) we find that

$$L_p \approx 7 \times 10^{40} \frac{E_{51}^{0.85} R_1^{0.78}}{\kappa_{0.2}^{0.60} M_1^{0.60}} \text{ erg s}^{-1}, \quad (5)$$

is roughly the plateau luminosity.

To conclude, the detection of the shock-heated cooling phase in core-collapse SNe I is very challenging given its low luminosity and short duration, but the rewards are also high. Photometry alone of the rising phase provides tight constraints on the explosion time, and both the optical rising and subsequent plateau phase provide constraints on the progenitor radius (if not overcast by radioactive heating as we discuss next). A convenient method to identify that a rising optical emission is due to shock-heated cooling emission (and not

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4 The luminosity pre-factor of [Nakar & Sari 2010] is divided here by a factor of 2.5 following the numerical result of [Katz et al. 2012].

5 Applying the same line of arguments to a red supergiant with $R_\star = 500 R_\odot$, $M = 15 M_\odot$, and $\kappa = 0.34 \text{ cm}^2 \text{ g}^{-1}$, results in $t_p \approx 10 \text{ days}$ and $L_p \approx 10^{42} \text{ erg s}^{-1}$, compatible with observations of SNe II-P.
radioactively powered phase), is that the optical emission rises (as predicted by eq. (3)), while the bolometric luminosity drops. This can be seen, for example, by having simultaneous UV and optical coverage in the time that the progenitor sits between these two frequency windows.

2.3. $^{56}\text{Ni} \text{ Shallower than the Diffusion Depth}$

If there was no radioactive energy input, this would be the end of the story for the electromagnetic signal from SNe Ia, and most SNe I would be too dim to have ever been detected. But eventually the radioactive decay of $^{56}\text{Ni}$ starts heating the expanding ejecta, and this is where their lightcurves begin rising in earnest.

When the thermal diffusion wave reaches the shallowest deposits of $^{56}\text{Ni}$, the energy generation from $^{56}\text{Ni}$ roughly goes directly into the observed bolometric luminosity (as shown with the red curves in Figure 1), so that

$$L_{56} \approx M_{56} \epsilon,$$

where $M_{56}$ is the mass of $^{56}\text{Ni}$ that is exposed by the diffusion wave, and the specific heating rate from $^{56}\text{Ni}$ is

$$\epsilon(t) = \epsilon_{\text{Ni}} e^{-t/\tau_{\text{Ni}}} + \epsilon_{\text{Co}} e^{-t/\tau_{\text{Co}}} - e^{-t/\tau_{\text{Bin}}} ,$$

where $\epsilon_{\text{Ni}} = 3.9 \times 10^{10}$ erg g$^{-1}$ s$^{-1}$, $\tau_{\text{Ni}} = 8.76$ days, $\epsilon_{\text{Co}} = 7.0 \times 10^{8}$ erg g$^{-1}$ s$^{-1}$, and $\tau_{\text{Co}} = 111.5$ days. It should be pointed out that in our more detailed analysis in Appendix C we show that if $L_{56}$ is the observed luminosity, then equation (6) is only accurate up to a factor of $\sim 2$. Even with this small uncertainty, the rough level of $^{56}\text{Ni}$ mixing that can be deduced from the observations is still robust as we will discuss.

During this phase the depth of the diffusion wave is important because it tells us what part of the exploding star is being probed by the observations. The depth in mass is related to the time after explosion by (see Appendix C)

$$\Delta M_{\text{diff}} \approx 8 \times 10^{-2} \frac{L_{56}^{0.44}}{\epsilon_{\text{Ni}}^{0.58} \epsilon_{\text{Co}}^{0.32}} \left(\frac{t}{1 \text{ day}}\right)^{1.76} M_{\odot},$$

where $\epsilon_{\text{Ni}} = \epsilon / 0.1$ cm$^2$ s$^{-1}$ is the canonical value we use for the radioactively powered phase, since the gas is partially ionized. For the estimates in this paper, we use quantities that roughly correspond to the SNe Ib/c, but the same general arguments apply to SNe Ia (as well as the rare SNe II that are radioactively powered) with different prefactors. To see how much these can vary for other progenitors see Appendix C. Also note that equation (8) becomes less accurate closer to the peak of the lightcurve, when $\Delta M_{\text{diff}}$ grows more slowly with time.

When powered by radioactive heating, the observed luminosity is no longer sensitive to the progenitor radius as can be seen by the lack of an explicit dependence on $R_{*}$ in equation (6). Instead, it provides a direct measurement of the amount of $^{56}\text{Ni}$ (via eq. (6)) at the location of the diffusion wave. If the explosion time is known, equation (8) provides $\Delta M_{\text{diff}}$ into which this $^{56}\text{Ni}$ is mixed. From this the mass fraction $X_{56} \approx M_{56}/\Delta M_{\text{diff}}$ can be inferred as a function of the depth $\Delta M_{\text{diff}}$. Thus, in principle, detection of the rise of the $^{56}\text{Ni}$ lightcurve should provide an estimate of the distribution of $^{56}\text{Ni}$ (as attempted for the SN Ia 2011fe by Piro (2012), which is helpful for understanding the nature of the explosive burning and the outer structure of the progenitor.

2.4. $^{56}\text{Ni} \text{ Deeper than the Diffusion Depth: “Diffusive Tail”}$

In practice, the exercise described above is not as straightforward as one might think. The reason is that if there is a steep increase in the $^{56}\text{Ni}$ abundance, then the assumption that the observed emission is generated only by the composition at the location of the diffusion wave may not be valid. Instead a “diffusive tail” of the energy released in $^{56}\text{Ni}$-rich layers deeper than the diffusion depth (shown as purple curves in Figure 1) may actually dominate over the energy released in $^{56}\text{Ni}$-poor layers shallowly than the diffusion depth (shown as red curves in Figure 1). Next we calculate the contribution of this diffusive tail.

Consider a deposit of $^{56}\text{Ni}$ at some depth $d$ in the star. The $^{56}\text{Ni}$ decays to produce gamma-rays, which are absorbed and create thermal photons. The photons then spread due to diffusion, creating roughly a Gaussian distribution around the $^{56}\text{Ni}$. At each time $t$, the Gaussian has a width $\sqrt{\kappa t}$, where $\kappa$ is the diffusion coefficient, which is related to the diffusion time by $t_{\text{diff}} = d^2/\kappa$. For such a distribution, there is a small, but non-zero, fraction of photons that escape from the star because they have diffused by a distance $> d$ from the $^{56}\text{Ni}$ depth. The fraction of photons that reach the surface at time $t$ is

$$\text{Escaping fraction} \approx \text{erfc}\left(\frac{\sqrt{\kappa t}}{2}\right),$$

where erfc is the complementary error function. Therefore, if at some time $t'$ the diffusion wave reaches a layer of $^{56}\text{Ni}$ that would be producing a luminosity $L_{56}(t')$ according to equation (6), this implies for previous times $t < t'$ that the diffusive tail from this layer also produces a luminosity. Since $t_{\text{diff}} \propto \rho r^2 / t$, this diffusive tail luminosity is

$$L_{\text{tail}}(t < t') = L_{56}(t') \frac{\epsilon(t)}{\epsilon(t')} \frac{\text{erfc}(t'/\sqrt{\kappa t})}{\text{erfc}(1/\sqrt{\kappa})},$$

A simpler approximation to this scaling is

$$L_{\text{tail}}(t < t') \approx L_{56}(t') \frac{\epsilon(t)}{\epsilon(t')} \frac{1}{1 + t'/t},$$

which is accurate within $\approx 20\%$ for $t'/t < 1 < t'$ even though $L_{\text{tail}}$ changes by a factor of $\approx 9$ over this range of times.

From this discussion one can see what we meant earlier in this section when we mentioned “a steep increase in the abundance of $^{56}\text{Ni}$.” Consider the two $^{56}\text{Ni}$ distributions shown in Figure 1. In the top panel the $^{56}\text{Ni}$ has a rather shallow distribution which produces the $L_{56}(t)$ (shown by the red curve). Therefore if we consider the luminosity $L_{56}$ at some time $t'$, and then trace back the diffusive tail implied from that depth using equation (10) (shown as a purple curve), the diffusive tail always falls below the $L_{56}$ lightcurve. This is a case where the shallow $^{56}\text{Ni}$ prevents the diffusive tail from having a noticeable impact. In the bottom panel the $^{56}\text{Ni}$ has a steeper distribution. Now, when the diffusive tail is drawn back from a point at time $t'$, it exceeds the $L_{56}$ lightcurve. In this latter case the diffusive tail will dominate the observed rise.

Therefore to evaluate whether the diffusive tail is important at a given depth the key quantity to consider is the slope of $L_{\text{tail}}$. Using equation (11), the logarithmic derivative is

$$\frac{d \ln L_{\text{tail}}}{d \ln t} \approx 1 + \left(\frac{t'}{t}\right)^2.$$

For $t \ll t'$, this shows that $L_{\text{tail}}(t)$ can become rather steep,
the thermal diffusion wave begins backing its way into the expanding ejecta, and from this at all later times it is roughly known what depths of the exploding star are being probed by the observations via equation (8).

When the time of explosion is not known from direct detection of shock breakout or shock heating of the surface layers, our discussion of the early lightcurve should also provide some reason for caution. To illustrate the problem, in the top and middle panels of Figure 1 we compare the lightcurves for different 56Ni depositions. In the top panel, the 56Ni is deposited into rather shallow layers, therefore the timescale between the beginning of the explosion and the rise of the lightcurve is fairly short. In this case, the time of explosion could be reasonably well-approximated by extrapolating the 56Ni lightcurve back in time. In the middle panel, the 56Ni is deposited in deeper layers, and correspondingly the delay between the shock heating and rising lightcurve is longer. In this case, extrapolating the 56Ni lightcurve back in time would provide a poor estimate for the time of explosion. We are lead to following important conclusion: one cannot simply estimate the time of explosion by extrapolating the rising lightcurve back in time because its position relative to the moment of explosion depends on the depth of radioactive heating. This means that for events like SN 2011fe, the constraints on $R_\epsilon$ cannot be as tight as previously reported (Bloom et al. 2012) when the explosion time is not known.

The earliest detection of the rising lightcurve from 56Ni does not probe the shallowest layers of the star, but merely the shallowest deposits of radioactive heating. For example if the diffusive tail only reaches 56Ni after traveling $\sim 0.1M_\odot$ below the exploding star’s surface, then it will take $\sim 2$ days to detect this depth (using eq. (8)). Depending on the specific parameters for a given SN, this timescale can even be a few times larger. Similar delays between the moment of explosion and the rising lightcurve are seen in the numerical work of Dessart et al. (2011).

### 2.6. Clues about the Depth of 56Ni

When the shock breakout and shock heating are not detected, additional information from color, or even better spectroscopic, observations can be used to break the degeneracy between the depth of 56Ni and the time of explosion. We summarize some of the properties of SNe that are most useful for doing this in this section and the next.

In the bottom panel of Figure 1 we schematically show the time-dependent radius of the photosphere (orange curve) during a SN. For a polytropic index of $n = 3$, the photospheric radius is (Rabinak & Waxman 2011).

$$r_{ph}(t) \approx 3 \times 10^{14} \frac{0.1}{\log_{10} \frac{0.39}{M_1^{0.28}}} \left( \frac{t}{1 \text{ day}} \right)^{0.78} \text{ cm},$$

(13)

where we have suppressed the dependence on the structure density factor $f_\rho$ and set it to 0.01 (see Appendix B). Even though $r_{ph}$ is moving into the star as the ejecta expands, it is always moving away from an Eulerian frame. The observed color temperature is

$$T_c \approx \left( \frac{L_{\tau_c}}{4 \pi r_c^2 \sigma_{SB}} \right)^{1/4} \geq \left( \frac{L}{4 \pi r_{ph}^2 \sigma_{SB}} \right)^{1/4},$$

(14)

where $r_c$ is the color radius and $\tau_c$ is the optical depth at the color (thermalization) depth. The inequality comes from the

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**Figure 2.** Schematic diagram showing how the slope of the 56Ni distribution determines the relative importance of the diffusive tail.

As should be expected because of the exponential dependence of the thermal diffusion wave, the slope of the lightcurve at any given time $t$ is roughly $1 + (t'/t)^2 \approx 2$. For $t \approx t'$, the slope of $L(t')$ is at its shallowest with roughly $L_{\text{diff}} \propto t^2$. From this we conclude that a rising lightcurve that is shallower than $\sim t^2$ indicates that direct heating from 56Ni is dominating, as shown in the upper panel of Figure 2. In such a case, $M_{56}$ can be approximated from the observations and $X_{56} \approx M_{56}/\Delta M_{\text{diff}}$ can be inferred as a function of time if the explosion time is well constrained. Otherwise, a rising lightcurve steeper than $\sim t^2$ indicates that the diffusive tail is having an impact as shown in the bottom panel of Figure 2 and this diffusive tail must be accounted for in order to derive $X_{56}$.

This conclusion is only approximate since we are focusing on the the diffusive tail from a single depth. This is likely not too bad of a simplification, since the diffusive tails from 56Ni at even larger depths are exponentially suppressed. In detail though, the luminosity of the diffusive tail should depend on the sum of contributions from all depths. We calculate and discuss the impact of this in Appendix B. The conclusion is that even in instances when local radioactive heating dominates, the expected luminosity given by equation (6) is not more accurate than a factor of $\sim 2$.

### 2.5. The Importance of the Time of Explosion

Our discussion thus far makes it clear that it is very important to know the time of explosion. It sets the time at which

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fact that \( r_c \geq 1 \) and \( r_c \leq r_{ph} \). For typical parameters \( r_c \) is not much larger than unity (see Appendix \( B \)) and so is \( r_{ph}/r_c \), so this inequality is also a fair approximation of \( T_c \). Although \( r_{ph} \) roughly always scales as a power-law with time (eq. \( 14 \)), \( L(t) \) can vary greatly depending on the \( ^{56}\text{Ni} \) deposition. Thus, the color evolution during the rising phase provides the first clue on the \( ^{56}\text{Ni} \) depth. For deep \( ^{56}\text{Ni} \), the initial rising phase of \( L \) is exponential, so from equation \( 14 \) we see that \( T_e \) should be rising as well. On the other hand, if \( ^{56}\text{Ni} \) is shallow then \( L \) rises more gradually and \( T_e \) is roughly constant or even slowly decreasing during the lightcurve rise.

In the bottom panel of Figure \( 1 \) we also show the photospheric velocity (green curve). Taking \( v_{ph} = r_{ph}/t \), we find

\[
v_{ph}(t) \approx 35,000 \frac{0.11 E_{51}^{0.39}}{M_1^{0.28}} \left( \frac{t}{1 \text{ day}} \right)^{-0.22} \text{ km s}^{-1}.
\]

Unlike the photospheric radius, the photospheric velocity is decreasing with time. This is because the photosphere is moving into the star in a Lagrangian sense, and reaching material that is moving at successively lower velocities. Furthermore, the time derivative of the photospheric velocity is \( dv_{ph}/dt \approx t^{-1.22} \). Therefore we expect the change in the photospheric velocity to become more gradual with time.

Using the above arguments, we conclude that there are four main ways in which the depth of \( ^{56}\text{Ni} \) heating may be qualitatively inferred. All other things being equal, a larger \( ^{56}\text{Ni} \) depth implies the following.

1. The photospheric radius is larger when the \( ^{56}\text{Ni} \) heating gets out, and therefore the temperature will be lower.
2. The luminosity at early times is increasing faster than the radius expands, causing temperature to evolve bluer.
3. The photospheric velocities are lower.
4. The photospheric velocities evolve more slowly with time.

2.7. The Minimum Explosion Time

In addition to the qualitative correlations listed above, a single measurement of the photosphere velocity and color temperature during the rise can provide a strict, model independent, upper limit to the time of explosion before that measurement. This is seen by first rewriting equation \( 14 \) as

\[
L \approx 4 \pi r_c^2 \sigma_{SB} T_e^4 \left( \frac{t}{r_c} \right),
\]

and then setting \( r_c \approx v_c t_{exp} \) where \( t_{exp} \) is the time since the explosion began, to estimate that

\[
t_{exp} \approx \left( \frac{L T_e}{4 \pi r_c^2 \sigma_{SB} T_c^4} \right)^{1/2}.
\]

In general \( T_c \) can be measured directly from the observations, but any velocity that can be measured will be from an absorption feature shallower than the photosphere and generally \( v_{ph} \lesssim v_c \). Furthermore, \( \tau_c \) will be greater than unity but also difficult to infer just from the observation. It is therefore useful to have a quantity that can simply be estimated directly in terms of observable quantities,

\[
t_{min} \equiv \left( \frac{L}{4 \pi v_{ph}^2 \sigma_{SB} T_c^4} \right)^{1/2} = \frac{t_{exp}}{r_c} \left( \frac{v_c}{v_{ph}} \right) \lesssim t_{exp}.
\]

Substituting typical values we find

\[
t_{min} = 4.3 \left( \frac{L}{10^{52} \text{ erg s}^{-1}} \right)^{1/2} \left( \frac{T_c}{10^4 \text{ K}} \right)^{-2} \times \left( \frac{v_{ph}}{10^4 \text{ km s}^{-1}} \right)^{-1} \text{ days}.
\]

Therefore \( t_{min} \) is an observable quantity that provides a model independent lower limit to the time of explosion. A second reason that \( t_{min} \) is especially useful is that it only requires that the velocity and temperature of the SN be obtained at a single time. Thus when resources are limited, using equation \( 19 \) is a helpful technique for learning a lot about the SN with minimal additional investment.

This concludes our discussion of the qualitative features of rising SN I lightcurves. In general specific events may have details that we have not addressed, like non-spherical explosions or complicated velocity profiles. But by clearly spelling out the chain of reasoning behind our conclusions we provide rules of thumb that can be used to build intuition, even in cases that are somewhat more complicated. To illustrate how our arguments can be used in practice, in the next section we apply them to a recently-discovered, well-studied SNe Ic.

3. PTF 10GvV

PTF 10GvV is a SN Ic that was discovered on 2010 September 14.1446 (UTC time) with the Palomar Oschin Schmidt 48 inch telescope (P48), by the PTF survey \( \text{Corsi et al. 2012} \). In a previous image taken on 2010 September 12.4830, it was not seen down to a limiting magnitude of \( R > 20.2 \). Following detection, the \( R \)-band luminosity rises quickly to a peak \( \approx 10 \) days later. A single spectrum was taken \( \approx 2 \) days after detection. In light of what we have been discussing, it would have been ideal if the emission from shock heating could have been identified. It would have provided a measurement of the progenitor radius and the time of explosion, which could have been used to directly probe the \( ^{56}\text{Ni} \) deposition during the rising lightcurve. Although it is unfortunate that this was not available, this event is ideal as a test case for exploring what can be learned when the time of explosion is not known.

3.1. Radius Constraints

Even though shock-heated cooling was not observed, PTF 10GvV did have an early detection of the rising lightcurve and upper limits in the time before this. Using this information, interesting constraints on the progenitor’s radius are still possible \( \text{Corsi et al. 2012} \). In Figure \( 3 \) we plot the observed \( R \)-band absolute magnitude of PTF 10GvV (circles), along with upper limits (triangle). These all assume a distance modulus of 34.05 and galactic extinction of \( A_R = 0.445 \text{ mag} \) \( \text{Schlegel et al. 1998} \). We then calculate the shock-heated cooling according to equation \( 1 \), which transitions to the plateau phase given by equation \( 5 \). We infer the corresponding \( R \)-band absolute magnitude using equation \( 2 \), or a plateau temperature of \( \approx 0.6 \text{ eV} \), along with P48-calibrated bolometric corrections provided by Eran O. Ofek (private communication). The resulting lightcurves are plotted alongside the data from PTF 10GvV in Figure \( 3 \) for a range of explosion times and progenitor radii. The conclusion is that \( \alpha \)

\[
6 \text{ In Appendix E we derive the value of } \tau_c \text{, and show that it is not much larger than unity. For example, even for high temperature of } 20,000 \text{ K and } v_{ph} = 20,000 \text{ km s}^{-1} \text{ at 10 days after the explosion } 1 < \tau_c < 6, \text{ implying } t_{min} < t_{exp} < 2.5 t_{min}. \text{ For lower temperature and velocity the range is even smaller. Therefore } t_{min} \text{ is not only a lower limit but also a rough approximation of } t_{exp}.
\]
What Can We Learn from the Rising Lightcurves of SNe?

3.2. Bolometric Lightcurve

Next we consider what can be learned about the mass and distribution of $^{56}$Ni from the observed lightcurve. Before this can be done, there are two considerations that must be made. First, we need to convert the observed $R$-band magnitudes to a bolometric luminosity. We solve for this using

$$L \approx 4\pi r_c^2 \sigma_{SB} T_c^4 / \tau_c \approx 10^{4-M_{peak}-BC}/2.5 L_\odot. \quad (20)$$

Since $BC$ depends on $T_c$, we can self-consistently find $T_c$ and thus $L$ at any given time. For our estimates here we simply take $\tau_c \approx 1$ and $r_c \approx r_{ph}$, as given by equation (13) to produce the thick, solid curves in Figure 4. Although not physically accurate, these estimates are adequate since the bolometric luminosity is found to be fairly robust. Once again, we must take into account that the time of explosion is not known, so we consider three example explosion times.

The second consideration we must make before attempting to associate the bolometric lightcurve with the $^{56}$Ni distribution is the impact of a diffusive tail. Assuming that the bolometric lightcurves in Figure 4 are representative of the $^{56}$Ni distribution, we can then ask, for a given luminosity at a given time what is the implied diffusive tail at earlier times? The dotted lines in Figure 4 show the diffusive tails originating from various times using equation (10). From this comparison we conclude that if the SN occurred recently (top panel), then only the earliest heating may be explained by a diffusive tail and the majority of the rising lightcurve directly probes the $^{56}$Ni distribution. On the other hand, if the explosion occurred further in the past (the middle and bottom panels), then the majority of the rising lightcurve can be explained as a diffusive tail due to deep deposits of $^{56}$Ni. This example demonstrates the degeneracy between the time of explosion and the depth of $^{56}$Ni if only a photometric lightcurve is available.

from $^{56}$Ni (as shown in the top diagram in Figure 2). On the other hand, if the dotted line sits above the solid curve, then the diffusive tail from that point overestimates the earlier bolometric lightcurve and luminosity at that time cannot be from direct heating of $^{56}$Ni at the diffusion depth (as shown in the bottom diagram in Figure 2).

The conclusion from this comparison is that it is difficult to determine which of these three explosion times are more accurate from just this information. If the SN occurred recently (top panel), then only the earliest heating is explained by a diffusive tail and we would infer that the majority of the rising lightcurve directly probes the $^{56}$Ni distribution. On the other hand, if the explosion occurred further in the past (the middle and bottom panels of Figure 4), then the majority of the rising lightcurve is explained as a diffusive tail due to deep deposits of $^{56}$Ni. This is consistent with our discussion in (10) that there is a degeneracy between these two limits unless more information is available.

Nevertheless, we can still try to constrain the mass and distribution of $^{56}$Ni as a function of the explosion time, the results of which are shown in Figure 5. In the top panel we plot the mass of $^{56}$Ni inferred from equation (6). This is only plotted for depths at which the bolometric luminosity cannot be explained by a diffusive tail using Figure 4. In the middle panel we plot the mass fraction $X_{56}$, and in bottom panel the thermal...
diffusion depth using equation \((8)\).

3.3. Temperature and Velocity Constraints

We next consider what additional information can be learned about PTF 10vgv from temperature and velocity measurements. Unfortunately, the available data during the rise is rather limited. Corsi et al. (2012) mention a 16,000 km s\(^{-1}\) Si II absorption feature (which is typically associated with the photosphere. Tanaka et al. (2008) observed on 2010 September 16, which corresponds to \(\approx 2\) days after discovery. From the spectrum taken on that same day, by eye one can see that it roughly peaks at \(\approx 4300\) Å, which, assuming a blackbody spectrum, corresponds to a temperature of \(\approx 6700\) K. But we must be cautious. In the study of SN 1994I by Sauer et al. (2003), a spectrum with a similar peak is observed at 8 days post explosion, and their detailed models infer a temperature of \(\sim 10,000\) K. As we describe next, if the correct temperature were known, then tight constraints could be placed on the time of explosion and thus the other properties of PTF 10vgv. But given that the temperature cannot be extracted from the data without a detailed spectral modeling, which is beyond the scope of this paper, we separately consider both the cases of 6700 K and 10,000 K and discuss the implications.

Using equation \((19)\) we estimate the minimum time of the explosion. For \(T_e = 6700\) K, \(v_{ph} = 16,000\) km s\(^{-1}\), and \(L \approx 1.5 \times 10^{42}\) erg s\(^{-1}\) (taken from Figure\(4\)), we find \(t_{min} \approx 7\) days. Therefore, if the cooler temperature is the correct one then the explosion must have occurred \(\sim 5\) days or more before the first detection. On the other hand, for \(T_e = 10,000\) K we estimate \(t_{min} \approx 3\) days and the explosion must have occurred \(\sim 1\) day or more before the first detection.

To test these conclusions, we perform a detailed fit of the temperature, velocity, and bolometric luminosity constraints simultaneously. This is done by varying \(E, M\), and the time of explosion over a wide range of values and identifying which best fit the constraints. The result for either 6700 K or 10,000 K is that a fit can only be obtained when \(E \propto M^{0.72}\) because this fixes the normalization of the velocity (see eq. \((15)\)). But for a given temperature only a very narrow range of explosion times are consistent with the data. In Figures\(6\) and \(7\) we plot the results of our fitting. These show that for a temperature of 6700 K or 10,000 K, the explosion much have occurred \(\sim 5\) days or \(\sim 2\) days prior to first detection, respectively. These match the values of \(t_{min}\) estimated before.

Besides showing how the explosion time can be constrained, Figures\(6\) and \(7\) highlight many of the general trends that we discussed in \(2\) when considering how different explosion times impact various observables of the SN. For a more recent explosion time, the gradient in \(v_{ph}\) is greater and the temperature is higher and decreasing. In contrast, for an explosion time more distant in the past, \(T_e\) is lower and increasing for a longer time. If merely a couple of data points were available, these trends could be identified in the observations, and even tighter constraints could be placed on PTF 10vgv. Nevertheless, it is powerful that even a single velocity and temperature measurement provides such stringent constraints. SNe Ia tend to have more early data available than SNe Ib/c, and we consider some of these events in forthcoming work.

3.4. Progenitors

Our conclusions from the previous sections show just how greatly our inferences about the progenitor can vary depending on the explosion time. These are summarized as follows.

1. If the temperature at 2 days past first detection is \(\sim 6700\) K, then the explosion occurred \(\sim 5\) days or more before first detection. The radius is constrained

%prog
to be $\lesssim 1 R_\odot$ and the $^{56}$Ni must be located much deeper in the ejecta. Also the ejecta is likely of higher mass since it takes longer to reach the SN peak.

2. If the temperature at 2 days past first detection is $\sim 10,000$ K, then the explosion occurred merely $\sim 2$ days before first detection. The radius is constrained to be $\lesssim 6 R_\odot$ and the $^{56}$Ni must be located fairly shallowly. Also the ejecta is likely of lower mass.

So we are left with two seemingly opposite conclusions about almost all the aspects of the ejecta just from opposite assumptions about the time of explosion. We hope that a proper modeling of the available spectrum, which we do not carry out here, can provide the correct temperature and decide between these two options. We next discuss the implications for the progenitor star that produced PTF 10vgv in each case.

Since the progenitors of such SN Ic are hydrogen-stripped stars, the models of Yoon et al. (2010) are a helpful starting place. The most striking feature of these progenitors is the inverse relationship between mass and radius that is shown in their Figure 12. If PTF 10vgv had a radius $\lesssim 1 R_\odot$, it is strongly inconsistent with any of the binary models below $M_f \lesssim 7 M_\odot$, where $M_f$ is the final mass at the time of explosion, which all have radii larger than $\sim 2 R_\odot$. Their helium star models (i.e., Wolf-Rayet progenitors) have smaller radii and may be consistent with $M_f \gtrsim 6$–$8 M_\odot$, depending of the mass loss prescription employed. Other progenitors that are consistent with such small radii are the gamma-ray burst models of Yoon et al. (2006) with $M_f \gtrsim 12 M_\odot$, but these probably are not applicable to PTF 10vgv.

Combining the generally large mass inferred for a small-radius progenitor with an ejecta mass of $\sim 3$–$5 M_\odot$ (estimated from $\Delta M_{\text{diff}}$ at peak), the remnant mass must be $\gtrsim 2$–$3 M_\odot$. This is near the range of the maximum mass of normal NSs (Lattimer & Prakash 2001). Would this imply that this event led to the formation of a black hole? Given the number of estimates that have been made throughout our analysis, it is difficult to make this conclusion with such certainty. But this does demonstrate how our analysis can make interesting connections between observations on one hand and the detailed simulations of progenitors on the other.

For the case where $R_\ast \lesssim 6 R_\odot$, Yoon et al. (2010) show that such radii are naturally expected for progenitors in a mass range of $M_f \sim 4$–$5 M_\odot$, and it is easier to reconcile the ejecta mass estimate with formation of a neutron star remnant. A less massive progenitor with a large radius has been predicted to have mixing of $^{56}$Ni into the outer layers by Rayleigh-Taylor instability during the supernova explosion (Hachisu et al. 1991; Joggerst et al. 2009), similar to what we infer for a recent explosion (Fig. 5). So even in this case there are interesting correlations and implications for the explosion mechanism that can be investigated.

Finally, it is worth discussing the inferences that can be made from the fact that PTF 10vgv was classified as a SN Ic. Dessart et al. (2012) studied the impact of $^{56}$Ni mixing on the ejecta of SNe Ib/c, and found that it strongly impacts the observed spectral features. Chief among these are the He I lines, which require non-thermal electrons that are excited by $\gamma$-ray lines from $^{56}$Ni and $^{56}$Co when the helium mass fraction is $\lesssim 0.5$ (Dessart et al. 2011). This means that the same ejecta with a significant amount of helium can produce either a SN Ib if the $^{56}$Ni mixing is strong or a SN Ic if the $^{56}$Ni mixing is weak. If PTF 10vgv were constrained to have a recent explosion with strong $^{56}$Ni mixing, we would be forced to conclude that its outer layers are very helium-poor. In the future it would be informative to repeat our analysis on a SN Ib to see if shallow $^{56}$Ni can be inferred to make the He I visible.

4. CONCLUSIONS AND DISCUSSION

We have discussed the photometric rising lightcurves of SNe I to investigate what can and cannot be learned from these observations. We provided a detailed summary of the various stages that determine the early lightcurve, and how their relative contributions are impacted by $^{56}$Ni mixing and uncertainties in the explosion time. We then looked at a particular SN Ic event PTF 10vgv in some detail. Even with no direct constraint on the explosion time, and from just an $R$-band photometric lightcurve and a single estimate of the velocity and temperature of the ejecta, we argued that the explosion time could be constrained, which would have had a number of implications for the progenitor that we discussed.

Our investigation demonstrates the kinds of connections that can be made between observations, explosion calculations, and progenitor models. To facilitate such connections in the future, we summarize our conclusions below. We also highlight observational information that we deem especially important, and which should be considered of high priority, when investigating a rising lightcurve on limited resources.

1. A SN may exhibit a dark phase between the moment of explosion and the rise of the $^{56}$Ni lightcurve. This means that extrapolating the $^{56}$Ni lightcurve back in time is not a reliable method for estimating the time of explosion, and that without a known explosion time,
constraints on $R_*$ (such as for SN 2011fe, Bloom et al. 2012) are less stringent.

2. Even though shock breakout may only emit at short wavelengths and may be too short-lived for detection in most circumstances, the UV/optical detection of shock-heated cooling phase can be just as useful for putting constraints on the progenitor radius.

3. If caught when rising (eq. (3)), shock-heated cooling can also identify the time of explosion. Conversely, if the UV/optical rise is steeper than $\sim t^{1.5}$, then this argues that the shock-heated cooling is not being observed, and the explosion time is not well constrained. Due to a possible dark phase, data right before the 56Ni lightcurve may not be sufficient to catch this rise that instead may occur $\sim 1-6$ days earlier.

4. If the time of explosion is well constrained, then the 56Ni mixing can be extracted from the bolometric luminosity in the following way. First use equation 6 to find $M_{56}(t)$ assuming there is no contribution from the diffusive tail. Then at any time $t'$ find the diffusive tail contribution from $M_{56}(t)$ at $t < t'$ (eq. (10)). If this contribution falls below the actual luminosity then $M_{56}(t)$ is a reliable estimate (to within a factor of $\approx 2$) of the 56Ni mass that mixed into $\Delta M_{\text{diff}}(t)$. Otherwise it is not. An upper limit on $M_{56}(t)$ can be derived then by finding the largest 56Ni mass that produces a diffusive tail that falls below the observed luminosity. A useful rule of thumb is that if the luminosity rises faster than $\sim t^{1.5}$, then it is dominated by the diffusive tail.

5. If the time of explosion is unknown, having even a single temperature and velocity measurement during the rise can go a long way toward supplementing the photometric data and provide strong constraints on the time of explosion using equation (19).

6. Having $\gtrsim 2$ temperature and velocity measurements during the rise will allow the time evolution of each to be constrained in more detail. Knowing the time evolution of the temperature is especially useful because it provides a check of any conclusions about 56Ni mixing.

7. A future goal of observations should be to obtain similar coverage of SNe Ib/c with strong 56Ni mixing. Such events would likely be classified as SNe Ib unless completely devoid of helium (Dessart et al. 2012) and should never show a shock-heated cooling phase. This would be an important test of whether all the telltale signs of 56Ni mixing are present as we have outlined.

Although the scalings we have used are fairly robust, the numerical prefactors come from semi-analytic work that is only approximately correct. A useful future project would therefore be to calibrate our prefactors against more detailed numerical calculations, in particular the photospheric velocity, color temperature, and diffusion depth. As highlighted by Dessart et al. (2012), the opacities for these ejecta can become complicated, so the various fits may only be applicable over different subsets of parameter space in composition and temperature. Nevertheless, the resulting collection of relations would be useful tools for quickly estimating properties of the explosions and the progenitors for future observations.

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APPENDIX

A. SHOCK-HEATED COOLING IN THE RAYLEIGH-JEANS TAIL

At early times, when the lightcurve is dominated by shock-heated cooling of ejecta and before 56Ni heating has become important, the lightcurve can rise in the optical/UV even if the bolometric luminosity is declining. This is possible as long as the UV/optical detection of shock-heated cooling phase can be just as useful for putting constraints on the progenitor radius. Therefore when a lightcurve rises faster than $t^{1.5}$ it is strong evidence that something other than shock heating is provided the observed energy (although this conclusion is not foolproof, because in principle velocity gradients may differ from $v_\rho \propto r^{-0.19}$ as we assume).

B. THE TOTAL CONTRIBUTION FROM THE DIFFUSIVE TAIL

As mentioned in §2.3 and §2.4, the total luminosity from the diffusive tail in detail is not just from a single location. Instead it is integral over the diffusive tails from all depths. Such an integral can be approximated as:

$$L_{\text{tail, total}}(t) = \int_t^{t_{\text{peak}}} X_{56}(t') \frac{\partial \Delta M_{\text{diff}}}{\partial t'} \epsilon(t') \frac{\text{erfc}(t'/\sqrt{2})}{\text{erfc}(1/\sqrt{2})} \, dt',$$  

(A1)

where $r_c$ is the color radius and $\tau_c$ is the optical depth at the color depth. Using the scalings from equations (20) and (21) of Nakar & Sari (2010), we find that $r_c \propto t^{0.83}$ and $\tau_c \propto t^{-0.44}$ for a polytropic index of $n = 3$. Combining this with $\tau_c$ from equation (2) above, this gives $L_{\text{opt/UV}} \propto t^{1.5}$ as summarized in §2.2. Therefore when a lightcurve rises faster than $t^{1.5}$ it is strong evidence that something other than shock heating is provided the observed energy (although this conclusion is not foolproof, because in principle velocity gradients may differ from $v_\rho \propto r^{-0.19}$ as we assume).
Fig. 8.— Comparisons on the lightcurves (shown in each top panel) for different distributions of $^{56}\text{Ni}$ (shown in the bottom panel), with the inclusion of the integrated diffusive tail according to equation (B2). In the middle panels we plot the ratio of the total tail component to the luminosity directly from local $^{56}\text{Ni}$.

where we take the limit of the integration to be the time of peak $t_{\text{peak}}$ since this roughly corresponds to when the diffusion wave has made its way completely through the ejecta. Making use of the time-dependence of the diffusion depth from equation (8), the integral can be rewritten as

$$L_{\text{tail,total}}(t) \approx 1.76 L_{56}(t) \int_{t_{\text{peak}}}^{t} t' \frac{X_{56}(t')}{X_{56}(t)} \left( \frac{t'}{t} \right)^{1.76} \frac{\text{erfc}(t'/\sqrt{2}t)}{\text{erfc}(1/\sqrt{2})} dt' ,$$  \hspace{1cm} (B2)

In Figure 8 we plot the results of this integral for various distributions of $X_{56}$. In all cases we use a constant luminosity from $^{56}\text{Ni}$ heating (no exponential decay) for simplicity. In the first three plots we keep the shape of the increase of $X_{56}$ the same, but vary its depth, increasing in depth from the top left to the top right to the bottom left. The top panel of each plot shows the various contributions to the lightcurve as labeled. The middle panels show the ratio of the tail contribution to the direct heating with a dotted line at a ratio of unity. In the bottom panel shows the $X_{56}$ distribution. In the bottom right plot we consider a shallower distribution of $^{56}\text{Ni}$, as can be seen in the bottom panel.

These examples confirm our general conclusion that when the $^{56}\text{Ni}$ is more shallow or has a shallower slope, the impact of the diffusive tail is diminished. These examples also show that in the shallow outer layers exposed at early times, the diffusive tail is always at least comparable to the direct $^{56}\text{Ni}$ luminosity, so that $L_{56}$ cannot be inferred from the observations more accurately than
a factor of \(\sim 2\). This uncertainty is not overly problematic because of the number of other estimates in this work and estimates of \(M_{56}\) within an order of magnitude will still facilitate our goals. Such a limitation should not be surprising, because the diffusive tail always leaks at least a little and contaminates \(L_{56}\). Our investigation of this effect also shows that the ratio \(L_{\text{tail,total}}/L_{56}\) strongly depends on the prefactor in equation (82), which comes from the power-law scaling of \(\Delta M_{\text{diff}} \propto r^n\). We generally find that this ratio is weaker with smaller \(\eta\), demonstrating that \(L_{56}\) can be estimated more reliably in such circumstances.

C. THE DIFFUSION DEPTH

Here we derive the depth of the diffusion wave as a function of time, as was used in equation (8). This roughly matches the derivation by Rabinak & Waxman (2011), with a few changes of numerical factors.

The pre-explosion density profile of the star is approximated as

\[
\rho_0(r_0) = \rho_{1/2} \delta^n,
\]

where \(\rho_{1/2}\) is the density at half the radius, \(\delta = 1 - r_0/R_\star\) is the fractional radius, and \(n\) is the polytropic index. The fraction of ejecta mass lying above a radius \(r_0\) is

\[
\delta_m = \frac{1}{M} \int_{(1-\delta)R_\star}^{R_\star} 4\pi r^2 \rho_0(r_0) dr_0 \approx \frac{3f_\rho}{n+1} \delta^{n+1},
\]

so that the mass depth is \(\Delta M = \delta_m M\).

The velocity profile from the passage of the SN shock is taken from Matzner & McKee (1999), who use an interpolation between the Sedov–von Neuman–Taylor and the Gandel’Man–Frank–Kamenetski–Sakurai self-similar solutions (Von Neumann 1947; Sedov 1959; Taylor 1950; Gandel’Man & Frank-Kamenetskii 1956; Sakurai 1960). Near the surface of the star this gives

\[
v_s = A_v \left( \frac{E}{M} \right)^{1/2} \left( \frac{4\pi}{3f_\rho} \right)^{\beta/2} \delta^{-\beta m},
\]

where \(A_v \approx 0.8, \beta = 0.19\), and \(f_\rho = \rho_{1/2}/\rho_\star\) is the ratio of the half-radius density to the average density \(\rho_\star = 3M/4\pi R_\star^2\). In Appendix D we derive \(f_\rho\) for some appropriate density profiles for the SN I progenitors we are interested in. Matzner & McKee (1999) show that the final velocity of the fluid is given by \(v_f(r_0) = f_\rho(r_0)v_s(r_0)\), where \(f_\rho \approx 2\).

The thermal diffusion length scale is

\[
D(\delta_m, t) \approx \sqrt{\frac{ct}{3\kappa \rho(\delta_m, t)}},
\]

where the density is given from continuity to be

\[
\rho = \frac{M}{4\pi r^2 t} \left( \frac{dv_f}{d\delta_m} \right)^{-1} \approx \frac{1+1/n}{\beta} \frac{\Delta M}{4\pi t^3 v_f^3}.
\]

The diffusion depth is approximated as the location where \(D\) is roughly equal to the thickness of a given shell of material

\[
\Delta r = -t \frac{dv_f}{d\delta_m} \approx \frac{\beta}{1+1/n} \frac{v_f}{\delta_m}.
\]

Setting \(\Delta r = D\), we find

\[
\delta_m = \left[ \frac{1+1/n}{\beta} f_\rho A_v \frac{1}{3\kappa} \frac{4\pi^2}{M} \left( \frac{E}{M} \right)^{1/2} \left( \frac{4\pi}{3f_\rho} \right)^{\beta/2} \frac{3f_\rho}{n+1} \right]^{(1+1/n)/(1+1/n+\beta)}.
\]

This expression is then used to derive equation (8). Note that \(\Delta M_{\text{diff}} = M \delta_m \propto f_\rho^{-0.04}\) for \(\beta = 0.19\) and \(n = 3\). In Appendix D we find that \(f_\rho\) ranges from \(\approx 0.01\) for SN Ib/c progenitors (this is what is used for the numerical factor in eq. (8)) to \(\approx 1\) for SN Ia progenitors, so the prefactor of equation (8) varies by \(\approx 20\%\) between these two very different scenarios.

D. THE DENSITY STRUCTURE FACTOR

Even though the diffusion depth is relatively insensitive to the dimensionless density structure factor \(f_\rho\), it is worth approximating just how greatly it varies for different SN I progenitors. Using Matzner & McKee (1999), for a radiative envelope of constant opacity

\[
f_\rho = \frac{\pi}{144} \frac{\beta^4}{1-\beta} \left( \frac{\mu m_p}{k_B} \right)^4 G^3 M^2 a
\]

where \(1 - \beta = L_\star/L_{\text{edd}}\) is the ratio of the stellar luminosity to the Eddington limit. For a Thomson opacity with hydrogen-deficit material this ratio is

\[
1 - \beta = 0.51 \left( \frac{L}{10^{38}L_\odot} \right) \left( \frac{M}{3M_\odot} \right)^{-1}.
\]
This then gives $f_p \approx 0.015$ for a SN Ib/c progenitor.

For degenerate electrons as in a WD progenitor for SNe Ia, the equation of state is $P = K \rho^{1+1/n}$. For the case of non-relativistic electrons $n = 3/2$ and $K = 9.91 \times 10^{32} \mu_e^{-5/3}$, and for relativistic electrons $n = 3$ and $K = 1.23 \times 10^{15} \mu_e^{-4/3}$, where $\mu_e$ is the molecular weight per electron and $K$ is in cgs units. In this case,

$$f_p = \frac{4\pi R^3}{3M} \left[\frac{GM}{(n+1)KR_c}\right]^n \approx 1.3 \left(\frac{M}{1.4M_\odot}\right)^2,$$

where in the last expression we have assumed relativistic electrons and $\mu_e = 2$.

### E. THE THERMALIZATION DEPTH

Here we approximate the optical depth at the thermalization depth as a function of the observed color temperature, $T_c$, the photospheric velocity and the time since explosion. The thermalization depth is defined as the outermost region which is in thermal equilibrium. In case that scattering dominates the opacity, and assuming that free-free dominates the photon generation (and opacity), this is the location where free-free emission generates within a diffusion time just enough photons to sustain thermal equilibrium (see Nakar & Sari 2010 for a detailed discussion). Namely, $n_{\text{BB}}(T_c) = \tau_{\text{diff}} n_{\text{ph},\text{ff}}(T_c)$ where $n_{\text{BB}}(T_c) \approx a T_c^4 / 3k_b T_c$ is the blackbody photon density and $n_{\text{ph},\text{ff}} \approx 3 \times 10^{36} s^{-1} \text{cm}^{-3} \rho^2 T_c^{1/2} x_i$ is the volumetric rate at which photons with temperature $T_c$ are generated by free-free (in cgs units), $a$ is the radiation constant, $k_b$ is Boltzmann’s constant, $\rho$ is the mass density at the thermalization depth and $x_i$ is the average number of ionized electrons per atom. Now, the diffusion time is $\tau_{\text{diff}} \approx (\tau_c r_c/c) \beta/(1 + 1/n)$ while $\tau_c = \kappa \rho r_c \beta/(1 + 1/n)$. Therefore, taking $\beta = 0.19$ and $n = 3$ and neglecting the difference between $v_c$ and $v_{ph}$ which introduces completely negligible correction, we obtain:

$$\tau_c \approx 2 \left(\frac{T_c}{10^4 \text{K}}\right)^{7/6} \left(\frac{v_{ph}}{10^4 \text{km/s}}\right)^{1/3} \left(\frac{\tau}{\text{day}}\right)^{1/3},$$

(E1)

where we approximate $x_i^{-1/3} = 1$ and $\kappa_0^{2/3} = 1$, which are reasonable values for the typical early temperatures of $T_c = 10,000 – 20,000$ K. For lower $T_c$ both $\chi$ and $\kappa$ (which are linear at each other) are smaller and so is $\tau_c$. This estimate of $\tau_c$ assumes that bound-free absorption is negligible, otherwise $\tau_c$ is smaller. Finally, $\tau_c > 1$ is always satisfied. If equation (E1) indicates otherwise, then its assumptions are not valid (e.g., scattering does not dominate the opacity) and $\tau_c \approx 1$.

### REFERENCES

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