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$K \to \pi e^+ e^-$ in the six-quark model

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The decay $K \to \pi e^+ e^-$ is considered in the six-quark model. The effective Hamiltonian applicable to such decays is calculated in leading-logarithmic approximation and attention is focused on the magnitude of CP-violating effects.

I. INTRODUCTION

The decay $K \to \pi e^+ e^-$ was suggested some time ago as a process in which effects due to (virtual) heavy quarks may play an important role. In particular, in the four-quark model Gaillard and Lee[1] considered this decay as occurring mainly through a mechanism involving an effective $sd\gamma$ vertex whose origin was in diagrams with a virtual u or c quark coupled to the photon.

For the decay $K^+ \to \pi^+ e^+ e^-$ the width predicted from such considerations is of the right order of magnitude to agree with its measured value,[2] which was often regarded as a success of the theory. However, later calculations[3-5] of the quantum-chromodynamics (QCD) corrections to the free (with respect to strong interactions) quark result of Ref. 1 show that they not only change its magnitude, but, with typical choices of parameters, its sign as well. Furthermore, the analysis in Ref. 3 would indicate that other diagrams give contributions to the amplitude for $K^+ \to \pi^+ e^+ e^-$ which are just as important as the mechanism of Ref. 1. From the point of view of making accurate predictions it is discouraging that some contributions enter with opposite signs.

In this paper we will reconsider the decay $K \to \pi e^+ e^-$ in the six-quark model with QCD corrections. Because of the entrance of virtual heavy quarks into the calculation of the effective interaction, the additional b and t quarks of the six-quark model could be of some importance, as could the change in QCD corrections due to their presence.

Of even more concern to us here, and what led us to undertake this investigation, is the question of CP violation. The decay $K_2 \to \pi^0 e^+ e^-$ is forbidden with only one intermediate γ (or Z^0) if CP were conserved. The physical situation, where CP is not conserved, permits contributions to $K_L^0 \to \pi^0 e^+ e^-$ both from the K^0-$\bar{K}^0$ mass matrix and from CP-violating decay amplitudes. As already noted by Ellis *et al.*[6] in the six-quark model without QCD corrections, the presence of a virtual t quark in the diagram leading to an effective $sd\gamma$ vertex can yield a CP-violating contribution from the decay amplitude comparable to that from the mass matrix. We investigate this question here within the context of full QCD corrections to both the CP-conserving and CP-violating amplitudes.

In Sec. II we present the general method of calculating an effective Hamiltonian for processes of the type $K \to \pi e^+ e^-$, with QCD corrections performed in the leading-logarithmic approximation. The technique we adopt is somewhat different than the one used previously,[4] but equivalent. Section III is devoted to examining the results of applying the method to the specific case of six quarks, with numerical results given for both the CP-conserving and CP-violating parts. The dependence of QCD corrections on the choice of parameters is discussed and the physical reasons for their sign and magnitude is established. This is applied to CP-violating effects in $K_L \to \pi^0 e^+ e^-$, where QCD corrections are found to cause a change in relative sign between the real and imaginary parts of that portion of the decay amplitude arising from the $s \to d\gamma \to de^+e^-$ single-quark transition.

II. THE EFFECTIVE HAMILTONIAN FOR $K \to \pi e^+ e^-$

The task of deriving an effective Hamiltonian for weak decays of the type $K \to \pi e^+ e^-$ is accomplished in a very similar manner to that for the effective Hamiltonian for $\Delta S = 1$ nonleptonic processes, which we have recently considered in the six-quark model. The only notable but obvious change is that the matrix elements of the effective Hamiltonian are generally to be evaluated to order e^2, the lowest relevant order in electromagnetism. In the particular case of a six-quark model of the weak and electromagnetic interactions, one successively considers the W boson, t quark, b quark, and c quark as heavy, and eliminates them from explicitly appearing. The resulting effective Hamiltonian contains only u-, d-, and s-quark fields and can be written as a sum of Wilson coefficients times local four-fermion operators, $\sum_i C_i Q_i$.

This sum contains exactly the same six local

four-quark operators as the previously calculated nonleptonic effective Hamiltonian[7] (which are order e^0 and hence their matrix elements must be evaluated to order e^2 for $K\to\pi e^+e^-$) plus one more operator

$$Q_7=\left[\frac{e^2}{4\pi}\right]\bar{s}\gamma_\mu(1-\gamma_5)d\bar{e}\gamma_\mu e\,,$$

which is of order e^2 and hence its matrix element need only be evaluated to order e^0 to get an amplitude correct to order e^2.

Furthermore, on recognizing $\bar{s}\gamma_\mu(1-\gamma_5)d$ as the quark representation of an isospin rotation of the usual strangeness-changing weak current, its matrix elements between K and π can be identified with those involved in the decay $K\to\pi e\nu$. The magnitude of $\Gamma(K\to\pi e^+e^-)$ arising from just the new term C_7Q_7 in the effective Hamiltonian is then directly proportional to $C_7{}^2\Gamma(K\to\pi e\nu)$, with completely known factors in the proportionality.

Having laid out the general features of this problem, especially in comparison with the earlier construction of an effective Hamiltonian for $\Delta S=1$ decays to order e^0, we proceed to the actual calculation relevant to this paper. Most of the details will be relegated to an appendix and we will draw upon some results derived in our earlier work,[7] stressing only new aspects of the calculation.

In the standard SU(2)⊗U(1) model of weak and electromagnetic interaction, the six quarks are assigned to right-handed singlets

$$(u)_R,\quad (d)_R,\quad (c)_R,\quad (s)_R,\quad (t)_R,\quad (b)_R$$

and left-handed doublets

$$\begin{bmatrix}u\\d'\end{bmatrix}_L,\quad \begin{bmatrix}c\\s'\end{bmatrix}_L,\quad \begin{bmatrix}t\\b'\end{bmatrix}_L$$

of the weak SU(2) gauge group. The primed fields are not mass eigenstates. However, they can be related to mass eigenstates by a unitary transformation. With the standard choice for the phases of the quark fields this transformation has the form

$$\begin{bmatrix}d'\\s'\\b'\end{bmatrix}=\begin{bmatrix}c_1 & -s_1c_3 & -s_1s_3\\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta}\\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta}\end{bmatrix}\begin{bmatrix}d\\s\\b\end{bmatrix}\,, \tag{1}$$

where $c_i\equiv\cos\theta_i$, $s_i\equiv\sin\theta_i$, $i\in\{1,2,3\}$. The signs of the quark fields are chosen so the θ_1, θ_2, and θ_3 lie in the first quadrant. Then the quadrant of the CP-violating phase δ has physical significance and cannot be chosen by convention. The measured phase of the CP-violation parameter ϵ can be used to determine[8] that δ lies in the upper half plane if s_1 and s_3 can be treated as small quantities.

In the absence of strong interactions, an effective Hamiltonian for $K\to\pi e^+e^-$ can be derived by treating the charged W boson as very heavy and keeping only the leading contribution in $1/M_W{}^2$. Then[9]

$$\mathcal{H}_{\rm eff}=-\frac{G_F}{2\sqrt{2}}[A_c(O_c^{(+)}+O_c^{(-)})+A_t(O_t^{(+)}+O_t^{(-)})]+\text{H.c.}\,, \tag{2}$$

where

$$O_q^{(\pm)}=[(\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A}\pm(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}]-[u\to q]\,, \tag{3}$$

and

$$A_c=s_1c_2(c_1c_2c_3-s_2s_3e^{-i\delta})\,, \tag{4a}$$

$$A_t=s_1s_2(c_1s_2c_3+c_2s_3e^{-i\delta})\,. \tag{4b}$$

In Eq. (3) the color subscripts α and β are summed over $\{1,2,3\}$ when repeated, normal ordering of the four-fermion operators is understood, and the notation

$$(\bar{\psi}\psi)_{V-A}(\bar{\psi}\psi)_{V-A}\equiv[\bar{\psi}(0)\gamma_\mu(1-\gamma_5)\psi(0)]\times[\bar{\psi}(0)\gamma^\mu(1-\gamma_5)\psi(0)] \tag{5}$$

is used. Since the electromagnetic fine-structure constant is small, the $K\to\pi e^+e^-$ matrix element of the effective Hamiltonian is to be evaluated to the lowest possible order in electromagnetic interactions (i.e., order e^2).

Now introduce the strong interactions in the form of quantum chromodynamics (QCD). The effective Hamiltonian in Eq. (2) with the W boson removed is replaced by[9]

$$\mathcal{H}_{\rm eff}=-\frac{G_F}{2\sqrt{2}}\left\{\left[\frac{\alpha(M_W{}^2)}{\alpha(\mu^2)}\right]^{a^{(+)}}(A_cO_c^{(+)}+A_tO_t^{(+)})+\left[\frac{\alpha(M_W{}^2)}{\alpha(\mu^2)}\right]^{a^{(-)}}(A_cO_c^{(-)}+A_tO_t^{(-)})\right\}+\text{H.c.}\,, \tag{6}$$

where the leading-logarithmic approximation has been used. In Eq. (6), $\alpha(M^2)$ is the running fine-structure constant for strong interactions and

$$a^{(+)}=6/(33-2N_f)\,,\qquad a^{(-)}=-12/(33-2N_f)\,, \tag{7}$$

where N_f is the number of quark flavors, which equals six at this stage. The $K\to\pi e^+e^-$ matrix element of the effective Hamiltonian in Eq. (6) is to be evaluated to all orders in the strong inter-

action, to zeroth order in the weak interactions, and to order e^2 in the electromagnetic interaction.

The next step is to consider the t quark as very heavy and eliminate it from explicitly appearing in the effective Hamiltonian for the decay $K \to \pi e^+ e^-$. What happens to the operator $0_c^{(\pm)}$ is identical with the case of $\Delta S = 1$ weak nonleptonic decays,[9] so we focus our attention on $0_t^{(\pm)}$.

Assuming that m_t is much greater than all the other quark masses and the momenta of the external states allows us to expand the matrix element of $O_t^{(\pm)}$ in terms of matrix elements of operators not explicitly containing the heavy-t-quark field,[9]

$$\langle | O_t^{(\pm)} | \rangle = \sum_i B_i^{(\pm)}(m_t/\mu, g) \langle | O_i | \rangle' + O(1/m_t^2) . \tag{8}$$

The primed matrix elements are to be evaluated in an effective theory of strong (and electromagnetic) interactions with five quark flavors, strong coupling g', and mass parameters $m'_u, m'_d, \ldots, m'_b$. That is,

$$\langle | O_i | \rangle' \equiv \langle | O_i | \rangle (g', e, \mu, m'_u, \ldots, m'_b) . \tag{9}$$

To carry out the expansion in Eq. (8) in the leading-logarithmic approximation, seven linearly independent operators are necessary. The first six may be chosen[7] just as in the absence of electromagnetism,

$$\begin{aligned}
O_1 &= (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A} , \\
O_2 &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V-A} , \\
O_3 &= (\bar{s}_\alpha d_\alpha)_{V-A} [(\bar{u}_\beta u_\beta)_{V-A} + \cdots + (\bar{b}_\beta b_\beta)_{V-A}] , \\
O_4 &= (\bar{s}_\alpha d_\beta)_{V-A} [(\bar{u}_\beta u_\alpha)_{V-A} + \cdots + (\bar{b}_\beta b_\alpha)_{V-A}] , \\
O_5 &= (\bar{s}_\alpha d_\alpha)_{V-A} [(\bar{u}_\beta u_\beta)_{V+A} + \cdots + (\bar{b}_\beta b_\beta)_{V+A}] , \\
O_6 &= (\bar{s}_\alpha d_\beta)_{V-A} [(\bar{u}_\beta u_\alpha)_{V+A} + \cdots + (\bar{b}_\beta b_\alpha)_{V+A}] ,
\end{aligned} \tag{10}$$

and close under strong-interaction renormalization in the absence of electromagnetism. In the presence of electromagnetism a seventh operator must be added; it is chosen most straightforwardly to be[4]

$$O_7 = \frac{e^2}{4\pi} (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{e} e)_V . \tag{11}$$

We choose instead to define for calculational purposes

$$O'_7 = \left[\frac{e^2}{g'^2} \right] (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{e} e)_V . \tag{12}$$

The utility of our definition will become clear shortly.

With either definition of the seventh operator, the Wilson coefficients $B_j^{(\pm)}(m_t/\mu, g)$, as defined in Eq. (8), are independent of the electromagnetic coupling e if we work to order e^2 in the overall amplitude. They satisfy the renormalization-group equation[9]

$$\sum_{j=1}^{7} \left[\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_t(g) m_t \frac{\partial}{\partial m_t} + \gamma^{(\pm)}(g) \right) \delta_{ij} - \gamma'^T_{ij}(g') \right] B_j^{(\pm)}(m_t/\mu, g) = 0 . \tag{13}$$

Here $\beta(g)$ is the QCD β function, which has the perturbation expansion

$$\beta(g) = -(33 - 2N_f) \frac{g^3}{48\pi^3} + O(g^5) , \tag{14}$$

with $N_f = 6$. The $\gamma^{(\pm)}(g)$ are the anomalous dimensions of $O^{(\pm)}$, with perturbation expansion

$$\gamma^{(+)}(g) = \frac{g^2}{4\pi^2} + O(g^4) , \tag{15a}$$

$$\gamma^{(-)}(g) = -\frac{g^2}{2\pi^2} + O(g^4) . \tag{15b}$$

The anomalous-dimension matrix $\gamma'_{ij}(g')$ is that of the operators O_i and is to be evaluated in the effective theory of five quark flavors (t quark removed). It is defined by

$$\gamma'_{ij}(g') = \sum_{k=1}^{7} Z^{-1}{}_{ik} \mu \frac{d}{d\mu} Z_{kj} , \tag{16}$$

where the $Z(g')$ relate unrenormalized to renormalized operators via

$$O_i^{(0)} = \sum_j Z_{ij}(g') O_j . \tag{17}$$

With the more straightforward definition of O_7 in Eq. (11), $\gamma'(g')$ has the structure[4]

$$\begin{pmatrix} \gamma'_{11} & \cdots & \gamma'_{16} & \gamma'_{17} \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \vdots & & \vdots & \vdots \\ \gamma'_{61} & \cdots & \gamma'_{66} & \gamma'_{67} \\ 0 & \cdots & 0 & 0 \end{pmatrix}$$

In a perturbative expansion the γ'_{ij} with $1 \leq i, j \leq 6$ all start in order $(g')^2$, but the γ_{i7} start in order $(g')^0$ and this difference in powers of g' means that these two pieces of the anomalous-dimension matrix must be split off and treated separately in solving the renormalization-group equations at each stage of removing another heavy-quark field.[4] On the other hand, with the definition in Eq. (12) which we will use, $\gamma'(g')$ has the structure

$$\begin{pmatrix} \gamma'_{11} & \cdots & \gamma'_{16} & \gamma'_{17} \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \vdots & & \vdots & \vdots \\ \gamma'_{61} & \cdots & \gamma'_{66} & \gamma'_{67} \\ 0 & \cdots & 0 & \gamma'_{77} \end{pmatrix} .$$

Now *all* entries start their perturbation expansion in order $(g')^2$. The solution of the renormalization-group equations is a standard one which may simply be extended from the 6×6 matrix case of Ref. 7 to the 7×7 case of interest here. After that it is just a matter of brute-force computation. Both definitions of O_7 of course give the same final result, as is illustrated by an explicit calculation in Appendix A.

Notice that in either case the elements γ'_{ij} for $1 \le i,\ j \le 6$ are identical to those that would be present in the absence of electromagnetism. An inspection of the renormalization-group equation [Eq. (13)] then shows that the B_j for $1 \le j \le 6$ obey exactly the same equations as they would in the absence of electromagnetism. Thus, the Wilson coefficients of the operators $O_1, \ldots, O_6$ will be the same as calculated previously for the effective $\Delta S = 1$ nonleptonic Hamiltonian.

The actual 7×7 matrix $\gamma'_{ij}(g)$ to order g'^2 is given in Appendix B. Eigenvectors of γ'^T, according to Eq. (13), correspond to combinations of operators which are simply multiplicatively renormalized (i.e., do not mix with other operators). We denote by V the matrix which diagonalizes γ'^T to give eigenvalues $\gamma'_j(g')$ with $1 \le j \le 7$. The solution to the diagonalized version of Eq. (13), the renormalization-group equation for the $B_j^{(\pm)}(m_t/\mu, g)$, involves a running coupling $\bar{g}(m_t/\mu, g)$ defined as in Ref. 7, from which we define

$$\alpha_s(m_t^2) \equiv [\bar{g}(m_t/\mu, g)]^2/4\pi\ , \tag{18a}$$

$$\alpha_s(\mu^2) \equiv g^2/4\pi\ . \tag{18b}$$

More precisely, in leading-logarithmic approximation the $B_j^{(\pm)}(m_t/\mu, g)$ are related in the solution to linear combinations of the $B_i^{(\pm)}(1, \bar{g})$ weighted by the V's and powers of $\alpha_s(m_t^2)/\alpha_s(\mu^2)$. Values for the $B_i^{(\pm)}(1, \bar{g})$ are obtained by noting that at $m_t/\mu = 1$ the strong-interaction fine-structure constant is small and no large logarithms are generated when the renormalization point is equal to the top quark mass. The coefficients $B_i^{(\pm)}(1, \bar{g})$ can be replaced by their free- (no strong interactions) field values. A direct calculation of the effective Hamiltonian to order g^0 and e^2 with the W boson and t quark removed from the theory gives

$$\begin{aligned}\mathcal{H}_{\text{eff}} = &-\frac{G_F}{2\sqrt{2}} A_c\,[O_c^{(+)} + O_c^{(-)}] \\ &- \frac{G_F}{2\sqrt{2}} A_t[(O_1 + O_2) + (-O_1 + O_2)] \\ &- \frac{G_F}{\sqrt{2}}\left(\frac{2}{9}\right)\left(\frac{e^2}{4\pi^2}\right) A_t \ln\left(\frac{m_t^2}{\mu^2}\right)(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{e}e)_V\ ,\end{aligned} \tag{19}$$

where μ is the renormalization point for electromagnetic interactions and the calculation has been done in a mass-independent subtraction scheme chosen so that the coefficient of O_7 has no pieces independent of m_t/μ. From Eq. (19) we read off the free-field values

$$\begin{aligned} &B_1^{(\pm)} = B_1^{(\pm)}(1,0) = \pm 1\ , \\ &B_2^{(\pm)} = B_2^{(\pm)}(1,0) = +1\ , \\ &B_3^{(\pm)} = B_4^{(\pm)} = B_5^{(\pm)} = B_6^{(\pm)} = 0 \end{aligned} \tag{20}$$

as in Ref. 7, and, with our definition of O_7',

$$\begin{aligned} B_7^{(\pm)} &= B_7^{(\pm)}(m_t/\mu = 1, \bar{g}) \\ &= \frac{2}{9}\left(\frac{\bar{g}^2}{4\pi^2}\right)\ln\left(\frac{m_t^2}{\mu^2}\right)\bigg|_{\mu = m_t} = 0\ . \end{aligned}$$

This last result follows even though O_7' contains $1/g'^2$, since the logarithm vanishes when the renormalization point equals the t-quark mass.

Our final aim is to derive an effective Hamiltonian independent of the heavy-W-boson, t-quark, b-quark, and c-quark fields. To do this, the b quark and c quark must still be considered as heavy and removed from explicitly appearing in the effective Hamiltonian. However, the key differences between this calculation and that of deriving the nonleptonic effective Hamiltonian for $\Delta S = 1$ weak decays have already been illustrated in our discussion of how the t quark is removed from explicitly appearing in the theory. From here on we follow the path discussed in detail in Ref. 7, defining effective couplings g'' and g''' in the four- and three-quark theories, respectively, diagonalizing the transpose of the anomalous dimension matrices $\gamma''(g'')$ and $\gamma'''(g''')$ with matrices W and X, respectively, and treating all the solutions of the renormalization-group equations in leading-logarithmic approximation. The actual matrices are found in Appendix B. At the last step of removing the c quark, some care must be taken because there are only six (instead of the naively expected seven) linearly independent four-fermion operators. With the brute-force portion of the calculation completed, we revert from our choice for the seventh operator, Q_7', to Q_7, with an appropriate factor in the corresponding Wilson coefficient.

In the light-three-quark sector the effective Hamiltonian for the decay $K \to \pi e^+ e^-$ is the following sum of Wilson coefficients times local four-fermion operators:

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} s_1 c_2 c_3 \sum_i C_i Q_i + \text{H.c.}\ , \tag{21}$$

where the sum is over $i = 1, 2, 3, 5, 6, 7$ and

$$
\begin{aligned}
Q_1 &= (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}\,,\\
Q_2 &= (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}\,,\\
Q_3 &= (\bar{s}_\alpha d_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V-A}+(\bar{d}_\beta d_\beta)_{V-A}\\
&\qquad +(\bar{s}_\beta s_\beta)_{V-A}]\,,\\
Q_5 &= (\bar{s}_\alpha d_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V+A}+(\bar{d}_\beta d_\beta)_{V+A}\\
&\qquad +(\bar{s}_\beta s_\beta)_{V+A}]\,,\\
Q_6 &= (\bar{s}_\alpha d_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V+A}+(\bar{d}_\beta d_\alpha)_{V+A}\\
&\qquad +(\bar{s}_\beta s_\alpha)_{V+A}]\,,\\
Q_7 &= \frac{e^2}{4\pi}(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{e}e)_V\,.
\end{aligned}
\tag{22}
$$

This is to be compared to the corresponding effective Hamiltonian in the "free-quark model," i.e., no strong interactions, which with the W, t, b, and c quarks removed from explicitly appearing is

$$
\begin{aligned}
\mathcal{H}_{\rm eff} = &-\frac{G_F}{\sqrt{2}}s_1c_1c_3(\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A}\\
&-\frac{G_F}{\sqrt{2}}\frac{2}{9\pi}\frac{e^2}{4\pi}\left[A_c\ln\left(\frac{m_c^2}{\mu^2}\right)+A_t\ln\left(\frac{m_t^2}{\mu^2}\right)\right]\\
&\times(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{e}e)_V\\
&+\text{H.c.}
\end{aligned}
\tag{23}
$$

The matrix elements of the operators Q_i in the effective Hamiltonian (21) are to be evaluated to all orders in the effective theory of strong interactions with three quark flavors and coupling g''' (since strong-interaction perturbation theory is probably not valid), and to the lowest possible order in electromagnetic interactions. The Wilson coefficients C_i are independent of the electromagnetic coupling e. Each Wilson coefficient has a real and imaginary part, being of the form $a+b\tau$, where $\tau = s_2^2+s_2c_2s_3e^{-i\delta}/c_1c_3$ and a, b are real numbers. The imaginary part arises from diagrams with virtual heavy quarks. The operators Q_1, Q_2, Q_3, Q_5, and Q_6 occur in the effective Hamiltonian for $\Delta S=1$ nonleptonic weak decays and their Wilson coefficients are the same as in that problem,[7] where the operator Q_7 (which arises due to electromagnetic interactions) is absent.

III. NUMERICAL RESULTS AND DISCUSSION

We now proceed to choose a parametrization for $\alpha(M^2)$ and values for the W-boson and heavy-quark masses in order to evaluate the coefficients of the local four-fermion operators in the effective Hamiltonian.

TABLE I. Coefficient of the operator Q_7 [defined in Eq. (22)] in the effective Hamiltonian, $\mathcal{H}_{\rm eff} = (-G_F s_1c_1c_3/\sqrt{2})(\sum_i C_iQ_i)$, for decays like $K\to\pi e^+e^-$ in the six-quark model. $\tau\equiv s_2^2+s_2c_2s_3e^{-i\delta}/c_1c_3$.

Parameters			
$\alpha(\mu^2)$	m_t (GeV)	Λ^2 (GeV^2)	C_7
0.75	15	0.1	$-0.036+0.056\tau$
1.00	15	0.1	$-0.066+0.058\tau$
1.25	15	0.1	$-0.091+0.059\tau$
0.75	30	0.1	$-0.034+0.12\tau$
1.00	30	0.1	$-0.064+0.12\tau$
1.25	30	0.1	$-0.088+0.13\tau$
0.75	15	0.01	$-0.055+0.10\tau$
1.00	15	0.01	$-0.089+0.10\tau$
1.25	15	0.01	$-0.12+0.10\tau$
0.75	30	0.01	$-0.051+0.18\tau$
1.00	30	0.01	$-0.085+0.18\tau$
1.25	30	0.01	$-0.11+0.18\tau$

In leading-logarithmic approximation, within which we are working,

$$
\alpha_s(Q^2)=\frac{12\pi}{33-2N_f}\frac{1}{\ln(Q^2/\Lambda^2)}\,.
\tag{24}
$$

We shall take $\Lambda^2=0.1$ or 0.01 GeV^2, thereby including roughly the range of value suggested in recent experimental and theoretical analyses.[10] If the leading-logarithmic approximation is in fact valid, the calculation is insensitive to the precise value of Λ^2. We take m_c to be 1.5 GeV, $m_b=4.5$ GeV, and $m_t=15$ or 30 GeV, and do not differentiate between, for example, m_b and m_b', the b-quark masses in the effective strong-interaction theories with six and five quarks, respectively. $M_W=85$ GeV. Finally, since $\alpha_s(\mu^2)$ is to be of order unity, we let $\alpha_s(\mu^2)=0.75$, 1.0, and 1.25.

Values of the coefficients of the local operators Q_1, Q_2, Q_3, Q_5, Q_6, and Q_7 in Eq. (21) were calculated for the values of the parameters given above. As mentioned in the previous section, the coefficients of all but Q_7 are exactly the same as in the absence of electromagnetism and are found in Ref. 7. Values of C_7 for $\Lambda^2=0.1$ and 0.01 GeV^2, $m_t=15$ and 30 GeV, and $\alpha_s(\mu^2)=0.75$, 1.0, and 1.25 are found in Table I. Choosing one "typical" case [$\Lambda^2=0.1$ GeV^2, $m_t=15$ GeV, and $\alpha_s(\mu^2)=1$] and combining the results here with those in Ref. 7, we have

$$
\begin{aligned}
\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}}s_1c_1c_3[&(-0.87+0.036\tau)Q_1+(1.5-0.036\tau)Q_2+(-0.021-0.012\tau)Q_3\\
&+(0.011+0.007\tau)Q_5+(-0.047-0.072\tau)Q_6+(-0.066+0.058\tau)Q_7]\,,
\end{aligned}
\tag{25}
$$

TABLE II. Coefficient of the operator Q_7 [defined in Eq. (22)] in the effective Hamiltonian, $\mathcal{H}_{\rm eff} = (-G_F s_1 c_1/\sqrt{2})(\sum_i C_i Q_i)$, for decays like $K \to \pi e^+ e^-$ in the six-quark model with $\theta_2 = \theta_3 = 0$ and mixing with the penguin-type operators Q_3, Q_5, and Q_6 neglected.

Parameters			
$\alpha(\mu^2)$	m_t (GeV)	Λ^2 (GeV^2)	C_7
0.75	15	0.1	−0.037
1.00	15	0.1	−0.069
1.25	15	0.1	−0.096
0.75	30	0.1	−0.036
1.00	30	0.1	−0.067
1.25	30	0.1	−0.093
0.75	15	0.01	−0.063
1.00	15	0.01	−0.10
1.25	15	0.01	−0.13
0.75	30	0.01	−0.060
1.00	30	0.01	−0.097
1.25	30	0.01	−0.13

where $\tau = s_2^2 + s_2 c_2 s_3 e^{-i\delta}/c_1 c_3$.

To check on the sensitivity to varying other aspects of the calculation, Table II contains the values of C_7 that follow in the six-quark model with $\theta_2 = \theta_3 = 0$ (so that $\tau = 0$) and with mixing of the "penguin operators" Q_3, Q_5, and Q_6 with Q_1, Q_2 and Q_7 neglected. In Table III, on the other hand, values of C_7 are given which pertain to the four-quark model, again with mixing of the penguin operators Q_3, Q_5, and Q_6 with Q_1, Q_2, and Q_7 neglected. Comparison of Tables I, II, and III quickly shows that for a given value of Λ^2 and $\alpha_s(\mu^2)$, changing m_t from 15 to 30 Gev, dropping mixing between penguin operators and the usual operators, or going from a six-quark to a four-quark model, each results in less than a factor ~1.5 change in ReC_7. Of course, setting $\theta_2 = \theta_3 = 0$, or going from the six- to four-quark model (Tables II and III, respectively) makes

$$\tau = s_2^2 + s_2 c_2 s_3 e^{-i\delta}/c_1 c_3 = 0$$

and therefore forces Im$C_7 = 0$. Changing Λ^2 from 0.1 to 0.01 GeV^2 or $\alpha_s(\mu^2)$ from 0.75 to 1.25 makes a somewhat larger effect—over a factor of 2 for either Λ^2 or $\alpha_s(\mu^2)$ in some cases. Note that in the case of changing Λ^2, *both* ReC_7 and, when applicable, ImC_7 (proportional to the coefficient of τ) change, and that they both change magnitude in the same direction. Thus, their ratio is somewhat less sensitive to changes in parameters than each is individually.

More interesting is to compare the results with QCD corrections to those of the free-quark model, i.e., $g = 0$. In this latter case we read off from Eq. (23) that

$$C_7 = +\frac{2}{9\pi}\left[\ln\left(\frac{m_c^2}{\mu^2}\right) + \tau \ln\left(\frac{m_t^2}{m_c^2}\right)\right]. \tag{26}$$

We need to choose the scale parameter μ in Eq. (26), which refers to the renormalization point of electromagnetic interactions (QCD is turned off), if we are to make a comparison with the QCD-corrected values of C_7. The obvious choice of μ is such that $\alpha_s(\mu^2)$ has the appropriate value when the strong interactions are turned on [i.e., such that $\alpha_s(\mu^2) = 0.75$, 1.00, or 1.25 in Table I]. In Table IV we then list the free-quark values of C_7 for $\Lambda^2 = 0.1$ and 0.01 GeV^2, $m_t = 15$ and 30 GeV, and $\alpha_s(\mu^2) = 0.75$, 1.00, and 1.25, using this prescription for μ in Eq. (26). Comparison of Table IV with Table I immediately reveals that ReC_7 changes sign due to QCD corrections, but ImC_7 (proportional to the coefficient of τ) does not.

TABLE III. Coefficient of the operator Q_7 [defined in Eq. (22)] in the effective Hamiltonian, $\mathcal{H}_{\rm eff} = (-G_F \sin\theta_C \cos\theta_C/\sqrt{2})(\sum_i C_i Q_i)$, for decays like $K \to \pi e^+ e^-$ in the four-quark model when the mixing with the penguin-type operators Q_3, Q_5, and Q_6 is neglected.

Parameters		
$\alpha(\mu^2)$	Λ^2 (GeV^2)	C_7
0.75	0.1	−0.045
1.00	0.1	−0.081
1.25	0.1	−0.11
0.75	0.01	−0.10
1.00	0.01	−0.15
1.25	0.01	−0.19

TABLE IV. Coefficient of the operator Q_7 [defined in Eq. (22)] in the free-quark-model (i.e., no strong interactions) effective Hamiltonian, $\mathcal{H}_{\rm eff} = (-G_F s_1 c_1 c_3/\sqrt{2})(\sum_i C_i Q_i)$, given in Eq. (23). For given values of $\alpha(\mu^2)$ and Λ^2, the renormalization point is determined by $\mu^2 = \Lambda^2 \exp[12\pi/27\alpha(\mu^2)]$.

Parameters				
$\alpha(\mu^2)$	Λ^2 (GeV^2)	m_t (GeV)	μ^2 (GeV^2)	C_7
0.75	0.1	15	0.64	$0.089 + 0.33\tau$
1.00	0.1	15	0.40	$0.12 + 0.33\tau$
1.25	0.1	15	0.31	$0.14 + 0.33\tau$
0.75	0.1	30	0.64	$0.089 + 0.42\tau$
1.00	0.1	30	0.40	$0.12 + 0.42\tau$
1.25	0.1	30	0.31	$0.14 + 0.42\tau$
0.75	0.01	15	0.064	$0.25 + 0.33\tau$
1.00	0.01	15	0.040	$0.28 + 0.33\tau$
1.25	0.01	15	0.031	$0.30 + 0.33\tau$
0.75	0.01	30	0.064	$0.25 + 0.42\tau$
1.00	0.01	30	0.040	$0.28 + 0.42\tau$
1.25	0.01	30	0.031	$0.30 + 0.42\tau$

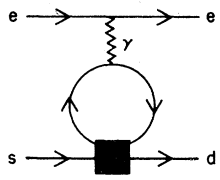

FIG. 1. Diagram contributing to C_7. The black box represents W exchange plus all strong-interaction corrections.

This result, which is at first glance surprising, may be understood physically as follows.

If we again neglect strong-interaction-induced penguin-type operators (which we have seen make little numerical difference in C_7), then the operator Q_7 comes about from heavy-quark loops, as illustrated graphically in Fig. 1. It is an "electromagnetic penguin operator," whose strength is determined by the integration over the momentum carried around the quark loop and the strength of the four-quark weak interaction arising due to W exchange with QCD corrections. Analytically, we may rewrite the last integration in the formula for $C_7^{(\pm)}$ in Appendix A [i.e., Eq. (A14)] in terms of momentum instead of the running coupling. Then we find, as in Ref. 3, that

$$C_7 \propto \int \frac{dq^2}{q^2}\left[2C^{(+)}(q^2) - C^{(-)}(q^2)\right], \tag{27}$$

with $C^{(+)}(q^2)$ and $C^{(-)}(q^2)$ the coefficients of the operators

$$O_+ = (\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A} + (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$

and

$$O_- = (\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A} - (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$

in the effective Hamiltonian for nonleptonic weak interactions at a mass squared scale characterized by q^2. The coefficients of $C^{(+)}$ and $C^{(-)}$ in Eq. (27) stem from color. In the free-quark model $C^{(+)} = C^{(-)} = 1$ and the integration in Eq. (27) leads to the characteristic logarithms in Eq. (26). In the integration range from μ^2 to m_c^2, the c- and t-quark contributions (with coefficients A_c and A_t, respectively) add, leading to the first term in Eq. (26) proportional to $\ln(m_c^2/\mu^2)$, while between m_c and m_t only the t quark contributes, leading to the second term in Eq. (26) proportional to $\tau \ln(m_t^2/m_c^2)$.

With strong interactions turned on, $C^{(+)}(q^2)$ is suppressed, while $C^{(-)}(q^2)$ is enhanced, having forms $[\alpha_s(q^2)/\alpha_s(M_W^2)]^{-6/25}$ and $[\alpha_s(q^2)/\alpha_s(M_W^2)]^{12/25}$, respectively, with four quarks operative. With reasonable choices of strong-interaction parameters the enhancement of $C^{(-)}(q^2)$ and suppression of $C^{(+)}(q^2)$ for values of $q^2 \lesssim m_c^2$ more than makes up for the extra factor 2 in the first term in the integrand in Eq. (27), and the integral from μ^2 to m_c^2 has the opposite sign from the free-quark model. On the other hand, for reasonable QCD parameters the enhancement [suppression] of $C^{(-)}(q^2)$ $[C^{(+)}(q^2)]$ is considerably less at $q^2 \approx m_t^2$, and the integration range from m_c^2 to m_t^2, which contributes the term in C_7 proportional to τ, turns out to yield a result which has the same sign as the free-quark-model result. We then understand in a fairly physical manner the change in sign or lack of it in applying QCD corrections to the free-quark-model result for C_7.

Precisely because of the cancellation between the two terms in the integrand, the values of C_7 after QCD corrections found in Table I are in general considerably smaller in magnitude than those of the free-quark model which are given in Table IV. In any situation where an operator is induced only through mixing with other operators, its Wilson coefficient will be the sum of several terms which partially cancel against one another. However, in the case of both $\mathrm{Re}C_7$ and $\mathrm{Im}C_7$ it is a particularly delicate cancellation and we are wary about taking the QCD-corrected results for C_7 too seriously. Also $\mathrm{Re}C_7$, like $\mathrm{Re}C_6$, for example, depends primarily on integration from μ^2 to m_c^2, where the leading-logarithmic approximation is most dubious.

With these limitations on the accuracy of the calculation in mind, let us now look at the predicted rates for $K \to \pi e^+ e^-$, and especially $K_L \to \pi^0 e^+ e^-$ which to order e^2 proceeds through violation of CP. First, let us calculate the contribution arising through the electromagnetic penguin operator Q_7 in the effective Hamiltonian Eq. (21). For $K^+ \to \pi^+ e^+ e^-$ we find an amplitude

$$A(K^+ \to \pi^+ e^+ e^-) = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \frac{e^2}{4\pi} C_7 f_+ (K+\pi)_\mu \bar{e}\gamma_\mu e\,, \tag{28}$$

where K_μ and π_μ represent the K and π four-momenta, respectively, and f_+ is the form factor in K_{l3} decay. For comparison, this latter process has an amplitude

$$A(K^+ \to \pi^0 e^+ \nu) = \frac{G_F}{\sqrt{2}} s_1 c_3 \frac{f_+}{\sqrt{2}} (K+\pi)_\mu \bar{\nu}\gamma_\mu (1-\gamma_5) e\,. \tag{29}$$

Similarly, the amplitude for $K_s \to \pi^0 e^+ e^-$ (neglecting CP-violating effects)

$$A(K_S \to \pi^0 e^+ e^-) = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \frac{e^2}{4\pi} C_7 (-f_+)(K+\pi)_\mu \bar{e}\gamma_\mu e\,. \tag{30}$$

As already noted, to order e^2 the amplitude for $K_L \to \pi^0 e^+ e^-$ involves only CP-violating effects.

Recall that

$$|K_L\rangle = \frac{(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle}{[2(1+|\epsilon|^2)]^{1/2}} . \tag{31}$$

The value of ϵ depends on the phase convention used for the $|K^0\rangle$ and $|\bar{K}^0\rangle$ states. We adopt the usual phase convention where the K^0 ($\bar{K}^0$) states have a quark content $e^{-i\xi}\bar{s}d$ ($e^{i\xi}\bar{d}s$) and ξ is chosen to make the $K^0 \to \pi\pi$ ($I=0$) amplitude real, apart from final-state $\pi\pi$ strong interactions. Then $\epsilon \approx (2\times 10^{-3})e^{i\pi/4}$ and

$$A(K_L \to \pi^0 e^+ e^-) = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \frac{e^2}{4\pi}(-f_+) \times [(\epsilon - i\xi)\mathrm{Re}C_7 + i\,\mathrm{Im}C_7](K+\pi)_\mu \bar{e}\gamma_\mu e . \tag{32}$$

In Ref. 7 it was shown that

$$\xi = f\,\mathrm{Im}C_6/\mathrm{Re}C_6$$

when the fraction of the $K^0 \to \pi\pi$ ($I=0$) amplitude arising from the operator Q_6 in the effective Hamiltonian for $\Delta S=1$ weak nonleptonic decays, f, is large. The only measured decay of the form $K \to \pi e^+ e^-$ is $K^+ \to \pi^+ e^+ e^-$ with a branching ratio[2] of $(2.6 \pm 0.5)\times 10^{-7}$. If this were all to arise from the $C_7 Q_7$ piece of the effective Hamiltonian it would imply

$$|C_7| \approx 0.3 . \tag{33}$$

The values of C_7 which are found in Tables I–III (with QCD corrections) are all a factor 2 or more smaller than this.[11] In addition to those of Q_7, possible contributions, illustrated in Fig. 2(a) and 2(b), have been estimated by Vainshtein *et al.*[3] In our effective Hamiltonian, formalism contributions such as these[12] arise from taking matrix elements of Q_1, Q_2, Q_3, Q_5, and Q_6 to order e^2. With the "vacuum-insertion" method of estimating matrix elements they find comparable contributions of differing signs for the decay $K^+ \to \pi^+ e^+ e^-$.

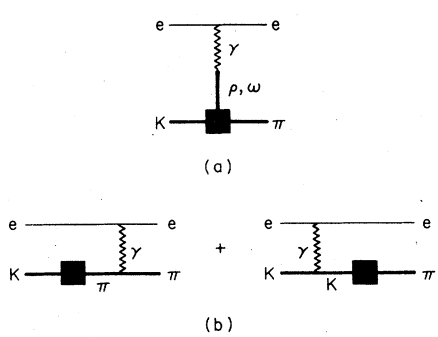

FIG. 2. Some diagrams contributing to $K \to \pi e^+ e^-$. The black box represents the action of the effective Hamiltonian for $\Delta S=1$ nonleptonic weak decays.

It is particularly interesting that for Fig. 2(b) they find that the penguin term $C_6 Q_6$ in the effective Hamiltonian gives comparable contributions to the normal $(V-A)\times(V-A)$ terms $C_1 Q_1 + C_2 Q_2$. This is unlike the case for ordinary nonleptonic $\Delta S=1$ weak decays (such as $K \to \pi\pi$), where matrix elements of Q_6 are argued to be strongly enhanced over those of Q_1 and Q_2. In the case at hand such an enhancement factor again arises, but is compensated by a factor $\langle r^2\rangle_{K^+} - \langle r^2\rangle_{\pi^+}$ in terms involving Q_6 versus just $\langle r^2\rangle_{K^+}$ in those involving Q_1 and Q_2. Note that in applying this method of estimating matrix elements to either $K_S^0 \to \pi^0 e^+ e^-$ or $K_L \to \pi^0 e^+ e^-$, the relevant electromagnetic charge radius squared that enters is that of the K^0 which, as expected, is measured[13] now to be much smaller than that of the K^+ or π^+. This gives some indication that the terms in the matrix element of $C_1 Q_1 + C_2 Q_2$ involving the charge radius will be less important in $K^0 \to \pi^0 e^+ e^-$ than they are in $K^+ \to \pi^+ e^+ e^-$.

One should also note at this point that the matrix elements of $\sum_1^6 C_i Q_i$ taken to order e^2 must be μ dependent. This follows since C_7 is μ dependent while the matrix elements of Q_7 are not,[14] so that matrix elements of $C_7 Q_7$ are explicitly μ dependent. However, the total amplitude obtained from the effective Hamiltonian cannot depend on the renormalization point, whose choice is arbitrary. Therefore, the matrix elements of the remaining terms $\sum_1^6 C_i Q_i$ must somehow compensate for that of $C_7 Q_7$.

This fact can be seen most strikingly in the free-quark-model effective Hamiltonian of Eq. (23). Here only the operators Q_2 and Q_7 enter. The logarithmic coefficient of Q_7 is explicitly dependent upon μ, while matrix elements of the operator Q_7 itself are not. The opposite pertains to Q_2: Its coefficient is explicitly μ independent, and hence its matrix element must be μ dependent in just such a way as to exactly compensate for the coefficient of Q_7. Explicit calculation verifies this.

The same kind of situation occurs in the case of $\Delta S=1$ nonleptonic decays.[7] There the Wilson coefficient of the penguin operator Q_6 is explicitly μ dependent, but μ dependence in the matrix element of Q_6 as well as in the matrix elements of $\sum_1^5 C_i Q_i$ must compensate for that of C_6 in just such a way as to make the net physical amplitude independent of the choice of renormalization point μ (at least if the Wilson coefficients are calculated exactly). On top of this, the accuracy of the vacuum-insertion approximation, which is usually used to evaluate the matrix elements, must be μ dependent since it separates a renormalized local four-quark operator into a product of re-

normalized quark bilinears. Consequently, both in our previous application of the $\Delta S = 1$ nonleptonic effective Hamiltonian[7] to $K \to \pi\pi$ decays and in the present paper, we are wary[15] of making quantitative predictions using this method for evaluating the matrix elements of $Q_1, \ldots, Q_6$.

If we stick to just Q_7, whose matrix element (but not Wilson coefficient) is unambiguous, we see from Eqs. (28) and (32) that

$$\frac{A(K_L \to \pi^0 e^+ e^-)}{A(K^+ \to \pi^+ e^+ e^-)} = -\left(\epsilon - i\xi + i\frac{\mathrm{Im}C_7}{\mathrm{Re}C_7}\right). \tag{34}$$

Since $\mathrm{Im}C_7$ arises from $\mathrm{Im}\tau = -s_2 s_3 c_2 \sin\delta$, we see that Table I has $\mathrm{Im}C_7/\mathrm{Re}C_7 = C s_2 s_3 c_2 \sin\delta$, with C a positive number with a magnitude roughly in the range one to four. The sign of C is opposite to that in the free-quark model, which was used previously to estimate it.[6] In the six-quark model the K^0-$\bar{K}^0$ mass matrix gives a contribution to ϵ of roughly[9] (3 to 6)$\times s_2 s_3 c_2 \sin\delta e^{i\pi/4}$, so that the ϵ and $i\,\mathrm{Im}C_7/\mathrm{Re}C_7$ terms in Eq. (34) give comparable contributions to the net CP-violating amplitude and $\mathrm{Im}\epsilon$ interferes constructively with $\mathrm{Im}C_7/\mathrm{Re}C_7$ and $-\xi$ when θ_3 can be treated as a small quantity.

The branching ratio for $K_L \to \pi^0 e^+ e^-$ may be calculated from

$$\begin{aligned} B(K_L \to \pi^0 e^+ e^-) &= \frac{\Gamma(K_L \to \pi^0 e^+ e^-)}{\Gamma(K_L \to \mathrm{all})} \\ &= \frac{\Gamma(K^+ \to \mathrm{all})}{\Gamma(K_L \to \mathrm{all})}\,\frac{\Gamma(K^+ \to \pi^+ e^+ e^-)}{\Gamma(K^+ \to \mathrm{all})} \\ &\quad \times \frac{\Gamma(K_L \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^0 e^+ e^-)}, \end{aligned} \tag{35}$$

with the first two factors on the right-hand side taken directly from experiment.[2,16] To obtain a crude estimate, the ratio $\Gamma(K_L \to \pi^0 e^+ e^-)/\Gamma(K^+ \to \pi^+ e^+ e^-)$ is assumed to be as the ratio of contributions from $C_7 Q_7$ in Eq. (34). These assumptions yield

$$B(K_L \to \pi^0 e^+ e^-) \approx 0.5 \times 10^{-11} \left| \frac{\epsilon - i\xi + i\,\mathrm{Im}C_7/\mathrm{Re}C_7}{\epsilon} \right|^2. \tag{36}$$

The final factor on the right-hand side of Eq. (36) seems unlikely to gain us much more than a factor of 2. This is very far below the recent upper limit[17]

$$B(K_L \to \pi^0 e^+ e^-) \leq 2.3 \times 10^{-6},$$

and, moreover, at a level where it seems likely that the CP-conserving decay process through two intermediate photons, $K_L \to \pi^0\gamma\gamma \to \pi^0 e^+ e^-$, also contributes. In addition there are CP-violating contributions from the matrix elements of $Q_1, \ldots, Q_6$ which must be added to that of Q_7 to get the full $K_L^0 \to \pi^0 e^+ e^-$ amplitude. It would seem that the change in sign of $\mathrm{Im}C_7/\mathrm{Re}C_7$ due to QCD corrections will remain a theoretical curiosity incapable of being checked experimentally.

ACKNOWLEDGMENTS

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APPENDIX A

Here we derive the effective Hamiltonian for $K \to \pi e^+ e^-$ in the four-quark model when the mixing with penguin-type four-quark operators is neglected. The purpose of this appendix is to compare the result of the method used in Ref. 4 to that of the method used in Sec. II in a situation where analytic expressions can be stated in each case.

After the W boson is treated as heavy and removed from explicitly appearing in the theory, the resulting effective Hamiltonian density is

$$\mathcal{H}_{\mathrm{eff}} = \frac{-G_F}{2\sqrt{2}} \sin\theta_C \cos\theta_C \left\{ \left[\frac{\alpha_s(\mu^2)}{\alpha_s(M_W{}^2)}\right]^{-6/25} O_c^{(+)} + \left[\frac{\alpha_s(\mu^2)}{\alpha_s(M_W{}^2)}\right]^{+12/25} O_c^{(-)} \right\} + \mathrm{H.c.} \tag{A1}$$

The operators $O_c^{(\pm)}$ are defined in Eq. (3). The matrix elements of this Hamiltonian are to be evaluated to order e^2 in the electromagnetic interactions.

The next step is to treat the charm quark as heavy and remove it from explicitly appearing in the theory. The matrix elements of $O_c^{(\pm)}$ may be expanded in terms of matrix elements [evaluated in an effective three-quark theory with coupling $g'(m_c/\mu, g)$] of operators not explicitly involving the heavy-charm-quark field. When the mixing with penguin-type operators is neglected, this expansion has the form

$$\begin{aligned} \langle | O_c^{(\pm)} | \rangle &= L_+^{(\pm)}\left(\frac{m_c}{\mu}, g\right) \langle | O_+ | \rangle' \\ &\quad + L_-^{(\pm)}\left(\frac{m_c}{\mu}, g\right) \langle | O_- | \rangle' \\ &\quad + L_7^{(\pm)}\left(\frac{m_c}{\mu}, g\right) \langle | O_7 | \rangle', \end{aligned} \tag{A2}$$

where

$$\begin{aligned} O_\pm &= (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \\ &\quad \pm (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A} \end{aligned} \tag{A3}$$

and, using the definition of Ref. 4,

$$O_7 = \left(\frac{e^2}{4\pi}\right)(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{e}e)_V \,. \tag{A4}$$

The primed matrix elements in Eq. (A2) are evaluated in an effective three-quark theory with coupling g'. The factor of $e^2/4\pi$ is inserted into the definition of O_7 so that if we work to the lowest possible order in the electromagnetic interactions the operators O_j, $j \in \{+,-,7\}$ undergo a renormalization

$$O_j^{(0)} = \sum_i Z_{ji} O_i \,, \tag{A5}$$

where the Z_{ji} are independent of the electromagnetic coupling e. A simple calculation of the renormalization of the operators O_i at the one-loop level gives[18]

$$Z_{ij}(g'^2) = \begin{bmatrix} 1 - 2g'^2/16\pi^2\epsilon & 0 & 8/9\pi\epsilon \\ 0 & 1 + 4g'^2/16\pi^2\epsilon & -4/9\pi\epsilon \\ 0 & 0 & \mu^{2\epsilon} \end{bmatrix}. \tag{A6}$$

The factor of $\mu^{2\epsilon}$ arises from the relation $e_0^2 = \mu^{2\epsilon}e^2$. From (A6)

$$\mu\frac{d}{d\mu}Z = \begin{bmatrix} 4g'^2/16\pi^2 & 0 & 0 \\ 0 & -8g'^2/16\pi^2 & 0 \\ 0 & 0 & 2\epsilon\mu^{2\epsilon} \end{bmatrix}. \tag{A7}$$

The term proportional to ϵ is not negligible since it combines with terms of order $1/\epsilon$ in Z^{-1} to give an anomalous-dimension matrix

$$\gamma'_{ij}(g') = \sum_k Z^{-1}{}_{ik}\mu\frac{d}{d\mu}Z_{kj}$$

$$= \begin{bmatrix} 2g'^2/8\pi^2 & 0 & -16/9\pi \\ 0 & -4g'^2/8\pi^2 & 8/9\pi \\ 0 & 0 & 0 \end{bmatrix}. \tag{A8}$$

The Wilson coefficients $L_j^{(\pm)}(m_c/\mu, g)$ satisfy the renormalization-group equations

$$\sum_j \left[\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_c(g)m_c\frac{\partial}{\partial m_c} + \gamma^{(\pm)}(g)\right)\delta_{ij} - \gamma'^T_{ij}(g')\right]L_j^{(\pm)}\left(\frac{m_c}{\mu}, g\right) = 0 \,. \tag{A9}$$

The $\gamma^{(\pm)}(g)$ are the anomalous dimensions for the operators $O_\pm$ and were calculated in Ref. 19. The renormalization-group equations (A9) may be solved with the aid of a running coupling constant $\bar{g}(y,g)$ defined by

$$\ln y = \int_g^{\bar{g}(y,g)} dx\left[\frac{1-\gamma_c(x)}{\beta(x)}\right], \quad \bar{g}(1,g) = g \,. \tag{A10}$$

Because of the structure of the matrix $\gamma'(g')$ in Eq. (A8), the renormalization-group equations for $L_+^{(\pm)}$ and $L_-^{(\pm)}$ relate them only to themselves and they have a standard solution.[9] The equation for $L_7^{(\pm)}$ relates it to $L_+^{(\pm)}$, $L_-^{(\pm)}$, and itself, and has the solution

$$\begin{aligned} L_7^{(\pm)}\left(\frac{m_c}{\mu}, g\right) = {} & \left[\exp\int_g^{\bar{g}} dx\frac{\gamma^{(\pm)}(x)}{\beta(x)}\right] \\ & \times\left\{\int_{g'}^{g'(1,\bar{g})} dz\left(\frac{16}{9\pi\beta'(z)}\right)\left[\exp\int_z^{g'(1,\bar{g})} dx\left(\frac{-2x^2}{8\pi^2\beta'(x)}\right)\right]L_+^{(\pm)}(1,\bar{g})\right. \\ & \left. + \int_{g'}^{g'(1,\bar{g})} dz\left(\frac{-8}{9\pi\beta'(z)}\right)\left[\exp\int_z^{g'(1,\bar{g})} dx\left(\frac{4x^2}{8\pi^2\beta'(x)}\right)\right]L_-^{(\pm)}(1,\bar{g}) + L_7^{(\pm)}(1,g)\right\}, \end{aligned} \tag{A11}$$

where $\bar{g} = \bar{g}(m_c/\mu, g)$ and $g' = g'(m_c/\mu, g)$. Now defining

$$a^{(+)} \equiv \tfrac{6}{25}, \quad a^{(-)} \equiv -\tfrac{12}{25} \,, \tag{A12}$$

and utilizing

$$\beta(x) = \frac{-25}{48\pi^2}x^3 + O(x^5), \quad \beta'(x) = \frac{-27}{48\pi^2}x^3 + O(x^5) \quad \text{and} \quad g'(1,\bar{g}) \approx \bar{g} \,, \tag{A13}$$

the solution Eq. (A11) becomes

$$\begin{aligned} L_7^{(\pm)}\left(\frac{m_c}{\mu}, g\right) = {} & \left[\frac{\alpha_s(m_c^2)}{\alpha_s(\mu^2)}\right]^{-a^{(\pm)}}\left\{\left(\frac{2^7\pi}{3^4}\right)\left[-\int_{g'}^{\bar{g}} dz\, 2z^{-3-12/27}\bar{g}^{12/27}L_+^{(\pm)}(1,g)\right.\right. \\ & \left.\left. + \int_{g'}^{\bar{g}} dz\, z^{-3+24/27}\bar{g}^{-24/27}L_-^{(\pm)}(1,\bar{g})\right] + L_7^{(\pm)}(1,\bar{g})\right\}. \end{aligned} \tag{A14}$$

Noting that the $L_j^{(\pm)}(1,g)$ may be replaced by their free-field values (see Sec. II) in a leading-logarithmic calculation and using the solutions to the renormalization-group equations for $L_+^{(\pm)}$ and $L_-^{(\pm)}$ gives

$$\mathcal{H}_{\text{eff}}=\frac{-G_F}{2\sqrt{2}}\sin\theta_C\cos\theta_C\left(\left(\frac{\alpha_s'(\mu^2)}{\alpha_s(m_c^2)}\right)^{-6/27}\left(\frac{\alpha_s(m_c^2)}{\alpha_s(M_W^2)}\right)^{-6/25}O_+ + \left(\frac{\alpha_s'(\mu^2)}{\alpha_s(m_c^2)}\right)^{12/27}\left(\frac{\alpha_s(m_c^2)}{\alpha_s(M_W^2)}\right)^{12/25}O_-\right.$$
$$+\frac{32}{9\alpha_s(m_c^2)}\left\{\frac{1}{11}\left[1-\left(\frac{\alpha_s'(\mu^2)}{\alpha_s(m_c^2)}\right)^{-33/27}\right]\left(\frac{\alpha_s(m_c^2)}{\alpha_s(M_W^2)}\right)^{-6/25}\right.$$
$$\left.\left.-\frac{1}{10}\left[1-\left(\frac{\alpha_s'(\mu^2)}{\alpha_s(m_c^2)}\right)^{-15/27}\right]\left(\frac{\alpha_s(m_c^2)}{\alpha_s(M_W^2)}\right)^{12/25}\right\}O_7\right)+\text{H.c.} \tag{A15}$$

as the effective Hamiltonian density in the light-three-quark sector. This is the result in closed form derived starting from the definition of O_7 in Eq. (A4), which is similar to that in Ref. 4.

To perform the expansion in Eq. (A2) in a manner similar to that used in Sec. II we use an operator O_7' defined by

$$O_7'=\left(\frac{e}{g'}\right)^2(\bar{s}_\alpha d_\alpha)_{V-A}(e\bar{e})_V\,. \tag{A16}$$

This has the advantage of giving an anomalous-dimension matrix proportional to the coupling squared, for when O_7 is given by (A16)

$$\gamma_{ij}'(g')=\frac{g'^2}{8\pi^2}\begin{bmatrix}2 & 0 & -\frac{32}{9}\\ 0 & -4 & \frac{16}{9}\\ 0 & 0 & -\frac{27}{3}\end{bmatrix}. \tag{A17}$$

In this case the renormalization-group equations for the coefficients $L_j(m_c/\mu,g)$ are still given by Eq. (A9), but now with the anomalous-dimension matrix in Eq. (A17). The equations and solutions for $L_+^{(\pm)}$ and $L_-^{(\pm)}$ are as before. For $L_7^{(\pm)}(m_c/\mu,g)$ the solution is now

$$L_7^{(\pm)}\left(\frac{m_c}{\mu},g\right)=\left[\exp\int_g^{\bar{g}}dx\,\frac{\gamma^{(\pm)}(x)}{\beta(x)}\right]\left[\exp\int_{g'}^{g'(1,\bar{g})}dx\,\frac{+27x^2}{24\pi^2\beta'(x)}\right]$$
$$\times\left\{\int_{g'}^{g'(1,\bar{g})}dz\left(\frac{4z^2}{9\pi^2\beta'(z)}\right)\left[\exp\int_z^{g'(1,\bar{g})}dx\left(\frac{-33x^2}{24\pi^2\beta'(x)}\right)\right]L_+^{(\pm)}(1,\bar{g})\right.$$
$$\left.+\int_{g'}^{g'(1,\bar{g})}dz\left(\frac{-2z^2}{9\pi^2\beta'(z)}\right)\left[\exp\int_z^{g'(1,\bar{g})}dx\left(\frac{-15x^2}{24\pi^2\beta'(x)}\right)\right]L_-^{(\pm)}(1,\bar{g})+L_7^{(\pm)}(1,\bar{g})\right\}. \tag{A18}$$

Using the perturbation expansions for β, β', and $\gamma^{(\pm)}$ it is easy to show that Eq. (A18) together with the solution for $L_+^{(\pm)}(m_c/\mu,g)$ and $L_-^{(\pm)}(m_c/\mu,g)$ leads to an effective Hamiltonian density which is identical with that [in Eq. (A15)] derived starting with the other definition of O_7. Thus, although the anomalous-dimension matrices which correspond to the two definitions of O_7 are quite different, the effective Hamiltonian density in the light-three-quark sector is independent of the way O_7 is defined.

Finally we note that the results derived in this appendix are equivalent to those in Ref. 3. This is most easily seen by converting Eq. (A14) to an integration over the momentum variable q^2 through the substitution

$$z=[4\pi\alpha_s(q^2)]^{1/2}\,, \tag{A19}$$

with

$$\alpha_s(q^2)=\frac{12\pi}{27\ln(q^2/\Lambda^2)}\,.$$

APPENDIX B

In this section numerical results are given for various quantities which play a role in deriving the effective Hamiltonian for $K\to\pi e^+e^-$.

When the t quark is treated as heavy, and removed from explicitly appearing in the theory, a straightforward calculation of the renormalization of the operators $\{O_1,\ldots,O_7'\}$ [defined in Eqs. (10) and (12)] at the one-loop level gives an anomalous-dimension matrix

$$\gamma'_{ij}(g') = \frac{g'^2}{8\pi^2} \begin{bmatrix} -1 & 3 & 0 & 0 & 0 & 0 & -\frac{8}{3} \\ 3 & -1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & -\frac{8}{9} \\ 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & -\frac{4}{3} \\ 0 & 0 & \frac{22}{9} & \frac{2}{3} & -\frac{5}{9} & \frac{5}{3} & -\frac{4}{9} \\ 0 & 0 & 0 & 0 & 1 & -3 & -\frac{4}{3} \\ 0 & 0 & -\frac{5}{9} & \frac{5}{3} & -\frac{5}{9} & -\frac{19}{3} & -\frac{4}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{23}{3} \end{bmatrix} + O(g'^4) . \tag{B1}$$

The element $\gamma'_{77}(g')$ arises because the coupling constant in the definition of O'_7 [see Eq. (12)] gets renormalized. The matrix $\gamma'^T_{ij}(g')$ can be diagonalized by the transformation

$$\sum_{kl} V^{-1}{}_{il}\gamma'^T_{lk}(g')V_{kj} = \delta_{ij}\gamma'_j(g') , \tag{B2}$$

where

$$V_{kj} = \begin{bmatrix} 0 & 0 & 0.695\,89 & 0 & 0 & -0.706\,58 & 0 \\ 0 & 0 & -0.695\,89 & 0 & 0 & -0.706\,58 & 0 \\ 0 & -0.202\,36 & -0.231\,96 & 0.959\,85 & 0.171\,32 & 0.100\,94 & -0.402\,26 \\ 0 & 0.281\,03 & 0.231\,96 & -0.830\,58 & 0.083\,375 & 0.100\,94 & -0.776\,72 \\ 0 & -0.044\,316 & 0 & -0.079\,869 & 0.964\,45 & 0 & 0.313\,09 \\ 0 & -0.829\,89 & 0 & -0.163\,34 & -0.354\,39 & 0 & -0.264\,31 \\ 1 & 0.742\,68 & -0.281\,17 & -0.165\,37 & -0.158\,7 & 0.241\,33 & 0.053\,852 \end{bmatrix} \tag{B3}$$

and

$$\gamma'_j(g') = \frac{g'^2}{8\pi^2} \begin{bmatrix} -7.6667 \\ -6.8954 \\ -4 \\ -3.2429 \\ 1.1166 \\ 2 \\ 3.1327 \end{bmatrix} + O(g'^4) . \tag{B4}$$

Note that the last six of these eigenvalues are the same as those which occur in the diagonalization of the anomalous-dimension matrix in Eq. (A7) of Ref. 7, where the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays was discussed.

When the b quark is treated as heavy the renormalization of the operators $\{P_1, \ldots, P'_7\}$, where

$$\begin{aligned} &P_1 = (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A} , \quad P_2 = (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A} , \quad P_3 = (\bar{s}_\alpha d_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V-A} + \cdots + (\bar{c}_\beta c_\beta)_{V-A}] , \\ &P_4 = (\bar{s}_\alpha d_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V-A} + \cdots + (\bar{c}_\beta c_\alpha)_{V-A}] , \quad P_5 = (\bar{s}_\alpha d_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V+A} + \cdots + (\bar{c}_\beta c_\beta)_{V+A}] , \\ &P_6 = (\bar{s}_\alpha d_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V+A} + \cdots + (\bar{c}_\beta c_\alpha)_{V+A}] , \quad P'_7 = \left(\frac{e}{g''}\right)^2 (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{e}e)_V , \end{aligned} \tag{B5}$$

gives rise to the anomalous-dimension matrix

$$\gamma''_{ij}(g'')=\frac{g''^2}{8\pi^2}\begin{bmatrix} -1 & 3 & 0 & 0 & 0 & 0 & -\frac{8}{3} \\ 3 & -1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & -\frac{8}{9} \\ 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & -\frac{8}{3} \\ 0 & 0 & \frac{23}{9} & \frac{1}{3} & -\frac{4}{9} & \frac{4}{3} & -\frac{8}{9} \\ 0 & 0 & 0 & 0 & 1 & -3 & -\frac{8}{3} \\ 0 & 0 & -\frac{4}{9} & \frac{4}{3} & -\frac{4}{9} & -\frac{20}{3} & -\frac{8}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{25}{3} \end{bmatrix}+O(g''^4)\,. \tag{B6}$$

$\gamma''^T_{ij}(g'')$ is diagonalized by the transformation

$$\sum_{k,l} W^{-1}{}_{il}\gamma''^T_{lk}(g'')W_{kj}=\delta_{ij}\gamma''_j(g'')\,, \tag{B7}$$

where

$$W_{kj}=\begin{bmatrix} 0 & 0 & 0.655\,8 & 0 & 0 & 0.706\,43 & 0 \\ 0 & 0 & -0.655\,8 & 0 & 0 & 0.706\,43 & 0 \\ 0 & 0.144\,52 & -0.327\,9 & -0.780\,05 & 0.141\,4 & -0.117\,74 & -0.675\,61 \\ 0 & -0.202\,98 & 0.327\,9 & 0.712\,36 & 0.067\,442 & -0.117\,74 & -1.146\,8 \\ 0 & 0.032\,867 & 0 & 0.041\,364 & 0.967\,75 & 0 & 0.429\,31 \\ 0 & 0.725\,49 & 0 & 0.096\,564 & -0.350\,21 & 0 & -0.341\,87 \\ 1 & -0.726\,43 & -0.134\,52 & 0.258\,84 & -0.286\,98 & -0.202\,56 & 0.176\,41 \end{bmatrix} \tag{B8}$$

and

$$\gamma''_j(g'')=\frac{g''^2}{8\pi^2}\begin{bmatrix} -8.333\,3 \\ -7.042\,8 \\ -4 \\ -3.501 \\ 1.097\,4 \\ 2 \\ 2.890\,9 \end{bmatrix}+O(g''^4)\,. \tag{B9}$$

When the charm quark is treated as heavy and removed from explicitly appearing in the theory only the six operators Q_1, Q_2, Q_3, Q_5, Q_6, and Q'_7 ($Q'_7=[1/\alpha'''_s(\mu^2)]Q_7$) defined in Eq. (22) are required. Calculating their renormalization at the one-loop level gives the anomalous-dimension matrix

$$\gamma'''_{ij}(g''')=\frac{g'''^2}{8\pi^2}\begin{bmatrix} -1 & 3 & 0 & 0 & 0 & -\frac{8}{3} \\ \frac{8}{3} & -\frac{2}{3} & \frac{2}{9} & -\frac{1}{9} & \frac{1}{3} & -\frac{8}{9} \\ -\frac{11}{3} & \frac{11}{3} & \frac{22}{9} & -\frac{2}{9} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 \\ -1 & 1 & \frac{2}{3} & -\frac{1}{3} & -7 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{27}{3} \end{bmatrix}+O(g'''^4)\,. \tag{B10}$$

The matrix $\gamma'''^T_{ij}(g'')$ is diagonalized by the transformation

$$\sum_{k,l} X^{-1}{}_{il}\gamma'''^T_{lk}(g''')X_{kj}=\delta_{ij}\gamma'''_j(g''')\,, \tag{B11}$$

where

$$X_{kj} = \begin{bmatrix} 0 & 0.175\,24 & -0.695\,47 & -0.051\,101 & -0.812\,29 & 0.734\,89 \\ 0 & -0.175\,24 & 0.695\,47 & 0.051\,101 & -0.541\,53 & -0.734\,89 \\ 0 & -0.052\,12 & -0.030\,131 & 0.160\,7 & 0.270\,76 & -1.221\,2 \\ 0 & 0.029\,23 & 0.018\,233 & 0.975\,15 & 0 & 0.247\,09 \\ 0 & 0.814\,15 & 0.048\,407 & -0.346\,86 & 0 & -0.186 \\ 1 & -0.175\,23 & 0.235\,77 & +0.009\,015\,9 & 0.240\,68 & -0.111\,86 \end{bmatrix} \tag{B12}$$

and

$$\gamma_j'''(g''') = \frac{g'''^2}{8\pi^2} \begin{bmatrix} -9 \\ -7.2221 \\ -3.7559 \\ 1.0761 \\ 2 \\ 2.6797 \end{bmatrix} + O(g'''^4)\,. \tag{B13}$$

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[11]A similar situation occurs in the weak radiative decays of hyperons. There, diagrams at the one-loop level give rise to an effective magnetic-moment $sd\gamma$ vertex which is suppressed by a factor of $(m_c/m_w)^2$ compared to other contributions which come from the matrix elements of the operators $Q_1, \ldots, Q_6$ evaluated to order e in the electromagnetic interactions (see Ref. 1). Diagrams beyond the one-loop level induce a magnetic-moment operator in the effective Hamiltonian for weak radiative decays which is not suppressed by a factor of $(m_c/m_w)^2$. However, its Wilson coefficient as calculated by M. A. Shifman *et al.* [Phys. Rev. D 18, 2583 (1978)], is much too small to explain the observed $\Sigma^+ \to p\gamma$ decay width. In addition, it has been shown by F. J. Gilman and M. B. Wise [Phys. Rev. D 19, 976 (1979)] that an effective $sd\gamma$ vertex of arbitrary strength by itself is inconsistent with the present experimental data on the weak radiative decays of hyperons.

[12]In general, a given diagram in perturbation theory will contribute to both the Wilson coefficients C_j and to the matrix elements of the local four-fermion operators Q_j. For example, the electromagnetic penguin-type diagram with u, c, and t quarks in the loop shown in Fig. 1 contributes to the Wilson coefficient C_7 but it also gives contributions to the matrix elements of $Q_1, \ldots, Q_6$. Exactly how much of the diagram goes into C_7 and how much goes into the matrix elements of $Q_1, \ldots, Q_6$ is determined by the value of the renormalization point μ.

[13]W. R. Molzon *et al.*, Phys. Rev. Lett. 41, 1213 (1978).

[14]Although the hadronic part of Q_7 is a composite operator involving two quark fields at the same point, it does not require renormalization since it is a partially conserved current. Thus, its matrix elements are not μ dependent.

[15]In $\Delta S = 1$ weak nonleptonic decays, if the vacuum-insertion approximation is used to evaluate matrix elements then it appears that, with the Wilson coefficients calculated in Ref. 7, one does not account for the full $K \to \pi\pi$ amplitude. One can attribute this failure either to uncertainties in the matrix elements or to uncertainties in the real part of the Wilson coefficient for Q_6. The first approach was adopted in Refs. 7 and 8 together with the assumption that most of the $K \to \pi\pi$ amplitude is coming from matrix elements of Q_6. The second approach has been used by V. V. Prokhorov, Yad. Fiz. 30, 1111 (1979); B. Guberina and R. D. Peccei, Nucl. Phys. B163, 289 (1980). The latter approach generally leads to smaller values for the CP-violation parameter ϵ' than does the former.

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[18]This is the renormalization in the minimal subtraction scheme. As mentioned in Sec. II we are working in a closely related scheme where extra pieces not proportional to $1/\epsilon$ are put into the "infinite" part. However, such pieces give higher-order (in g'^2) contributions to $\gamma'(g')$ and can be neglected in the leading-logarithmic approximation.

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