Thermal Noise in Double Injection

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Noise measurements from 500 kHz to 22 MHz and at ambient temperatures \( T \) between 140 and 350\(^\circ\)K have been performed on a double-injection silicon diode as a function of operating point. The results indicate that at high frequencies, (i) the noise increases linearly with \( T \), and (ii) the noise also depends linearly on the differential conductance \( g \) at the same frequency. Within at most a 5\% error, the high-frequency noise is quantitatively represented by Nyquist's formula \( \langle P \rangle = 2kTg\Delta f \) throughout the experimental range. This proves the thermal nature of the high-frequency noise of double injection. Possible limits on this result and its comparison with alternative theories are discussed.

I. INTRODUCTION

A recent investigation has established that in a silicon double-injection diode operated at room temperature (298\(^\circ\)K) in the semiconductor regime \( (I \propto V^2) \) the noise \( \langle P \rangle \) is quantitatively represented by

\[
\langle P(\omega) \rangle = 4kTg(\omega)\Delta f
\]

for \( \omega > \tau^{-1} \), where \( \tau \) is the common high-level lifetime of the charge carriers, \( k \) is Boltzmann's constant, \( T \) is the absolute temperature, and \( g \) is the differential conductance of the device.\(^1\) Recombination mechanisms cause the nonlinearity of the device.\(^2\) At high frequencies \( \omega > \tau^{-1} \), however, recombination is ineffective. Electrons and holes each carry current independently. If the charge carriers remain in thermal equilibrium with the lattice in spite of a net current, the system then constitutes a resistance even for large signals. Hence, noise at high frequencies should be thermal.

A direct test of this model is to establish the proportionality of the high-frequency noise \( \langle P(\omega) \rangle \) with the absolute temperature \( T \) of the diode. The results of such an experiment are reported here.

II. DEVICE AND METHOD OF MEASUREMENT

The double-injection diode analyzed here is identical to that of Ref. 1. This diode consists of high-resistivity float-zone-grown \( p \)-type silicon with the dimensions given in Fig. 1. The \( p^+ \) contact is made by vacuum evaporation and subsequent alloying of Al, whereas the \( n^+ \) region is formed by Li diffusion.\(^3\) Electrical contact to the \( n^+ \) and \( p^+ \) regions is made with a 1:1 mixture of Ga and In on brass plates. The \( I-V \) characteristics measured in the dark from 140 to 350\(^\circ\)K are shown in Fig. 1. A quadratic behavior is observed over approximately one decade of current at each temperature. The deviation at high currents and 140\(^\circ\)K could possibly be due to diffusion and heating effects as discussed by Baron.\(^4\) Departures from Ohmic behavior at low cur-


III. NOISE MEASUREMENTS AND COMPARISON WITH THEORY

Noise spectra and small-signal conductances have been obtained from 500 kHz to 22 MHz for all operating

\[ \text{Fig. 1. } I-V \text{ characteristics of a silicon double-injection diode between 140 and 350\(^\circ\)K ambient temperature. The solid dots indicate the operating points at which noise measurements have been made.} \]
points marked by solid dots in Fig. 1. In addition, the current response to a differential voltage step at the highest operating points indicates that the diode is truly operating in the semiconductor regime throughout the temperature range.\(^1\)

Figure 2 illustrates typical spectra, taken at 220°K. A constant level is reached at all operating points above a limiting frequency. To determine the value of this white-noise level the functional dependence

\[ I_{\text{eq}} = c_1 + c_2 \frac{1}{f} \]  

is least-squares-fitted to the data (solid lines). From the constant \(c_1\), an equivalent noise resistance

\[ r_{\text{eq}} = 2kT/qc_1 \]  

is derived, with which the measured white noise of the diode at high frequencies is represented as

\[ \langle i^2 \rangle = 4kT(1/r_{\text{eq}})\Delta f, \]  

where \(q\) is the value of electronic charge.

The differential resistance \(r\) of the diode measured at the same operating points is constant over the frequency range in which the noise spectra are also constant. The average value of \(r\) obtained from eight measurements between 1 and 22 MHz is listed in Table I. Values of \(r_{\text{eq}}\) for corresponding operating points are tabulated there also.

The values of \(r\) and \(r_{\text{eq}}\) agree very closely. This means that the high-frequency noise is represented by

\[ \langle i^2 \rangle = \alpha \times 4kT(1/r)\Delta f, \]  

where \(\alpha\) is very close to unity and given by \(r/r_{\text{eq}}\). A better estimate of the mean value of \(\alpha\) is obtained from a plot of the white-noise level versus \(g = 1/r\) for each temperature, as shown in Fig. 3. The dashed lines correspond to the dependences

\[ \langle i^2 \rangle = 4kT(1/r)\Delta f \]  

predicted from the model of thermal noise (\(\alpha = 1\)). Departures from this theory, estimated by least-squares fitting of straight lines to the experimental data, yield \(\alpha\) values of 1.00, 1.00, 1.01, 1.01, 1.04, and 1.03 ±0.05 for 140, 175, 220, 273, 298, and 350°K, respectively. The uncertainty in the absolute-temperature value accounts for most of the error limits. Figure 3 therefore establishes the thermal origin of the high-frequency noise in double injection.

### IV. DISCUSSION

An alternative theory has been advanced by van der Ziel\(^8\) to explain the high-frequency noise in double injection. According to this model,

\[ \langle i^2 \rangle = \alpha \times 4kTg\Delta f, \]  

where

\[ \alpha = 4\mu_{\text{m+}} \mu_{\text{n+}}^2 / (\mu_{\text{m+}} + \mu_{\text{n+}})^2 \]  

\(^8\) A. van der Ziel, IEEE Trans. Microwave Theory Tech. 16, 308 (1968).
and $\mu_+ \text{ and } \mu_-$ are the electron and hole mobilities. With
the standard room-temperature values of $\mu_+ = 1350$
$\text{cm}^2/\text{V sec}$ and $\mu_- = 480 \text{cm}^2/\text{V sec}$, Eq. (8) gives $\alpha = 0.77$
From large-signal turn-on transients as described by
Dean, the actual values for the present diode are measured as $1280$
and $410 \text{cm}^2/\text{V sec}$, respectively, leading to $\alpha = 0.73 \pm 0.06$. Both quantities are clearly
inconsistent with the experimental value of $\alpha = 1.00$
$\pm 0.05$. In addition, measurements of the temperature
dependence of $\mu_+ \text{ and } \mu_-$ on the diode have established that $\mu_+ = 1280(298^\circ \text{K}/T)^{1.72}$
$\text{cm}^2/\text{V sec}$ and $\mu_- = 410$
$(298^\circ \text{K}/T)^{2.18} \text{cm}^2/\text{V sec}$. According to Eq. (8), $\alpha$ should then increase by approximately 0.13 when the temperature
varies from 350 to 140$^\circ \text{K}$. This definitely disagrees with the results of Fig. 3 and invalidates Eq. (8), with its associated model.

Liao has presented experimental results on a germanium
$p^+n^+n^-$ structure at room temperature which appear to support Eq. (8). The range over which $I \propto V^2$
extends from 0.2 to 0.5 V; the physical length of the device is 4.5 mm. It is possible that under those conditions
noise is still influenced by contact effects and diffusion. The information provided is insufficient to verify the presence of a true semiconductor regime, where such effects are negligible.

The results of Fig. 3 reveal that the thermal-noise
expression of Eq. (6) is valid even where the square-law
dependence $I \propto V^2$ does not hold. This conclusion does
not readily follow from the theory of double injection. It was experimentally established, however, that at
those operating points the current response to a differential step voltage satisfies the test $\eta = n$, where $n$ is
the power in the dc characteristic ($I \propto V^n$) and $\eta$ is the ratio of final to initial current in this response. It proves

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FIG. 3. Equivalent noise current $I_{eq, Mt}$ at high frequencies
versus the high-frequency conductance $g = 1/r$ of the silicon
double-injection diode at the operating points indicated in Fig. 1.
$I_{eq, Mt} = 2_0$ of Eq. (2).

that diffusion is negligible at these operating points. This suggests that as long as diffusion, recombination,
and contact effects are negligible, noise in double injection is thermal.

There is no reason why this conclusion should be limited to the temperature range of this experiment, as
long as the conditions above hold. A similar extension can be advanced for frequencies above 22 MHz. Neither
the transit of the carriers across the device nor the dielectric relaxation should alter the mechanism of
thermal noise. It is therefore proposed that Eq. (6) holds also at frequencies above $\omega = \tau^{-1}$. This prediction
could be tested by measuring the noise for frequencies above $1/\theta$, where $\theta$ is the dielectric relaxation time and
is of the order of $(V/I)C_0$, and $C_0$ is the geometrical capacitance of the structure biased at the operating
point $V, I$. The value of $\tau$ is known to change at this frequency.2

Below $1/\tau$, recombination determines the main
features of double injection. In that frequency range, noise varies as $1/f^\alpha$, with $1.7 \leq \alpha \leq 3.0$ as obtained from the least-squares fits. At fixed frequency, the noise also increases with the dc current. Both facts are consistent with the idea that generation recombination is the main source of noise at low frequencies.1,9,10 A dependence of the general form $\langle \delta I^2 \rangle \sim I^n/(1+\omega^2\tau^2)$, $m>1$, can be anticipated for that case. Very recent results support the essential features of the model.11

If the physical length of a double-injection diode is progressively shortened, the structure obtained in the limit is a $p^-n$ junction. It is known both theoretically and experimentally that noise of $pn$ junctions is shotlike. It would be interesting to investigate this transition and relate the change in the noise to that in the transport mechanisms involved.

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