Role of faults, nonlinear rheology, and viscosity structure in generating plates from instantaneous mantle flow models

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Abstract. Concentrated strain within plate margins and a significant toroidal component in global plate motion are among the most fundamental features of plate tectonics. A significant proportion of strain in plate margins is accommodated through motion on major tectonic faults. The decoupling influence of faulted plate margins primarily results from history-dependent lithospheric deformation rather than from instantaneous stress-weakening rheologies. For instantaneous mantle flow models, we argue that faults should be treated as preexisting mechanical structures. With models incorporating preexisting faults, a power law rheology with an exponent of 3, and slab pull and ridge push forces, we demonstrate that nonlinear interaction between weak faults and this power law rheology produces plate-like motion. Our models show that in order to produce plate-like motion, the frictional stress on faults needs to be small and the asthenosphere viscosity should be much weaker than that of lithosphere. While both plateness and the ratio of toroidal to poloidal velocities are reduced with increasing fault coupling, the viscosity contrast between the lithosphere and asthenosphere only influences plateness. This shows that both diagnostics, plateness and the ratio of toroidal to poloidal velocities, are necessary to characterize plate motion. The models demonstrate that weak transform faults can guide plate motion. This guiding property of transform faults and the decoupling of thrust faults result in oblique subduction where the strike of subducted slabs is oblique to transform faults. Subducted slabs beneath a dipping fault produce oceanic trench and fore bulge topography and principal stresses consistent with subduction zone observations.

1. Introduction

According to the theory of plate tectonics, plates are nearly rigid with negligible internal strain and with strain concentrated within plate margins [Wilson, 1965; Morgan, 1968]. Moreover, plate motion is approximately equally partitioned between polaroid (i.e., motion associated with the creation and destruction of plates) and toroidal (i.e., strike-slip motion and plate spin) velocities [Hager and O'Connell, 1979]. A significant amount of strain is accommodated in plate margins through earthquakes on major faults between plates [Davies and Brune, 1971; Kanamori, 1977]. An understanding of the dynamics of plate tectonics is a central goal of geodynamics, not only because plate tectonics is one of the most important features of the Earth but also because plates have a first-order influence on the dynamics of the deep interior [Hager and O'Connell, 1979; Davies, 1988; Gable et al., 1991].

Two classes of methodologies have been applied to the problem of coupling plates to mantle flow: one based on a force balance and another based on rheology. In the first method, plates with a preset geometry are assumed perfectly rigid; velocity for each plate is determined by balancing torques and forces that act on the plate from mantle flow [Hager and O'Connell, 1981; Ricard and Vigny, 1989; Gable et al., 1991]. These models are capable of reproducing plate motion by using seismically determined mantle buoyancy structures [Ricard and Vigny, 1989] or estimated subducted slab density anomalies based on the history of plate motion [Lithgow-Bertelloni and Richards, 1995]. Since plates are derived by definition in these models, they can only address questions of the plate driving mechanism and the influence of plates on the mantle [Hager and O'Connell, 1981] rather than answering the question of plate generation. This methodology simplifies the dynamics of plates and margins by using laterally homogeneous viscosity and prescribed boundary velocities for plates. Potentially important plate boundary forces [e.g., Forsyth and Uyeda, 1975] are often ignored in torque or force balances [Ricard and Vigny, 1989; Gable et al., 1991].

The second methodology does not assume perfectly rigid plates, rather a complex mantle rheology is used in the hope of producing plate-like surface motion from the solution of the momentum equation. Creep experiments on mantle rocks indicate that mantle rheology is both temperature- and stress-dependent. The overall form of effective viscosity can be expressed with

\[ \mu_{\text{eff}} = A \frac{1}{n} e^{\frac{1}{n-1}}. \]  (1)
where \( n \) is the exponent and is unity for a Newtonian rheology; \( A \) is the preexponent which depends on temperature, pressure, and composition; \( \dot{\varepsilon} \) is the second invariant of a strain rate tensor. For mantle rocks, the exponent \( n \) is about 3 (i.e., stress weakening) [e.g., Karato and Wu, 1993]. Equation (1) is recast from the stress-strain rate relationship (Figure 1). Stresses may be high in plate margins, and this is manifest as stress weakening [e.g., Karato and Wu, 1993].

Equation (1) is recast from the stress-strain rate relationship (Figure 1). Stresses may be high in plate margins, and this is manifest as stress weakening [e.g., Karato and Wu, 1993]. Earlier two-dimensional (2-D) models with prescribed weak zones, representing plate margins, enforce a localized deformation in weak zones [Kapitke, 1979; Schmeling and Jacoby, 1981; Jacoby and Schmeling, 1982]. The weak zones in these models were intended to mimic the stress weakening of the lithosphere and mantle inherent in equation (1), although additional weakening processes probably occur in plate margins, such as those dependent on lithology, volatiles [Lenardic and Kaula, 1994], and deformation history [Gurnis, 1997]. This weak zone formulation has been extensively used in modeling plates in two dimensions [Gurnis, 1988; Gurnis and Hager, 1988; Davies, 1989; King and Hager, 1996; Zhong and Gurnis, 1994a, Puster et al., 1993].

Considerable effort in modeling plate generation has been directed toward incorporating non-linear stress or stress-weakening rheology into dynamic models [Christensen, 1983; Weinstein and Olson, 1992; Bercovici, 1993, 1995; Solomatov and Moresi, 1997]. Models which utilize the power law rheology (equation (1)) with exponents in the range of 3 to 5 predict surface motion which has substantial internal strain but with only a few percent toroidal component [Christensen, 1983; Christensen and Harder, 1991]. By using thin plate models, Weinstein and Olson [1992] and Bercovici [1993] have shown that only when a much stronger nonlinear rheology is utilized can a concentrated strain or plate-like surface motion be achieved. The nonlinearity increases with \( n \), but the nonlinearity is strongest when the exponent is -1 as used in models considered by Bercovici [1993]. When \( n = -1 \), the power law rheology gives two possible branches of strain rate for a given stress, one normal branch with a small strain rate and the other branch with a larger strain rate [Bercovici, 1993]. For \( n \) as high as 20, the width of regions with concentrated strain (i.e., plate margins) is quite large [Weinstein and Olson, 1992], but models with \( n = -1 \) produce much narrower margins [Bercovici, 1993]. Recently, this \( n = -1 \) rheology has been implemented in a three dimensional model [Tackley, 1998]. Solomatov and Moresi [1997] used a pseudo-plastic rheology (i.e., \( n \) is infinite) with depth-dependent yield stress to model the plates, but the emphasis of their models is how a different yield stress can lead to a different style of convection (i.e., stagnant lid or mobile plate). Models with a strong nonlinear rheology apparently have the advantage over models with prescribed weak zones in that plate geometry and surface motion naturally emerge from the dynamics. However, there are significant implications of this methodology. Implicit to the stress weakening rheology is that the weakening processes are instantaneous. Essentially, the models imply that as long as the stress is high, weak fault zones will develop instantaneously, independent of deformation history. This may be inconsistent with observations.

According to the studies by Jeffreys [1970] and Artyushkov [1973], the large topography variation in regions near seamounts and orogenic belts induce large local stresses. With strongly nonlinear stress-weakening rheology (e.g., the pseudo-plasticity or with \( n = -1 \)), these high stress regions should develop weak fault zones and deform in the same way as plate margins. On the contrary, regions near large intraplate seamounts and old mountain belts do not show noticeable strain. On the other hand, transform fault plate margins, like the San Andreas, do not show high stresses [Lachenbruch and Sass, 1988], but significant deformation occurs within the fault zones. A traditional explanation, from Kanamori [1980], is that lithospheric strength itself is highly heterogeneous. Here the strength is defined as the deviatoric stress that the lithosphere can support without deforming [Jeffreys, 1970]. Kanamori [1980] has suggested that heterogeneous strength results from the distribution of weak faults which may be weak due to weak material within faults. Another possible explanation is to invoke the \( n = -1 \) rheology whose duality of strain rates can account for the apparently uncorrelated stress to strain rate. Physical mechanisms including void generation and volatile ingestion may lead to the \( n = -1 \) rheology [Bercovici, 1998]. However, more studies, both theoretical and experimental, are clearly needed in applying this rheology to lithospheric deformation. In summary, we make the following observational inferences: (1) the instantaneous stress weakening is not the primary cause for generating weak fault zones, although it may play a role, even a fundamental role, in lithospheric deformation; and (2) at any instant in time, the existence of weak fault zones reflects the deformation history rather than instantaneous stress.

With the recognition of these fundamental properties of lithospheric strength, it has become clear to us that a simple weak zone formulation, originally from Kapitke [1979], Schmeling and Jacoby [1981], and Jacoby and Schmeling [1982], has its own merits in capturing the fundamental features of history-dependent lithospheric rheology. Owing to its simplicity, however, the weak zone formulation has considerable drawbacks as a description of plate margin processes. Seismological studies suggest that most lithospheric deformation in converging margins is accommodated through great thrust earthquakes on thrust faults [Davies and Brune, 1971; Kanamori, 1977; Ruff and Kanamori, 1983]. Faults are discontinuities between plates. In the weak
zone formulation, the deformation is distributed over entire weak zones, typically a few hundred kilometers wide. Improving upon these weak zone models, Zhong and Gurnis [1994b, 1995, 1996] developed two- and three-dimensional finite element models of a mantle with preexisting faults directly incorporated into a mantle with a power law rheology. In these models, only faults are specified to be weak, although the stress may weaken the media surrounding the faults according to the power law rheology. In these models, faults are simulated as internal lines or planes across which normal velocities are continuous but tangential velocities may be discontinuous. Flow on either side of a fault can be coupled through frictional stress [Zhong and Gurnis, 1994b]. Moreover, only fault geometry is prescribed such that the sense of motion on faults (e.g., thrust or strike slip) is determined by the dynamics.

In their 2-D models, Zhong and Gurnis [1995] have shown that faulted converging plate margins may contribute to producing plate-like surface motion such that the dynamically determined motion of faulted margins is an essential ingredient to the evolution of plate size and subduction dynamics. 3-D models include both transform and thrust faults and suggest that the interaction between weak faults and a power law rheology is essential to giving rise to plate-like motion [Zhong and Gurnis, 1996]. This study is an extension of the report by Zhong and Gurnis [1996]. We include high resolution 2-D models with curved fault and pseudo-plastic rheology, and more 3-D models with a variety of vertical and horizontal viscosity structure, all of which were not presented by Zhong and Gurnis [1996]. First, we will present numerical techniques for modeling faults in two and three dimensions; second, we will briefly show the influence of a dipping fault on stress and flow fields in a 2-D model of subduction zone; third, we will detail the influence of fault geometry, fault strength, and vertical viscosity structure on plate generation; and finally, we will discuss the implications of our model results to mantle dynamics.

2. Description of Models and Numerical Methods

The governing equations for mantle flow are derived from the conservation of mass and momentum. Since the mantle has a very high Prandtl number, the inertial forces can be ignored in the equation of motion. With the assumption of incompressibility, the momentum and continuity equations are respectively,

\[ \sigma_{ij,j} + \rho g \delta_{ij} = 0, \]  
\[ u_{i,j} = 0, \]

where \( \sigma_{ij} \), \( u_i \), \( \rho \), and \( g \) are the stress tensor, the flow velocity, the density, and gravitational acceleration, respectively. The density may include contributions from temperature variation. The constitutive law is

\[ \sigma_i = -P \delta_i + 2\mu_{eff} \dot{e}_i, \]

where

\[ \dot{e}_i = (u_{i,j} + u_{j,i})/2, \]

\( P \) is the pressure, \( \delta_i \) is the unit tensor, and \( \mu_{eff} \) is the effective viscosity and is defined in equation (1). It is essential to recognize that (1) whether a model produces plate-like surface motion is entirely determined by the solution of equations (2) and (3) for a given rheology and density structure; (2) while history-dependent rheology may nominally be introduced through the preexponent of \( A \), the rheological equation (1) is only applicable to continua and cannot be used for faults of discontinuous nature. Boundary conditions of the models are free slip on both the bottom and top boundaries and reflecting on side walls.

Although we will concentrate on the influences of preexisting faults and \( n=3 \) power law rheology, we will also explore the influences of highly nonlinear stress-weakening rheologies on the generation of plates, particularly the influences of a pseudo-plastic rheology. We choose to use a pseudo-plastic rheology over other stress-weakening rheologies because the pseudo-plastic rheology gives a reasonable numerical convergence rate while containing the basic features of stress-weakening rheologies. For the pseudo-plastic rheology, the exponent \( n \) in equation (1) is assumed to be much larger than 3 when stress is larger than a yield stress (Figure 1). We refer to this rheology as "pseudo-plastic" because it differs fundamentally from the normal use of plasticity in rocks [Jaeger, 1969]. Normally, plasticity is used to describe the transitional deformation of material from elastic to plastic failure or flow regimes under a sufficiently large stress (i.e., yield stress) and is defined in terms of a stress-strain relationship [Jaeger, 1969], not a stress-strain rate relationship. Presumably, deformation in the plastic flow regime after the yield stress is reached can be described as viscous deformation or even linear viscous deformation. Pseudo-plastic rheology is based on a stress-strain relationship and implies that fluid viscosity is greatly reduced when stress is larger than a yield stress (Figure 1). In essence, pseudo-plastic rheology is a special form of a nonlinear viscous rheology. Similar pseudo-plastic rheologies have been applied to viscous flow models of the mantle by Sleep [1975] and more recently by Solomatov and Morelli [1997].

Our 3-D models in a Cartesian domain include a half-space cooling thermal boundary layer and cold subducted slabs (e.g., the geometry of one type of calculation is shown in Figure 2). The buoyancy is derived as if a region, \( \Omega \), is spreading from the left side boundary at a velocity of 3 cm \( \text{yr}^{-1} \) (Figure 2). In \( \Omega \), the surface age increases with distance from the spreading center, and outside of \( \Omega \), excluding the slabs, the thermal structure is identical to that with the oldest age (Figure 2). A 100 km thick slab extends from a depth of 100 km to 670 km with a 60° dip angle (Figure 2). Slab buoyancy is identical to that of the lithosphere just prior to subduction. The viscosity in the upper mantle including the top 100 km layer is determined by a power law rheology with \( n=3 \), while the viscosity in the lower mantle is Newtonian and equal to \( 2 \times 10^{22} \) Pa s. Temperature dependence of viscosity is simulated with the power law preexponent \( A \) such that a larger \( A \) is used for both slabs and the upper 100 km. At the spreading center, a narrow region within the top 100 km layer has a smaller \( A \) to account for the higher temperature and partial melting below a ridge. For most calculations undertaken, \( A \) for the top 100 km layer and slabs, the spreading centers, and the upper mantle are \( 2 \times 10^7 \), \( 2 \times 10^5 \), and 20, respectively. These preexponents are chosen such that the average effective viscosities for the top 100 km and the upper mantle are about \( 10^{13} \) Pa s and \( 2 \times 10^{10} \) Pa s, respectively, and that the average surface velocity in \( \Omega \) is close to the velocity used to derive the input buoyancy fields.
is zero, we explore nonzero frictional stress with prescribed values on faults.

We use a finite element method to solve equations (2) and (3) with nonlinear rheology and faults. The finite element software, Citcom, used in this study is based on a primitive variable formulation with two-level iterations to solve the pressure and velocity simultaneously [Moresi and Solomatov, 1995; Moresi and Gurnis, 1996]. Both multigrid and preconditioned conjugate gradient solvers are included in the inner level iteration in Citcom, while only a preconditioned conjugate gradient solver is used in this study because of the complex 3-D meshes resulting from the inclusion of faults. We have ported the software to massively parallel computers (see Appendix B). With the use of parallel computers, we are able to solve models with significantly higher resolution than presented by Zhong and Gurnis [1996]. For most 3-D calculations, we use about 1.5 × 10^5 linear elements (i.e., 8 nodes per element). The finite element mesh was refined near the faults, slabs, and top 100 km with horizontal and vertical spacing of about 15 km. The nonlinear momentum equation is solved iteratively until relative variation in velocity between two consecutive iterations is less than 1%.

To characterize plate-like surface motion, we define the ratio of toroidal to poloidal components $R_{T/P}$ [Gable et al., 1991] and plateness $P$ as

$$R_{T/P} = \frac{\sum_{i,m} k_x y V_{m}^{i}-k_x x V_{m}^{i}}{\sum_{i,m} k_x x V_{m}^{i}+k_x y V_{m}^{i}},$$

$$P = 1 - V_{rms}/V_{p},$$

$$V_{p} = \sqrt{\bar{u}_{x}^{2} + \bar{u}_{y}^{2}},$$

where $k_x = 2 \pi l/L_x$ and $k_y = 2 \pi m/L_y$ are the wavenumbers in $x$ and $y$ directions. $L_x$ and $L_y$ are the horizontal dimensions of the box in $x$ and $y$ directions; integer indices $l$ and $m$ cannot both be equal to zero; $V_{m}^{i}$ and $V_{n}^{i}$ are the spectra of surface velocity of $x$ and $y$ components associated with $l$ and $m$; $V_{p}$ is the magnitude of average velocity $(\bar{u}_{x} , \bar{u}_{y})$ in region $\mathcal{R}$; and $V_{rms}$ is the RMS deviation from $(\bar{u}_{x} , \bar{u}_{y})$ in $\mathcal{R}$ (Fig. 2). Plateness defined here is different from that used in 2-D studies by Weinert and Olson [1992]. Plateness is 1 for a perfectly rigid plate and is 0.52 for a sinusoidal variation in surface velocity along the spreading direction in region $\mathcal{R}$. Since $R_{T/P}$ is geometry-dependent [Olson and Bercovici, 1991], we define a normalized ratio of toroidal to poloidal components, $N_{T/P}$, which is $R_{T/P}$ normalized by the ratio of toroidal to poloidal components for a perfectly rigid plate within region $\mathcal{R}$.

3. Results

In this section, we will first show the influence of a dipping fault positioned directly above a subducted slab on plate velocity, stress, and strain rate. Then, we will focus on 3-D problems with multiple faults. For 3-D models, we will examine the roles played by viscosity structure, fault strength,
and fault geometry on plate generation. We will end with a discussion on the implications of our results to the Earth.

### 3.1. Two-Dimensional Models

Previously, the role which a dipping fault has on the dynamic support of outer rise, trench, and back arc topography in subduction zones has been investigated with 2-D viscous [Zhong and Gurnis, 1994b] and viscoelastic models [Gurnis et al., 1996]. Here, we build upon this earlier work and examine the influence of a dipping fault on stress, strain rate, and flow fields within lithosphere and mantle. The 2-D models should be viewed as cross sections dissecting the 3-D models of a subducted slab and spreading center described next (e.g., the cross section AA' in Figure 2). The rheological parameters and buoyancy structure are the same as those in 3-D models. Therefore the 2-D models can be directly compared with the 3-D models. With our higher-resolution 2-D models, we are able to directly explore the influence of fault geometry, especially the influence of a curved dipping fault with a smaller dip at a shallower depth. This latter model with a curved dipping fault is more realistic than our earlier models with a straight fault and allows better comparisons of stress and strain rate with seismic observations. A mesh with 192x64 bilinear elements is used and is refined near the subducted slab and within the top 100 km layer.

### 3.1.1. Models with and without a dipping fault. With no fault (case 1 in Table 1), the mantle flow induced by the slab and oceanic plate shows that (1) the horizontal velocity of the subducting plate is about 1.3 cm yr⁻¹, significantly greater than that of the overriding plate (Figures 3a and 4a); (2) a significant portion (~500 km in width) of the overriding plate is actively deforming and moving with a substantial horizontal velocity toward the overriding plate (Figures 3a, 3d and 4a); (3) although concentrated in the upper mantle, flow extends into the high viscosity lower mantle beneath the overriding plate (Figure 3a). In comparison with the upper mantle, the effective viscosities for the lithosphere and subducted slab are higher (Figure 3a). The averaged viscosities for the oceanic plate, slab, and upper mantle below the subducting plate are about 2x10¹⁰ Pa s, 4x10¹⁰ Pa s, and 4x10¹⁰ Pa s, respectively. In the top 100 km layer, besides a weak spreading center derived from a reduced exponent A, a weak zone develops above the slab with a viscosity of about 2x10¹⁰ Pa s (Figures 3a and 3d).

The strain rate is concentrated near the material boundaries (e.g., regions surrounding the slab and beneath the lithosphere) and in the weak zones in the top 100 km layer (Figures 3b and 3c). In the top 100 km layer excluding the weak spreading center, the largest strain rate (~2x10⁻³) occurs directly above the slab (Figure 3e). The deviatoric stress in the lithosphere, slab, and lower mantle near the slab is much larger compared to other regions and is largest within and above the slab (Figure 3e). The principal stresses show a horizontal compression in the overriding plate and a

#### Table 1. Model Parameters

<table>
<thead>
<tr>
<th>Case</th>
<th>Model Description</th>
<th>V_p, cm/yr</th>
<th>P</th>
<th>N_T/Tp</th>
<th>µ lith/µm</th>
<th>µ_\text{F}/\mu_\text{f}</th>
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<tr>
<td>1</td>
<td>2D1 M1 N</td>
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<td>2.8</td>
<td>3.5x10³</td>
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<td>2</td>
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<td>0.96</td>
<td>3.5</td>
<td>3.7x10³</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2D1 M1 Y</td>
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<td>0.96</td>
<td>3.5</td>
<td>4.1x10³</td>
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</tr>
<tr>
<td>4</td>
<td>2D1 M1P Y</td>
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<td>0.96</td>
<td>3.5</td>
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<tr>
<td>5</td>
<td>2D1 M1P Y</td>
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<td>0.96</td>
<td>3.5</td>
<td>2.1x10³</td>
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</tr>
<tr>
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<td>0.74</td>
<td>0.49</td>
<td>2.3x10²</td>
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</tr>
<tr>
<td>7</td>
<td>3D1 M1 Y</td>
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<td>0.87</td>
<td>3.1x10²</td>
<td></td>
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<td>2.8x10²</td>
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<tr>
<td>12</td>
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<td>0.65</td>
<td>2.7x10²</td>
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<tr>
<td>20</td>
<td>3D3 M2 Y</td>
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<td>0.88</td>
<td>0.90</td>
<td>3.5x10²</td>
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</table>

2D1 and 2D2 are for slab dips 60° and 30°, respectively; 3D1, 3D2, and 3D3 are for three different geometries that are shown in Figure 6 and 11. M1-M7 represent different rheology models: M1, A=(2x10⁷, 2x10⁷, 2x10⁷, 2x10⁷) and n=(3, 3, 3, 3); M2, A=(5.6, 1.8x10⁻², 5.6, 1.8x10⁻²) and n=(1, 1, 1, 1); M3, A=(5.6, 5.0x10⁻¹, 9x10⁻³, 1.8x10⁻²) and n=(1, 1, 1, 1); M4, A=(2x10⁷, 5x10⁻¹, 9x10⁻³, 1.8x10⁻²) and n=(3, 3, 1, 1); M5, A=(2x10⁷, 2x10⁷, 2x10⁷, 2x10³) and n=(3, 3, 3, 3); M6, A=(2x10⁷, 2x10⁷, 2x10³, 2x10³) and n=(3, 3, 3, 3); M7, A=(2x10⁸, 2x10⁸, 2x10⁸, 2x10³) and n=(3, 3, 3, 3), where the numbers in parentheses are the preexponents and exponents for the top 100 km layer, slab, spreading center, and upper mantle. M1P is for M1 rheology model with pseudo-plastic rheology for the top 100 km layer. F gives the frictional stress on faults. R_{T/Tp} for a perfectly rigid plate with geometry 3D1, 3D2, and 3D3 are 0.55, 0.55, and 0.63, respectively.
patterns of viscosity (Figure 5a), strain rate (Figure 5b), and stress (Figure 5c). However, we observe significant changes in the velocity (Figures 4a, 5a, and 5d), stress and strain rate within the top 100 km layer (Figures 5c, 5d, and 5e). The subducting plate velocity is now about 2.8 cm yr\(^{-1}\), more than twice the rate of the preceding case with no fault (case 1). The velocity above the slab is now substantially larger in the subducting plate compared to the overriding plate. The subducting plate velocity along the fault is nearly parallel to the tangent of the dipping fault (Figure 5d). Across the fault, the tangential velocity is discontinuous and the relative motion is comparable with convergence velocity between subducting and overriding plates (Figures 4a and 5d). This discontinuous tangential velocity arises from fault decoupling and differs from the preceding case with no fault (case 1) in which velocity everywhere is continuous (Figure 3d).

While horizontal compression persists in the overriding plate, the magnitude of the compressional stress is reduced
3.1.2. Models with a pseudo-plastic rheology. Given the important influence of a dipping fault, it is interesting to examine whether a highly nonlinear stress-weakening rheology alone can reproduce the stress, strain, and flow velocity that are seen in the case with a fault (case 2). We will use two models without faults (cases 4 and 5) and with a pseudo-plastic rheology for the lithosphere to demonstrate that the stress-dependent rheology alone may not reproduce the feature resulting from faults. In the pseudo-plastic rheology used here, the exponent \( n \) is assumed to be 20 when the deviatoric stress is larger than a yield stress, 20 MPa, while \( n \) is 3 for smaller stresses (Figure 1).

Case 4 is identical to the case without a fault (case 1) except that the pseudo-plastic rheology is used for the lithosphere. Compared with a purely \( n=3 \) rheology (case 1), the pseudo-plastic rheology leads to a decrease in both effective viscosity and deviatoric stress but an increase in strain rate in the region above the slab (Figures 6a and 6b for case 4 and Figures 3d and 3e for case 1). This is because the maximum deviatoric stresses in the lithosphere are limited to be around 20 MPa by the pseudo-plastic rheology, which effectively weakens the region above the slab where the stress would be significantly greater than 20 MPa if there was no pseudo-plastic rheology (e.g., case 1). The pseudo-plasticity does not seem to influence the pattern of stress, although the strain rate is more localized and surface velocity is increased (Figures 6b and 4b), compared with case 1 (Figures 3e and 4b).

Compared with the case with a fault and purely \( n=3 \) rheology (case 2), the magnitude of surface velocity for the case with pseudo-plasticity (case 4) is approximately the same (Figure 4b), but the stress and strain rate above the slab are...
different (Figures 6a, 6b, 5d, and 5c); the location of the "plate margin" where the horizontal velocity decreases rapidly is about 100 km closer to the overriding plate (i.e., converging velocity has a steep dip angle) (Figures 4b, 6a, and 5b). For this pseudo-plastic case without a fault (case 4), the location of "margin" as well as the stress and strain rate fields are controlled by the buoyancy of the slab. The location of the effective "margin" moves even further toward the overriding plate when slab dip is reduced from 60° in case 4 to 30° in another model (case 5) which is otherwise identical to the preceding case with pseudo-plasticity (case 4) (Figures 4b, 6c, and 6d). This is because the maximum of the slab-induced deviatoric stress moves further into the overriding plate with smaller slab dips (case 5). Both cases with the pseudo-plasticity (cases 4 and 5) show similar stress and strain rate fields (Figures 6b and 6d), although the flow velocity is greater for case 5 with a smaller dip, primarily because of the longer slab and larger buoyancy (the slabs are assumed to reach the same depth, i.e., 670 km).

The flow velocity, velocity jump across the fault, and stress and strain rate beneath the fault (case 2) reflect influence of faults (Figure 5). It seems difficult for models with only pseudo-plastic or stress-weakening rheology with exponent n=1 (i.e., cases 4 and 5 and other cases with different yield stresses and exponents) to simulate realistically these features, although the pseudo-plastic rheology can produce weak margins to mobilize surface plates (Figures 4b and 6).

3.2. Three Dimensional Models

While 2-D models are a powerful tool to explore the role of dipping faults, they cannot be used to understand the role of transform faults on plate motion, especially the generation of toroidal motion which is inherently three dimensional. In the following models, we will study the influence of faults in three dimensions.

3.2.1. Models with and without faults. For comparison, our first model (case 6) is the 3-D equivalent of case 1 containing no faults and with slab strike parallel to the spreading center (Figure 7a). The length (i.e., in the spreading direction) and width of the region R are 3000 km and 2250 km, respectively. Surface velocities in region R vary gradually in both the spreading parallel and perpendicular directions (Figures 7b, 8a, and 8b), indicating a large internal strain rate. The average velocity in region R, V_p, is about 0.98 cm yr⁻¹ and is perpendicular to the spreading center (Figure 7b). Although there is a sudden jump in thermal structure from R to the surrounding region along the ridge direction, surface flow is predominantly ridge-perpendicular (Figure 7b). The lack in ridge-parallel surface flow results from the following reasons. The main driving force in this case is the slab, and the buoyancy associated with the jump in thermal structure is secondary because of its short-wavelength and shallow depth. The slab which is parallel to the ridge always tends to excite flow perpendicular to its strike. This effect is further enhanced by the lateral variations in viscosity associated with the slab and ridge which are served as a stress guide. The ratio of toroidal to poloidal components of the surface velocity, RT/FP, is 0.27, compared to 0.55 for a perfectly rigid plate with this configuration (i.e., N/P=0.49), and plateness P is 0.74 (Table 1). The effective viscosity of both the diverging and converging "margins" are reduced. The effective viscosity in other surface regions varies gradually and reflects the variations in strain rate (Figure 7b).

The average viscosity in R and in the upper mantle beneath R are about 10⁶ Pa s and 5×10⁶ Pa s, respectively. Topography in the region R increases with age and distance from the spreading center, as expected, but over the slab there is a broad depression (~500 km in width) (Figure 7c), similar to that from the 2-D calculation (Figures 4c and 4d for case 1).

We now contrast this model (case 6) with a model (case 7) that includes faults with zero frictional stress but is otherwise identical to case 6. The strike of the dipping fault is parallel to the spreading center but perpendicular to the strike of vertical faults (Figure 7d), and the dip angle is constant. We find that surface velocity in region R bounded by faults are nearly constant (Figures 7e, 8a and 8b) and parallel to the strike of vertical faults. There are sharp velocity contrasts across the faults (Figures 8a and 8b). V_p is about 3.3 cm yr⁻¹, close to the velocity with which the buoyancy was derived. RT'FP and P are 0.41 (N/P=0.87) and 0.91, respectively (Table 1). The gross topography and viscosity are similar to those from the case with no faults (case 6) (Figures 7b, 7c, 7e, and 7f). Like the 2-D model, there is a prominent outer rise (~200 km in width and 0.5 km in amplitude) and a distinct trench (~100 km wide and 4 km deep) near the dipping fault (Figure 7f). In comparison to the fluid near the converging margin, the transform faults are relatively strong (Figure 7f). The average viscosity in R and in the upper mantle beneath R are about 10⁶ Pa s and 3.6×10⁶ Pa s, respectively. Compared with case 6 without faults, the case with faults (case 7) has a larger
3.2.2. Influence of viscosity structure and fault strength. The plate-like surface velocity from case 7 (Figure 7e) (we will refer case 7 as the standard case hereafter) results from a non-linear interaction between weak faults and the stress weakening rheology and weak coupling between plate and asthenosphere. In this section, we will use a series of models with different frictional fault forces and different viscosity structures to demonstrate that weak faults, stress weakening rheology, and weak coupling between plate and asthenosphere are all necessary in order to achieve the plate-like surface motion.

We first show that stress weakening rheology plays an essential role in generating plate-like motion. Case 8 uses a layered Newtonian viscosity structure (i.e., \( n=1 \)) without lateral variations in viscosity, but it includes the same buoyancy force and faults as in our standard case (i.e., case 7). The viscosities for the top 100 km layer, the upper mantle (i.e., 100 km to 670 km depth), and the lower mantle are the same as the average effective viscosities for each corresponding layer in the standard case (Table 1). For this case with Newtonian viscosity, surface velocities are greatly reduced in amplitude, and internal strain rate increases such that both plateness and \( N_{i/P} \) are greatly reduced (\( P=0.49 \) and \( N_{i/P}=0.16 \)) (Figure 9a and Table 1). This indicates that faults alone do not result in the plate-like surface velocity. Augmenting this case with some lateral variations in viscosity, we further include a weak spreading center and strong slab in another Newtonian calculation (case 9 and Table 1). For consistency, the spreading center and slab viscosities are the same as the average effective viscosities for each corresponding regions in the standard case. Compared with the standard case, this Newtonian case with some lateral variations in viscosity (case 9), yields a similar plateness but significantly less toroidal component (\( P=0.82 \) and \( N_{i/P}=0.36 \) in Table 1) (Figure 9b for surface velocity). Based on the preceding case (i.e., case 9), we now change the rheology for the top 100 km layer into the stress weakening rheology used in the standard case, but with other regions remaining Newtonian (case 10 in Table 1). Surface velocities with stress weakening rheology only in the lithosphere (case 10) show similar plate-like features as those in the standard case, with a plateness of 0.89 and \( N_{i/P} \) of 0.89 (Figure 9c), indicating that the stress weakening rheology for lithosphere is the essential component for producing plate-like features.

We now demonstrate the importance of weak faults. The standard case with faults (case 7) and our original 3-D model...
without faults (case 6) represent two extremes. With no frictional force across the faults, flow is fully decoupled; at the other extreme, where no faults are included, flow is fully coupled. Calculations have been done for cases (cases 11, 12, and 13 in Table 1) with the same fault and buoyancy distribution as for the standard case but with different frictional forces on faults. As the frictional force increases from 0 to 20 MPa, the surface velocity, $N_{T/P}$ and $P$ decrease and progressively resemble those from the case with no faults (Figure 8). These calculations indicate that plateness and toroidal velocity component are sensitive to the frictional force on faults and that plate-like surface motion only arises when there is a low frictional force on faults.

Finally, we show the influence of coupling between lithosphere and asthenosphere on surface motion. We compare our standard case (case 7) with those cases (cases 14, 15, and 16) with different pre-exponents $A$ for the top 100 km layer and the upper mantle to determine the influence of viscosity contrast between plate and the upper mantle (Table 1). These three cases are identical to the standard case (case 7) except for the pre-exponents $A$. In cases 14 and 15, the pre-exponents are increased only for the upper mantle and remain the same elsewhere, as in the standard case. For these two cases, the increased pre-exponents for the upper mantle increase the upper mantle viscosity and reduce the viscosity contrast (i.e., increase the coupling) between plate and upper mantle, compared with the standard case (Figure 10a and Table 1). The increased coupling between plate and upper mantle reduces the surface velocity in $\Re$ and plateness (Figures 10b and 10c). In case 16, while pre-exponent for the upper mantle is the same as in the standard case, pre-exponents for the top 100 km and slab are increased. Therefore, both the viscosity within plate and the viscosity contrast between plate and the upper mantle are increased (Figure 10a and Table 1). The surface velocity from this case is reduced and shows less variation in $\Re$ (i.e., a larger plateness), compared with the standard case (Figures 10b and 10c), and this is due to the increased lithospheric viscosity. These three cases (cases 14, 15, and 16) along with the standard case (case 7) show that in order to achieve a large plateness the viscosity contrast between lithosphere and the upper mantle should be large (Figure 10d). However, $N_{T/P}$ does not vary significantly with the viscosity contrast (Figure 10d).

3.2.3. Influence of fault geometry. Plates usually have much more complicated geometry in comparison to our standard case. For the Pacific plate, the subduction zones are approximately parallel to the strike of plate motion, the strike of Aluuentian subduction zone is nearly parallel to plate motion, the strike of the Aluuentian subduction zone and the slab are increased (Figure 10a and Table 1). While one side of region $\Re$ is 3562 km long, the other side is 1125 km shorter (Figure 11a). The width of region $\Re$ is 2250 km. For this case with no faults, velocities in $\Re$ vary gradually in both magnitude and direction, and velocity becomes nearly perpendicular to the strike of the slab near the slab (Figure 11b). Plateness is 0.74 and $N_{T/P}$ = 0.51 ($R_{l/p} = 0.28$; $R_{l/p} = 0.55$ for a perfect plate of this geometry). The average velocity in $\Re$ is 0.91 cm yr$^{-1}$. When faults are included in a model (case 18) (Figure 11a) which is otherwise identical to the preceding case (case 17), surface velocities in $\Re$ are nearly parallel to the vertical faults and are approximately uniform (Fig. 11c), showing plate-like

![Figure 10](image-url)
behavior with $p=0.89$ and $f_T/f_p=0.87$ with $V_p=2.9$ cm yr$^{-1}$ (Table 1). Oblique subduction is indicated by the surface velocities near the dipping fault (Figure 11c).

In order to further understand the influence of faults on the oblique subduction, we now show two additional calculations of another configuration of slab and spreading center (Figure 11d for cases 19 and 20). In these two cases, the strike of the slab is parallel to that of the spreading center, but the transform faults are oblique to both the slab and spreading center (Figure 11d). This configuration of faults and slab is not observed, but these two cases closely show the physics of fault interaction. Again, for case 19 with no faults, surface velocity displays a large internal strain rate and does not show oblique subduction (i.e., the velocities are perpendicular to the strike of slab) (Figure 11e). However, when faults are included (case 20), plate-like features including oblique subduction and uniform surface velocities emerge (Figure 11f).

### 3.3. Discussions

Studies of earthquakes show that most strain in subduction zones is accomplished through thrust faulting [Davies and Brune, 1971; Kanamori, 1977; Ruff and Kanamori, 1983], while some strain is released through normal faulting within subducting lithosphere [Kitamura and Kanamori, 1995]. Our 2-D model with a preexisting dipping fault and a stress-weakening rheology with $n=3$ (case 2) is consistent with these observed features of strain and stress. The relative motion across the dipping fault is about the same as the converging velocity between the subducting and overriding plates (Figure 5a), consistent with observations [Davies and Brune, 1971; Ruff and Kanamori, 1983]. In the subducting plate and below the dipping fault, there is significant strain and the principal stress is perpendicular to the fault (Figure 5e), consistent with observations of normal fault earthquakes [Kitamura and Kanamori, 1995]. Our models of dynamic compensation of the slabs beneath a dipping fault are consistent with outer rise and trench topography (Figure 4c) [Zhong and Gurnis, 1994b]. The excess depression in the back arc region evident in the model (Figure 4d) indicates that other processes, perhaps volcanism, may have significant influence on dynamic topography.

Subduction zone structure is difficult to be reproduced in models with a purely instantaneous stress-weakening rheology without faults (cases 1, 4 and 5). However, lithospheric weakening above the slab may give rise to realistic plate velocities, especially if a pseudo-plastic rheology is used (Figure 4b). Without a fault, the maximum deviatoric stress (and strain rate) in the lithosphere always occurs in the overriding plate and directly above the slab, independent of the exponent $n$ and yield stress (Figures 3e, 6b, and 6d). This is because the stress is primarily determined by the slab buoyancy. With stress-weakening rheology, either $n=3$ or our pseudo-plastic rheology, the maximum stress will weaken the region directly above the slab and produce a weak margin (Figures 3 and 6). The surface location of this margin depends on the geometry of slab and moves further towards the overriding plate for a shallower slab dip (Figure 6). This is inconsistent with the observation that converging margins tend to move more towards oceanic plates with respect to the location of slabs when the subduction dip becomes smaller [Jarrard, 1986]. Moreover, the stress and strain rate fields from these models (Figures 3 and 6 for cases 1, 4 and 5) are very different from the model with a fault (Figure 5 for case 2) and cannot explain observed features. These models suggest that the stress weakening rheology may not be the primary cause for the observed stress and strain patterns in subduction zones.

The 3-D models show that the interaction between a power-law rheology and weak faults produces the basic features of plate tectonics including small internal strain and significant toroidal motion. The models suggest that faults are probably weak, consistent with other inferences [Lachenbruch and Sass, 1988; Bird, 1978]. Weak faults alone may not be sufficient to produce plate-like surface motion and the stress-weakening rheology clearly plays a fundamental role, but the nonlinearity need not be strong. A weak coupling between the lithosphere and the upper mantle is also required to produce plate-like surface motion, suggesting that the upper mantle viscosity should be much smaller than the lithosphere. This seems to be compatible with the viscosity structure constrained by the Earth's geoid [Hager and Richards, 1989; Hager and Clayton, 1989]. However, some other inversion results from the geoid suggest a relatively strong upper mantle (from a depth of 100 km to 410 km) [e.g., King and Masters, 1992]. Increasing fault strength (i.e., the horizontal coupling between plates) reduces the ratio of toroidal to poloidal motion and increases.
the internal strain, while the viscosity contrast between the lithosphere and upper mantle (i.e., the vertical coupling) only influences the distribution of strain. This suggests that both the strain pattern and ratio of toroidal to poloidal motion are important in characterizing plate motion [Bercovici, 1995; Zhong and Gurnis, 1996].

Faults strongly influence the direction of plate motion. A transform fault tends to enhance plate motion along its strike; a dipping fault decouples subducting from overriding plates. Therefore the incorporation of these faults results in oblique subduction when the strike of transform faults is nonperpendicular to the strike of subducted slabs (Figures 8 and 9). The influences of transform faults on plate motion revealed from our models are consistent with some inference based on plate kinematics [Richards and Engelsbreton, 1994].

As we have shown, the interaction between preexisting weak faults and power law rheology produces the observed pattern of plate motion in instantaneous mantle flow models. Essentially, our models indicate the importance of the heterogeneity in lithospheric strength (i.e., weak faults versus surrounding media) to the observed plate motion. On the basis of the observations of surface stress and strain and our numerical models, we suggest that the instantaneous stress-weakening rheology may not be the primary cause for producing such heterogeneity in lithospheric strength. We think that this heterogeneity in lithospheric strength is likely an integrated effect of lithospheric deformation over geologic time, and lithospheric deformation history, instantaneous stress-weakening, and lithology may all contribute to the development of this heterogeneity. Initial attempts have been made in incorporating deformation history [Sleep, 1997] and self-lubrication mechanism based on void generation and volatile ingestion [Bercovici, 1998] into lithospheric rheology models, but it remains a challenging question as to how different processes contribute to the development of this heterogeneity.

4. Conclusions

On the basis of observations and numerical models, we argue that fault decoupling at plate margins primarily results from history-dependent lithospheric deformation rather than from instantaneous stress-weakening rheology. We suggest that faults should be treated as preexisting mechanical structure for instantaneous mantle flow models. With models incorporating preexisting faults, a power-law rheology (n=3), and realistic buoyancy forces including slab pull and ridge push forces, we have demonstrated that nonlinear interaction between weak faults and a power-law rheology explains a number of key features of plate motion, such as concentrated strain within plate margins and a significant toroidal component.

Our models show that weak transform faults enhance surface motion in the direction parallel to the strikes of transform faults. That is, transform faults appear to guide plate motion. This guiding influence and the decoupling of thrust faults may result in oblique subduction where the strike of subducted slabs is oblique to transform fault. Our subduction zone models with a dipping fault also produce short-wavelength features including oceanic trench and forebulge topography and principal stresses that are largely consistent with subduction zone observations.

Appendix A: Incorporation of Fault Planes Into 3D Finite Element Models

Constrained elements and matrix transformation are used to incorporate faults into 3D finite element models. These techniques are similar to those by Melosh and Williams [1989] and Barr and Housenman [1996] and are an extension of our previous penalty function method [Zhong and Gurnis, 1994b]. The general treatment for such problems is given by Cook [1981] and will be briefly described as follows.

Fault planes consist of elemental boundaries (Figure A1a). Those elements with boundaries which overlap a fault are called constrained elements. The constraints on faults are that velocities are continuous in the normal direction \( \vec{n} = (n_x, n_y, n_z) \), while tangential velocities are coupled with a specified frictional stress, where \( \vec{n} \) is defined in a global coordinate system \( x, y, \) and \( z \) (Figure A1a). We define two

![Figure A1](image-url)

**Figure A1.** (a) Three dimensional fault with constrained elements in both global \( (x, y, z) \) and local \( (n, t, s) \) coordinate systems. (b) CPU time and efficiency versus the number of processors for the parallelized 3D finite element software Cicon on the Intel Paragon with 512 processors. For the benchmark in Figure A1b, the CPU times are only for solution of inner loop iteration of velocity and mesh size is 160x64x80.
tangential directional vectors \( \hat{t} = (t_x, t_y, t_z) \) and \( \hat{s} = (s_x, s_y, s_z) \) such that \( \hat{t}, \hat{s}, \) and \( \hat{F} \) form an orthogonal coordinate system (i.e., local coordinate system) (Figure A1a). We may define such a local coordinate system for each faulted node within constrained elements.

For each constrained element, elemental matrix equations in the global coordinate system are

\[
KU + G^T P = F, \quad (A1)
\]

\[
GU = 0, \quad (A2)
\]

where \( K \) is a stiffness matrix with 24x24 entries (this is for a 3-D trilinear element with eight nodes); \( F \) and \( G^T \) are force and divergence vectors; \( P \) is the pressure which is constant within the element; \( U' = (u_x, u_y, u_z, u_{x'}, u_{y'}, u_{z'}, u_{x''}, u_{y''}, u_{z''}) \) where \( i \) is the local node index. If local node \( i \) is a faulted node, we will use velocities in the local coordinate system for this node, and this can be done through the following transformation: \( \hat{U}' = \Gamma_1 U, \) where \( \hat{U}' = (\hat{u}_x, \hat{u}_y, \hat{u}_z, \hat{u}_{x'}, \hat{u}_{y'}, \hat{u}_{z'}, \hat{u}_{x''}, \hat{u}_{y''}, \hat{u}_{z''}) \) is the new velocity vector; \( \Gamma_1 \) is a transformational matrix with 24x24 entries with only nonzero entries in eight 3x3 submatrices on the diagonal of \( \Gamma_1. \) Seven of the eight 3x3 submatrices are unit matrices, and the only nonunit submatrix \( \Gamma_1 \) is associated with node \( i. \) \( \Gamma_1 \) can be written as

\[
\Gamma_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

Applying the transformation to equations (A1) and (A2) leads to

\[
\Gamma_1 K \Gamma_1^T U' + \Gamma_1 G^T P = \Gamma_1 F, \quad (A4)
\]

\[
\Gamma_1 G \Gamma_1^T U' = 0. \quad (A5)
\]

If we define \( K' = \Gamma_1 K \Gamma_1^T, \) which remains symmetric and positive definite, \( G' = \Gamma_1 G, \) and \( F' = \Gamma_1 F, \) then the form of (A4) and (A5) are identical to (A1) and (A2). If there are other faulted nodes within this element, we consecutively apply this transformation to these nodes such that velocities associated with each faulted node in \( U' \) vector are based on the local coordinate system for that node.

Fault constraints are enforced when elemental matrix equations are assembled into global matrix equations. For each faulted node in the global matrix equations, we assign five degrees of freedom: one normal velocity, \( u_i, \) and four tangential velocities, \( u_{i'}, u_{i''}, u_{i'''}, \) and \( u_{i'''} , \) with the first two for the left side of the fault and the last two for the right (Figure A1a). The parameters \( u_{i'}, u_{i''}, \) and \( u_{i'''}, \) and \( u_{i'''} \) are used for the constrained elements on the left and right sides of the fault, respectively. After the global equations are solved, we obtain the velocity field in which the velocities on faulted nodes are based on the local coordinate system for each faulted node. We can apply the transformation \( U' = \Gamma_1^T U \) for each faulted node to retrieve the velocities in the global coordinate system.

Appendix B: Parallelization of 3D Finite Element Code Citcom With MPI

Parallel computing has been previously used in modeling mantle dynamics [Gurnis et al., 1988; Tackley, 1994; Bunge and Baumgardner, 1995]. Tackley [1994] used a message passing software native to a massively parallel processing supercomputer Intel Delta for a finite volume code and a spectral convection code [Glazmaier, 1988], while Bunjee and Baumgardner [1995] parallelized a finite element code [Baumgardner, 1985] with a message passing software PVM. In the last few years, as a result of steady improvement of both interprocessor communication speed and message passing softwares, parallel computing has become very effective.

We have applied Message Passing Interface software MPI [Snir et al., 1996] to parallelize the finite element code Citcom [Moresi and Solomatov, 1995; Moresi and Gurnis, 1996]. Since MPI is widely supported on many different parallel computers, including shared and distributed memory machines, the parallelized Citcom can be easily ported onto different parallel computers. The two-level Uzawa algorithm implemented in Citcom displays a significant locality and is suitable for parallelization. Computations of stiffness matrix, force vectors, and divergence, gradient, and Laplacian operators are performed at an element level and do not involve interprocessor communication. Communications are needed in computing global residue terms and assembling force vectors. Communications only involve information on boundaries of each computational domain. The parallelized Citcom achieves better than 80% efficiency on the massively parallel processing supercomputer Intel Paragon for a moderate size problem (Figure A1b).

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