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Experimental demonstration of a unidirectional reflectionless parity-time metamaterial at optical frequencies

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Derivation of Attenuation and Coupling Coefficients

To obtain analytical solutions, we average the introduced modulation $\Delta\varepsilon = \cos(qz) - i\delta\sin(qz)$ ($4n\pi/q + \pi/q \le z \le 4n\pi/q + 2\pi/q$) in the entire period as

$$\Delta\varepsilon = C_q \exp(iqz) + C_{-q}\exp(-iqz) + C_0,$$

where $C_0 = \frac{q}{4\pi}\int_{\frac{\pi}{q}}^{\frac{2\pi}{q}} (\cos(qz) - i\delta\sin(qz))dz = i\frac{\delta}{2\pi}$, $C_q = \frac{q}{4\pi}\int_{\frac{\pi}{q}}^{\frac{2\pi}{q}} (\cos(qz) - i\delta\sin(qz))\exp(-iqz)dz = \frac{1-\delta}{8}$, and

$C_{-q} = \frac{q}{4\pi}\int_{\frac{\pi}{q}}^{\frac{2\pi}{q}} (\cos(qz) - i\delta\sin(qz))\exp(iqz)dz = \frac{1+\delta}{8}$. In this form of modulation, the coupled mode equations can be easily derived as:

$$\begin{cases} \frac{dA(z)}{dz} = iC_0\alpha A(z) + iC_q\kappa B(z) \\ \frac{dB(z)}{dz} = -iC_0\alpha B(z) - iC_{-q}\kappa A(z) \end{cases}.$$

At the exceptional point where $\delta = 1$, the coupled mode equations can be written as

$$\begin{cases} \frac{dA(z)}{dz} = -\frac{\alpha}{2\pi}A(z) \\ \frac{dB(z)}{dz} = -i\frac{\kappa}{4}A(z) + \frac{\alpha}{2\pi}B(z) \end{cases}.$$

Therefore the corresponding transmission and reflection coefficients are $T = \exp\left(-\frac{\alpha L}{\pi}\right)$, $R_f = \frac{\pi^2\kappa^2}{4\alpha^2}\sinh^2\left(\frac{\alpha L}{2\pi}\right)\exp\left(-\frac{\alpha L}{\pi}\right)$, and $R_b = 0$. We also numerically calculated transmission and reflection in the forward direction with different modulation lengths of the $\mathcal{PT}$ metamaterial using FDTD simulations as shown in Fig. S1. With derived analytical formulas for transmission and reflection, the attenuation coefficient $\alpha = 0.61\ \mu\mathrm{m}^{-1}$ is determined by fitting the transmission curve and then the coupling coefficient $\kappa = 0.49\ \mu\mathrm{m}^{-1}$ is obtained by fitting the reflection curve with the best fitted attenuation coefficient.

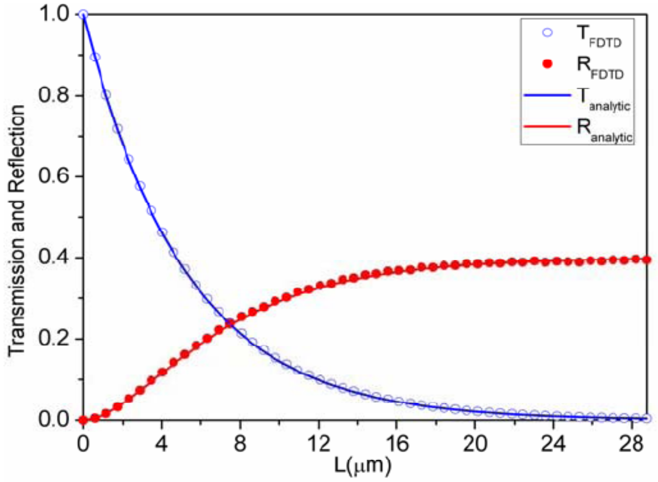

Figure S1. Numerically calculated transmission (blue dot) and reflection (red dot) spectra with different modulation lengths of $\mathcal{PT}$ metamaterial at its exceptional point. Blue and red curves denote the corresponding fits with the derived analytical formulas.

Comparison of Proposed Passive and Typical Balanced Gain/Loss $\mathcal{PT}$ Systems

Although the proposed $\mathcal{PT}$ system is completely passive and there is no gain to compensate the introduced loss, the obtained $\mathcal{PT}$ characteristic and its phase evolution as a function of δ are very similar to the typical balanced $\mathcal{PT}$ systems, which can be seen from the corresponding eigenvalues of both systems. The S-matrix of the typical balanced gain/loss $\mathcal{PT}$ system can be written as

$$S = \begin{pmatrix} \dfrac{1}{\cosh(\gamma_0 L)} & \dfrac{i\dfrac{1-\delta}{\sqrt{1-\delta^2}}\sinh(\gamma_0 L)}{\cosh(\gamma_0 L)} \\ \dfrac{i\dfrac{1+\delta}{\sqrt{1-\delta^2}}\sinh(\gamma_0 L)}{\cosh(\gamma_0 L)} & \dfrac{1}{\cosh(\gamma_0 L)} \end{pmatrix},$$

where $\gamma_0 = \dfrac{\kappa}{8}\sqrt{1-\delta^2}$. Its eigenvalues as well as their comparisons to those in the passive system can be found in Table S1.

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Table S1. Comparison of eigenvalues in passive and balanced $\mathcal{PT}$ systems.

	Passive $\mathcal{PT}$ systems	Balanced $\mathcal{PT}$ systems
$0 \le \delta < 1$	$s_n' = \dfrac{1 \pm i\dfrac{\kappa}{8\gamma}\sinh(\gamma L)\sqrt{1-\delta^2}}{\sqrt{1+\dfrac{1-\delta^2}{64}\dfrac{\kappa^2}{\gamma^2}\sinh^2(\gamma L)}}$ is unimodular.	$s_n = \dfrac{1 \pm i\sinh(\gamma_0 L)}{\cosh(\gamma_0 L)}$ is unimodular (exact $\mathcal{PT}$ phase).
$\delta = 1$	$s_n' = 1$ is degenerate (exceptional point).	$s_n = 1$ is degenerate (exceptional point).
$\delta > 1$	$s_n' = \dfrac{1 \pm \dfrac{\kappa}{8\gamma}\sinh(\gamma L)\sqrt{\delta^2-1}}{\sqrt{1+\dfrac{1-\delta^2}{64}\dfrac{\kappa^2}{\gamma^2}\sinh^2(\gamma L)}}$ is non-unimodular.	$s_n = \dfrac{1 \pm \sin(\dfrac{\kappa}{8}\sqrt{\delta^2-1}L)}{\cos(\dfrac{\kappa}{8}\sqrt{\delta^2-1}L)}$ is non-unimodular (broken $\mathcal{PT}$ phase).

It is evident that $\mathcal{PT}$ symmetry of the studied passive system is similar to the $\mathcal{PT}$ symmetric phase in the balanced gain/loss system but with an additional attenuation term $|a|$. Therefore these two systems share similar underlying physics and $\mathcal{PT}$ phase evolution as a function of δ. More intuitively, similarly to unidirectional invisibility in the typical balanced $\mathcal{PT}$ system, the achieved unidirectional reflectionless effect at the exceptional point in the passive system is also due to multiple reflections of guided light at different units and their phase cancellation.

Broadband Response of $\mathcal{PT}$ Characteristics

To study the broadband response of our $\mathcal{PT}$ metamaterial, frequency detuning is considered by adding the phase mismatch term into the coupled-mode equations:

$$\begin{cases} \dfrac{dA(z)}{dz} = -\dfrac{\delta}{2\pi}\alpha A(z) + i\dfrac{1-\delta}{8}\kappa B(z)e^{i2\Delta kz} \\ \dfrac{dB(z)}{dz} = \dfrac{\delta}{2\pi}\alpha B(z) - i\dfrac{1+\delta}{8}\kappa A(z)e^{-i2\Delta kz} \end{cases},$$

where $\Delta k = k_1 - k$ is the phase mismatch due to the frequency detuning. The transfer matrix M relates the complex amplitudes at two ends of $z = 0$ and $z = L$ as follows:

$$\begin{pmatrix} A(L) \\ B(L) \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A(0) \\ B(0) \end{pmatrix},$$

where $M_{11} = \left(\cosh(\gamma L) - (\frac{\delta}{2\pi}\alpha + i\Delta k)\sinh(\gamma L)\big/\gamma\right)e^{i\Delta kL}$, $M_{12} = i\frac{1-\delta}{8}\frac{\kappa}{\gamma}\sinh(\gamma L)e^{i\Delta kL}$, $M_{21} = -i\frac{1+\delta}{8}\frac{\kappa}{\gamma}\sinh(\gamma L)e^{i\Delta kL}$, $M_{22} = \left(\cosh(\gamma L) + (\frac{\delta}{2\pi}\alpha + i\Delta k)\sinh(\gamma L)\big/\gamma\right)e^{i\Delta kL}$. Therefore, it is apparent that at the exceptional point where $\delta = 1$ backward reflection $R_b = \left|\frac{M_{12}}{M_{22}}\right|^2$ is always 0 for any given Δk. Although the transmission and forward reflection coefficients change as a function of Δk with the frequency detuning, the corresponding contrast ratio is 1 in a broad band of frequencies, which is numerically confirmed as shown in Fig. S2 in next section.

Evolution of $\mathcal{PT}$ Metamaterial

In the design, the studied $\mathcal{PT}$ metamaterial is set to have 25 periods (each period is $4\pi/q = 575.5$ nm). For the $\mathcal{PT}$ metamaterial with the original optical modulations at the exceptional point $\Delta\varepsilon = \cos(qz) - i\sin(qz)$ (see Fig. 1a), the reflection spectra are numerically calculated using FDTD simulations as shown in Fig. S2a. The resonance peak in the forward direction is located around 1550 nm, while reflection in the backward direction is almost 0 within the entire studied range of wavelengths from 1520 nm to 1580 nm. It clearly shows the expected

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unidirectional reflectionless phenomenon corresponding to the exceptional point as the calculated contrast ratio is always close to 1 (Fig. S2b).

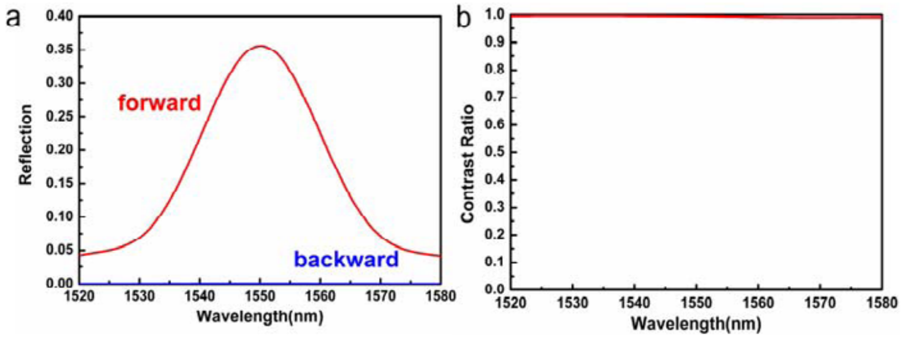

Figure S2. Simulated reflection spectra of the $\mathcal{PT}$ metamaterial in both directions (**a**) and the corresponding contrast ratio (**b**) from 1520 nm to 1580 nm.

Next, as stated in main text, to design a practical device, regions of $\Delta\varepsilon_{real}$ are extracted from the original $\mathcal{PT}$ optical potentials and then the corresponding cosine modulation in the real part is shifted $5\pi/2q$ in the z direction to become a positive sinusoidal modulation ($\Delta\varepsilon_{real} = \cos(qz) = \left|\sin(qz - 5\pi/2)\right|$), while the imaginary part modulation remains at the same location ($\Delta\varepsilon_{imag} = -i\sin(qz) = i\left|\sin(qz)\right|$) (Fig. S3a). Owing to the in-phase shift of the real part modulation, the modulated phase and amplitude of guided light accumulated from both real and imaginary part modulations remain the same after light propagates through an entire unit cell. Therefore, these in-phase arranged modulations create an equivalent unidirectional optical modulation similar to the original $\mathcal{PT}$ optical potentials as shown in the numerically calculated reflection spectra for both directions (Fig. S3b). Unlike the balanced modulation in the real part in the case of Fig. S2a, the only-positive real part modulation here results in a red-shift of the resonance peak to the wavelength around 1560 nm. However the corresponding contrast ratio close to 1 still manifests the expected unidirectional optical property over the studied band of wavelengths from 1520 nm to 1580 nm (Fig. S3c). This spontaneous $\mathcal{PT}$ symmetry breaking at the exceptional point is also visualized from mappings of light propagating inside the waveguide at the wavelength of 1550 nm: forward propagating light and its reflection forms strong interference, whereas reflection in the backward direction is close to 0 (Fig. S3d).

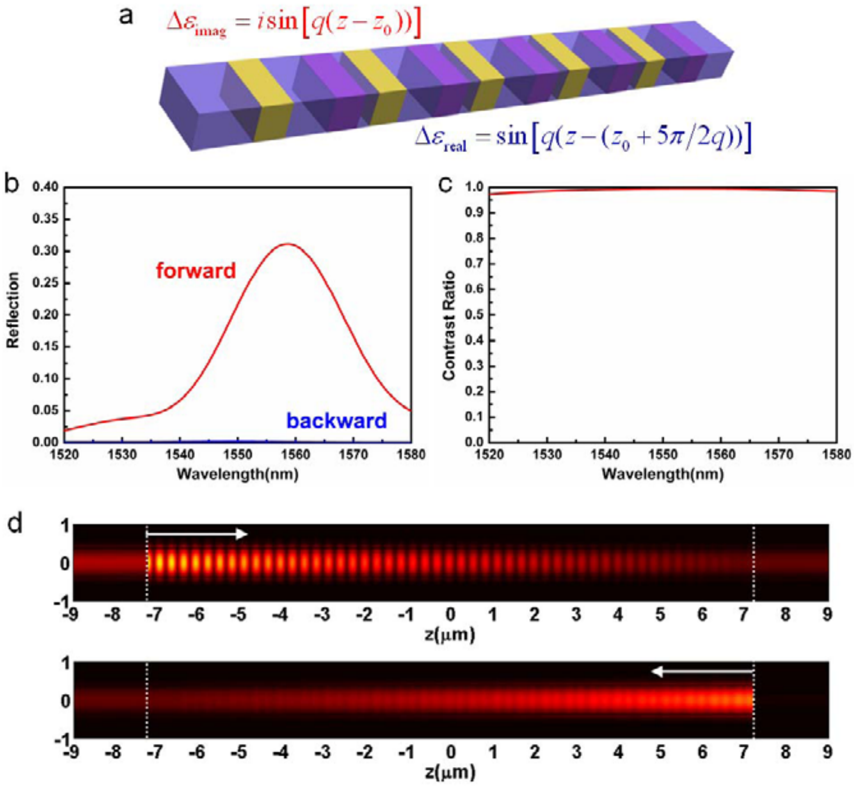

Figure S3. **a**, designed equivalent $\mathcal{PT}$ metamaterial with separate real and imaginary part optical modulations. **b** and **c** show simulated reflection spectra in both directions and the corresponding contrast ratio from 1520 nm to 1580 nm, respectively. **d**, simulated electric field amplitude distribution of light in a.

It is therefore concluded that the $\mathcal{PT}$ metamaterial with in-phase arranged individual real and imaginary part optical modulations can demonstrate the unidirectional optical properties owing to the exceptional point as well. Additional structures on top of the waveguide are then designed accordingly to mimic its associated quantum phenomenon as stated in the main text.

Design of On-Chip Waveguide Coupler

The 3D finite difference propagation method simulation (R-Soft Design Group, Inc.) has been implemented to design the 3 dB waveguide directional coupler that consists of two 400 nm-wide

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and 220-nm thick Si waveguides with a gap in between. Each single waveguide is a single mode waveguide. If the gap distance between two waveguides is short enough, guided light can be coupled between each other. As shown in Fig. S4, light is first launched inside the left waveguide and then coupled into the other that is 414 nm away due to evanescence coupling. The field intensity gets stronger in the right waveguide as light propagates in the z direction. After an interaction length of 40 μm, light intensities in two waveguides become the same (right panel of Fig. 3S) and a 3dB waveguide coupler is thus achieved. Therefore, the final waveguide coupler is composed of two 40 μm-long, 400 nm-wide and 220 nm-thick Si waveguides with a gap of 414 nm in between.

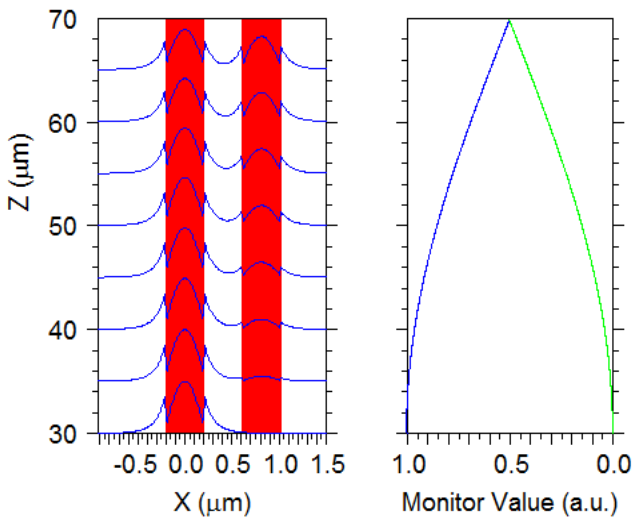

Figure S4. Simulated light coupling in the designed waveguide coupler, showing evolution of electric field envelopes (left) and intensities (right) in two waveguides as light propagates in the z direction.

Conversion from Unidirectional Reflection to Unidirectional Invisibility

Although the proposed $\mathcal{PT}$ metamaterial is completely passive and can only demonstrate unidirectional reflectionless properties at its exceptional point, unidirectional unviability can be simply achieved based on the presented structure by adding a suitable standard waveguide optical amplifier. Assuming a linear optical amplifier whose length is the same as the modulation length of our passive $\mathcal{PT}$ metamaterial, its corresponding transfer matrix is

$$\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} = \begin{pmatrix} \exp(gL) & 0 \\ 0 & \exp(-gL) \end{pmatrix},$$

where g is the corresponding gain coefficient of the added amplification waveguide. The transfer matrix of the combined system at the exceptional point is thus expressed as

$$\begin{pmatrix} M'_{11} & M'_{12} \\ M'_{21} & M'_{22} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} = \begin{pmatrix} \exp\left(\left(g - \frac{\alpha}{2\pi}\right)L\right) & 0 \\ -i\frac{\pi\kappa}{2\alpha}\sinh\left(\frac{\alpha}{2\pi}L\right) & \exp\left(\left(\frac{\alpha}{2\pi} - g\right)L\right) \end{pmatrix}.$$

If g is chosen to be $\alpha/2\pi$, this transfer matrix becomes $\begin{pmatrix} 1 & 0 \\ -i\frac{\pi\kappa}{2\alpha}\sinh\left(\frac{\alpha}{2\pi}L\right) & 1 \end{pmatrix}$. Transmission becomes unity in both directions, but reflection can only be observed in one direction, meaning that the device become unidirectional invisible in light intensities. Additional phase due to the added amplifier may be accumulated in transmitted light. Therefore, to get unidirectional invisibility in both amplitude and phase, the amplifier has to be designed to support phase accumulation of multiples of 2π. In practice, optical amplification waveguide can be implemented using III/V semiconductor materials. To completely diminish reflection due to index mismatch between III/V and Si waveguides, an adiabatically changed tapered waveguide might be necessary to gradually convert the guided mode. However, it is in principle feasible to achieve unidirectional invisibility by applying uniform linear optical gain on our passive structure, which provides a much easier approach compared to the typical balanced gain/loss $\mathcal{PT}$ system.