Abstract

Far-field noise radiated from coaxial cold underexpanded jet flows issuing from convergent two-nozzle configurations with coplanar and staggered-exits is investigated experimentally. The coaxial jets are operated in the "inverted" mode, i.e., the outer (annular) jet flow Mach number is higher than that of the inner (round) jet. Keeping all other geometrical and operating conditions the same, the exit-stagger of the inner (round) and the outer (annular) nozzles was varied. It is shown that the extent of the exit-stagger affects both the flows and the radiated noise from such coaxial underexpanded jet flows and that comparatively, the lowest noise levels are achieved when the coaxial nozzle-exits are coplanar. Moreover, the effectiveness of the co-flowing inner jet flow in reducing the noise radiated from either the annular or the coaxial underexpanded jet flows decreases noticeably as the exit-stagger is increased.

Nomenclature

- \( a \): Speed of sound
- \( A \): Area
- \( D \): Nozzle diameter
- \( f \): Frequency in Hz
- \( L \): Axial location downstream of the annular nozzle exit
- \( f_j \): Exit-stagger-length relative to the annular nozzle exit (Figs. 1 & 2)
- \( f_p \): The location of the intersection of the last Prandtl-Meyer expansion Mach wave with the outer surface of the extended inner nozzle
- \( M \): Flow Mach number
- \( m \): Mass flow rate
- \( oAPWL \): Overall acoustic power watt level, dB re: 10^-12 watts
- \( PWL \): Acoustic power watt level, dB re: 10^-12 watts
- \( P_R \): Reservoir pressure, psig
- \( P_o, P_a \): Ambient pressure, psia
- \( r \): Nozzle radius with reference to the central axis (Figs. 1 & 2)
- \( r_t \): Radial location of the pitot-tube with reference to the central axis
- \( SPL \): Sound pressure level, dB re: 2 x 10^-4 \( \mu \)bar
- \( t_e \): Lip Thickness at the inner nozzle-exit (Fig. 1)
- \( T \): Thrust

\[ T_R \]

Flow velocity calculated from thrust/mass flow rate

\[ V_j \]

Fully-expanded jet mean flow velocity calculated from P-M expansion of an underexpanded jet

\[ w \]

Exit width of the annular expansion (Figs. 1 & 2)

\[ W \]

Acoustic power in watts

\[ \beta \]

Angle between the direction of sound emission and the jet axis in the downstream direction

\[ \beta_j \]

Angle of divergence (spread) of free boundary of the jet flow

\[ \beta_a \]

Angle of the last Mach wave of the Prandtl-Meyer expansion with the outer surface of the extended inner nozzle

\[ \alpha \]

Angle of convergence of convergent nozzle (Figs. 1 & 2) and angle of divergence of convergent-divergent nozzle

\[ \rho_0 \]

Density of undisturbed medium

\[ \rho \]

Density of the flow

\[ \rho_f \]

Fully-expanded jet mean flow density

\[ \lambda \]

Density exponent

\[ \mu \]

Operating Pressure ratio,

\[ \mu = \frac{P_R + P_a}{P_a} \]

\[ \xi \]

International Standard Atmospheric Conditions

\[ j \]

Fully-expanded jet flow conditions

\[ e \]

Nozzle exit conditions

\[ o \text{ or } a \]

Ambient conditions

\[ t \]

Total pressure

\[ 1 \]

The inner nozzle or jet flow parameter

\[ 2 \]

The outer (annular) nozzle or jet flow parameter

Introduction

The noise radiated from a single round supersonic high specific thrust, turbulent heated jet flow, similar to the exhaust of a turbo-jet engine, is inherently too intense. Its reduction is possible if, instead, multi-stream coaxial jet exhaust flows are used to generate the same thrust. From considerations of the propulsion-efficiency and noise-suppression of jet-propelled subsonic aircraft, the use of the high by-pass ratio turbo-fan engine with its two-stream exhaust flow is preferred over an "equivalent" (generating the same thrust) single-stream heated turbo-jet exhaust. In this mode of operation of a turbo-fan engine exhaust, the inner heated turbo-jet flow of higher mean velocity is shrouded by an outer fan-flow stream of slower flow-velocity but of higher mass flow rate. However, to achieve the desired reduction in levels of noise radiated from underexpanded heated jet flows, this mode of operation of coaxial jet flows is not effective. Instead, the coaxial (or coannular) jet flows need to be operated such that the outer (annu-
lar) underexpanded jet is maintained at a comparatively higher pressure ratio (i.e., at a higher jet flow Mach number) and higher temperature and therefore at a higher specific thrust or flow velocity of the annular jet than that of the inner (round) jet flow. This mode of operation of the coaxial jets was first attempted by Dosanjh, Vu, and Abdelhamid 1,2,3 and was shown to embody many of the preferred attributes of an effective supersonic jet noise suppression approach. These earlier supersonic jet noise suppression studies with cold model jets were extended by Dosanjh, Ahuja and Bhutiani 4,7 to cold/heated coaxial model jets where the outer annular jet was heated to reservoir temperatures up to 1000°F and the inner jet was operated cold. It was shown that in comparison with an "equivalent" round underexpanded single jet of the same specific thrust, such use of coaxial jets results in substantial noise reductions. It has been shown by Bassioni and Dosanjh 8 that noise reductions from such coaxial jets are achieved over a wide range of operating pressure ratios (or jet flow Mach numbers) of the outer (annular) underexpanded jet, which is optimized by the potential of this approach for supersonic jet noise suppression and its possible applications to duct burning turbo-fan and advanced variable cycle turbo-jet engines, more recently the coaxial (co-annular) heated jets operated in such an "inverted" mode of operation have been investigated, among others, by Guiterrez 9, Lee 10 and Koszowski 11.

The basic reasons for the observed noise reductions from coaxial (or co-annular) jets with the "inverted" operating conditions, in general, are: (a) an annular underexpanded jet flow radiates lower levels of noise than an equivalent round jet of the same exit area and specific-thrust. Annular underexpanded jet flow decays to sonic speed more rapidly (i.e., over a shorter spatial distance downstream of the annular nozzle exit) than an "equivalent" round underexpanded jet. (b) Owing to the presence of the co-flowing inner (round) jet flow, the mean-flow shear at the inner jet boundary of the outer (annular) higher flow velocity jet is reduced just downstream of the nozzle exit. (c) The mixing, the spreading and the decaying characteristics of the two-stream coaxial high speed turbulent jet flows differ from those of an "equivalent" single round jet. (d) The repetitive wave structure of the individual (annular and round) underexpanded jets and the associated jet mean flows are modified due to the changed flow boundary conditions (i.e., both the pressure and flow directions are modified) at the interface of the co-flowing coaxial jets.

From investigations of the comparative acoustic performance of coaxial supersonic jets where some of these geometrical parameters were controlled, it was established that for maximizing the noise-reductions from coaxial supersonic jets, keeping the specific thrust the same, the use of the convergent underexpanded coaxial nozzles (as compared to the inverted or coannular convergent-divergent nozzles) is advantageous. 5 It was also shown that the noise reductions achieved by coaxial jets operated in the "inverted" mode are optimized (keeping all other operating conditions and factors the same) if the exit-areas of the outer (annular) and the inner (round) convergent nozzles are nearly equal. 5 Therefore from the considerations of the relative operating pressure ratios of the inner and outer jets and the nozzle exit areas commonly used for the coaxial jets in the "inverted" mode of operation, the ratio of the rate of mass flows of the outer to the inner jets, though higher than one, is not as high as normally used in the high by-pass ratio turbo-fan engines. Consequently, for the optimized noise reduction from the "inverted" mode of operation of coaxial jets, comparatively low by-pass ratios ought to be used.

From some earlier exploratory studies, the radiated noise levels from coaxial supersonic jets issuing from coaxial nozzles with staggered exits 1 were found to be higher than those from the coaxial jets issuing from coaxial nozzles with coplanar exits. Since the motivation of our earlier studies with coaxial jets operated in the "inverted" mode was to select the operating conditions and the coaxial flow configurations to optimize noise reductions from supersonic jet flows, the use of the coaxial nozzles with coplanar exits was preferred. 3,5,7 In more recent noise studies with "inverted" coannular jets undertaken by others in relation to the possible development of the duct-burning turbo-fan or advanced variable cycle turbo-jet engines, 10,11 coaxial (coannular) nozzle configurations with staggered exits seem to be preferred. Therefore a comparative study and an assessment of the acoustic performance of the "inverted" coaxial underexpanded jets issuing from coaxial convergent nozzles with either coplanar- or staggered-exit configurations, was undertaken.

Coaxial Nozzles, Experimental Facilities, Procedures and Data Reduction

Coaxial Nozzles

The typical model coaxial two-nozzle configuration with its nozzle exit areas, exit size (radius of the inner round nozzle, R_1 and the width of the annular nozzle W), exit-stagger L and other pertinent dimensions are shown in Fig. 1 and tabulated in Table 1. In the coaxial nozzle configuration shown in Fig. 1, both the inner (round) and the outer (annular) nozzles are convergent. The exit lip of the inner (round) nozzle is fairly sharp (thickness t_0 = 0.04 cm) with the ratio of the lip thickness to inner nozzle exit radius t_0 / R_1 = 0.06. To straighten the converging flow at each nozzle exit such that the jet flow emerges parallel to the nozzle axis, zero convergence angles are used over a short length (0.15 cm) just upstream of the individual nozzle exits (see Fig. 1). The ratio of the exit areas A_0 / A_2 = 1.17. The stagger length is between the outer (annular) and the inner (round) nozzle exit (or the ratio of stagger length to annular nozzle width, L/W) was
controlled by moving only the outer annular nozzle relative to the inner nozzle where in this nozzle arrangement (Fig. 1), the inner round nozzle is kept the same for all the coaxial-nozzle configurations with different exit-staggers. Because of the convergence angle \( \alpha = 12.83° \) of the outer surface of the inner convergent nozzle, moving the exit of the annular nozzle relative to the fixed inner nozzle therefore would have resulted in a different nozzle exit area \( A_{e2} \) for different exit staggers \( t/w \).

Therefore, four different end pieces of the outer convergent nozzle were used, each designed to keep the annular exit area \( A_{e2} \) and therefore \( A_{e2}/A_{e1} \) the same for each of the coaxial nozzle configurations when the exit staggers are varied in four steps between \( t/w = 0 \) to \( t/w = 2.7 \). However, because of the divergence angle \( \alpha = 12.83° \) of the outer surface of the inner round nozzle, the local wall thickness of the inner nozzle increases slightly upstream from the sharp lip of the inner nozzle exit. Therefore, when the exit staggers are varied from \( t/w = 0 \) to \( t/w = 2.7 \) in coaxial nozzle configuration shown in Fig. 1, even though area ratios \( A_{e2}/A_{e1} \) are kept fixed, the annular width decreased from 0.28 to 0.24 cm (Table 1).

To ensure that for different exit staggers \( t/w \), both the exit area ratios \( A_{e2}/A_{e1} \) and the annular nozzle width \( w \) are maintained constant, the model coaxial nozzle configuration shown in Fig. 2 was also used. The pertinent dimensions of this coaxial nozzle arrangement are tabulated in Table 2. The coaxial nozzles can be arranged to have either coplanar exits or staggered exits. In this coaxial nozzle arrangement, the inner (round)

and the outer (annular) nozzles are both convergent. The lip of the inner nozzle is fairly sharp, i.e., \( \ell_{e1}/\ell_{el} = 0.6 \) and is commonly designated as \( t/r = 0 \).

The inner nozzle is straight over a short length upstream of its exit. Therefore to adjust the nozzle exits for different \( t/w \)'s, the annular nozzle component is moved relative to the straight portion of the inner nozzle, thus keeping both the individual exit areas, their \( A_{e2}/A_{e1} \) as well as the annular nozzle width \( w \), the same for coaxial nozzle configurations with different exit-staggers.

In addition, the comparison of the acoustic performance of coplanar and a staggered-exit coaxial jets was also investigated when the inner (conical) nozzle was convergent-divergent with divergence angle \( \alpha = 12° \). Also, the role of the lip thickness at the exit of the inner nozzle was studied by using both a sharp lip \( (\ell_{e1}/\ell_{el} = 0) \) and a somewhat thicker lip \( (\ell_{e1}/\ell_{el} = 0.13) \) inner nozzle where either both nozzles were convergent or when the coaxial nozzle comprised an inner convergent-divergent nozzle and the outer (annular) convergent nozzle. The experimental results for these nozzle configurations are reported in Ref. 12. For these coaxial nozzle arrangements configurations operated in the "inverted" mode, the nature of the observed noise reductions and the influence of the exit-staggers is similar to the ones observed for convergent nozzles with sharp lips. Similar to the results from earlier studies with coplanar-exit configuration3 of coaxial jets, the maximum noise reductions observed with the finite lip thickness and staggered-exit configurations were somewhat higher than those attained with the sharp lip configuration. For the coaxial nozzle configuration with the inner convergent-divergent nozzle, the over-expanded mode of operation of the inner C-D nozzle involves some thrust loss and therefore the corresponding acoustic efficiency is somewhat higher and the use of conical convergent-divergent coaxial nozzles with either coplanar or staggered exits does not recommend itself. Therefore, here only the experimental results for coaxial jets operated in the "inverted" mode issuing from coaxial convergent nozzle configurations with staggered exits (varied from coplanar exits, \( t/w = 0 \) to \( t/w = 2.7 \)) shown in Fig. 1 are presented. For comparison, a few experimental observations of the noise and flow behavior of coaxial jets issuing from the coaxial nozzles with staggered-exit configurations shown in Fig. 2, are also included.

Optical Records and Flow Measurements

To record the general features of the individual underexpanded annular and round jet flows as well as the behavior of the coaxial jet flows at the "inverted" operating conditions, with different exit-staggers, the spark shadowgraphs of the coaxial flows were recorded using the coaxial nozzle configuration shown in Fig. 2.

To delineate the subsonic and supersonic regions of the combined coaxial jet flows, especially at the "minimum-noise" conditions as explained under Experimental Results, the stagnation pressures were surveyed by a pitot-tube (O.D. = 0.2" and I.D. = 0.1") traversed across and along the coaxial flows by a traversing mechanism capable of three dimensional displacements with an accuracy of .001". With the pitot-tube positioned at selected
Fig. 2 Details and Specifications of an Alternate Arrangement of Coaxial Convergent Two-Nozzle Arrangement with Staggered-Exits.

TABLE 2

Dimensions and Specifications of Coaxial Nozzles with Staggered-Exits Shown in Fig. 2:

<table>
<thead>
<tr>
<th>Exit</th>
<th>Annular Stagger Width</th>
<th>( \frac{k}{w} )</th>
<th>( \frac{r_{el}}{r_{e1}} )</th>
<th>( \frac{r_{e2}}{r_{el}} )</th>
<th>( A'_{e2} )</th>
<th>( A'_{el} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.25</td>
<td>0.61</td>
<td>0.88</td>
<td>0.06</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>0.32</td>
<td>0.25</td>
<td>1.26</td>
<td>0.61</td>
<td>0.88</td>
<td>0.06</td>
<td>1.09</td>
</tr>
<tr>
<td>0.48</td>
<td>0.25</td>
<td>1.9</td>
<td>0.61</td>
<td>0.88</td>
<td>0.06</td>
<td>1.09</td>
</tr>
</tbody>
</table>

locations in different regions of the coaxial flows, the corresponding shadowgraphs were also recorded. From the presence or absence of the standing shock ahead of the pitot-tube tip, it is possible to deduce whether the local flow is respectively supersonic or subsonic. From the shadowgraphic and pitot-tube data, the qualitative effects of the exit-stagger \( \frac{l}{w} \) (other operating conditions kept the same) on the wave structure, the turbulent mixing, spreading and decaying of the coaxial flows from the nozzle configuration shown in Fig. 2, were deduced as discussed under Experimental Studies. Such optical and flow studies of coaxial jets with staggered exits with convergent nozzle configuration shown in Fig. 1, are currently under way.

Acoustic Data and Analysis

One-third octave sound pressure levels were recorded in an anechoic chamber of dimensions 10' x 13' x 9', at eight azimuthal positions 15° apart, from 15° to 120° with respect to the downstream axis of the jet flow on an arc of 6 ft. radius. The acoustic data acquisition system mainly comprised a B&K 1/8 inch condenser microphone, 2619 cathode follower, type 2107 analyzers with adequately extended frequency range up to 200 KHz, and type 2307A level recorder. Some of the acoustic data for the alternate nozzle arrangement shown in Fig. 2 was digitized and recorded. These acoustic data were corrected for the microphone pressure response; the free-field effects and the corrections for the atmospheric absorption which were necessary particularly because of the high-frequency spectral content owing to the small size of the model nozzles used. The 1/3 octave OASPL's and the OASPL's were calculated from the corrected 1/3 octave SPL's data. The overall power watt levels (OAPWL's) were calculated from the OASPL's instead of the SPL's at each angle. The jet and acoustic facilities were calibrated and validated to ascertain that the upstream flow disturbances and the background noise did not influence the recorded acoustic data. The acoustic efficiencies \( \eta \) were calculated from the OAPWL's where to normalize OAPWL's and to obtain log \( \eta \), the calculated values of the thrust and mass flow rate of the coaxial jets, were used.

Experimental Results and Discussion

Minimum-Noise Mode of Operation

Optimization of noise reductions from coaxial nozzle configuration shown in Fig. 1, where both the inner (round) and the outer (annular) nozzles are convergent, was studied for four different exit-staggers varied from \( \frac{l}{w} = 0 \) (coplanar exits) to \( \frac{l}{w} = 1.26, 1.92 \) and 2.7. In each of these coaxial nozzle configurations, the outer (annular) jet reservoir pressure \( P_{R1} \) (or pressure ratio \( \xi_{1} \)) was kept fixed at a preselected value \( \xi_{1} = 3.04 \) such that the annular jet flow is underexpanded. The inner jet reservoir pressure \( P_{R1} \) ratio \( \xi_{1} \) was varied from \( \xi_{1} \) = 0 or \( \xi_{1} = 1 \) (i.e., the annular jet operated alone without the inner jet flow) to \( \xi_{1} = 2.5 \). The variation of the OAPWL of the coaxial jets with the inner jet reservoir pressure \( P_{R1} \) or \( \xi_{1} \) (keeping \( P_{R2} = 30 \) psig or \( \xi_{2} = 3.04 \) fixed) for each of the four exit-staggers is shown in Fig. 3. The radiated overall power watt level (OAPWL) for each set of the operating pressure ratios \( \xi_{1}, \xi_{2} \) is observed to decrease and attains a minimum value at an inner jet reservoir pressure \( P_{R1} \) or \( \xi_{1} \) which is less than the outer jet reservoir pressure \( \xi_{2} \). The set of the inner and the outer jet reservoir pressure ratios \( \xi_{2}, \xi_{1} \) with \( \xi_{2} > \xi_{1} \) (keeping all other parameters and the coaxial nozzle configuration the same) that results in the least OAPWL, is considered to constitute the "minimum-noise condition" for the given nozzle configuration and the fixed outer (annular) jet operating reservoir pressure ratio \( \xi_{2} \). This means that the optimization of the noise reductions occurs when the Mach number of the outer annular jet stream is higher than that of the inner jet flow: Since both jets are operated cold, the outer (annular) jet flow velocity \( V_{j2} \) therefore is also higher than the inner jet flow velocity \( V_{j1} \).

As the exit stagger \( \frac{l}{w} \) is increased from \( \frac{l}{w} = 0 \) to 2.7, the OAPWL's at the minimum-noise mode of operation of coaxial jets with different exit-staggers, vary differently. The variation in OAPWL with the increasing \( \xi_{1} \), is fairly smooth for the coplanar-exit (\( \frac{l}{w} = 0 \)) configuration and a well-defined minimum OAPWL occurs at \( \xi_{1} \approx 3.04 \); \( \xi_{1} \approx 1.82 \); \( V_{j2}/V_{j1} \approx 1.11 \). For exit-stagger \( \frac{l}{w} = 1.26 \), the minimum noise occurs at a higher \( \xi_{1} \approx 2.22 \); \( V_{j2}/V_{j1} \approx 1.14 \) and OAPWL is 2 dB higher than that for the coplanar-exit configuration. However, for \( \frac{l}{w} = 1.92 \), two minimum values occur at \( \xi_{1} \approx 1.82 \) and 2.22 respectively and the OAPWL's change sharply between these values of \( \xi_{1} \). The first minimum OAPWL is 3 dB higher than the co-planar
operation. For exit-stagger \( \xi / w = 2.70 \), the minimum OAPWL is achieved at \( \xi_1 = 1.82 \); \( V_{j2} / V_{j1} = 1.31 \) and it is 3.5 dB higher than that for the coplanar-exit configuration. At the minimum-noise mode of operation of coaxial jets (keeping all other factors the same), the noise reductions are therefore the highest with the coplanar exit configuration and progressively decrease with increasing exit-stagger. Furthermore, if each of these coaxial nozzle configurations (at fixed \( \xi_2 \)) were to be operated at the inner jet pressure ratio \( \xi_1 \) which is either much lower or higher than the \( \xi_1 \) at which the minimum OAPWL's are achieved, then OAPWL's from this off-minimum mode of operation of coaxial jets are noticeably higher.

**Acoustic Efficiency of Coaxial Jets with Staggered-Exits**

Each of the two co-flowing individual jet flows of the coaxial jets is thrust producing. If the OAPWL's from coaxial jets operated at the same annular jet pressure ratio \( \xi_2 \) but with different exit-staggers, minimized at different inner jet pressure ratios \( \xi_1 \), then the total thrust produced by the coaxial jets at each configuration with a different exit-stagger would be different. To interpret the variations in OAPWL's and the acoustic performance of the coaxial jets with different exit-staggers, the total thrust should be considered. Therefore the respective OAPWL's for different exit-staggers should be normalized to incorporate the total thrust produced by the coaxial jets by each of the sets of operating pressure ratios \( \xi_2, \xi_1 \).

The "acoustic efficiency" is defined as the ratio of the radiated acoustic power \( W \) in watts to the mechanical power in watts. For a free jet, the mechanical power \( W = T \times V \) where \( V \) is the flow velocity and \( T \) is the thrust.

The acoustic efficiency

\[
\eta = \frac{W}{\frac{\xi_2}{m} \times \text{constant}}
\]

The constant is required to convert the units of the term \( \frac{\xi_2}{m} \) into watts. Therefore the acoustic efficiency determines the amount of sound radiated from any jet per unit mechanical power. The acoustic efficiency may be expressed as

\[
\log_{10} \eta = \frac{\text{OAPWL}}{10} - \log_{10} \frac{\xi_2}{10} - 13.67
\]

where \( w_{\text{ref}} = 10^{-12} \) watts, and thrust \( T \) is given in \( \text{lbf} \) and \( \dot{m} \) is \( 1 \text{hmv/sec} \).

Thus, for each mode of operation of coaxial jets, the acoustic efficiency at every value of the inner jet operating pressure ratio \( \xi_1 \) (for a given outer jet pressure ratio \( \xi_2 \)) can be calculated using the OAPWL's, the calculated total mass flow rate, and the calculated total thrust (neglecting some of the likely thrust losses). In Fig. 4, the log of the acoustic efficiency versus the inner jet reservoir pressure \( P_{R1} \) or pressure ratio \( \xi_1 \) for two exit-staggers (\( \xi / w = 0 \) and \( \xi / w = 2.70 \)) of the coaxial jets, is plotted. The acoustic efficiency is noticeably lower (i.e., the radiated noise levels are lower) for the coplanar-exit \( (\xi / w = 0) \) configuration than that for the staggered-exit \( (\xi / w = 2.7) \) configuration of the coaxial jets.

The typical spectral behavior of the sound pressure levels (SPL's) at azimuthal angles \( \theta = 30^\circ \) and \( \theta = 90^\circ \); the power watt levels (PWL's) and the overall sound pressure levels (OASPL's) for the coaxial jet \( (\xi / w = 0) \) and the staggered-exit \( (\xi / w = 2.7) \), configurations at their respective minimum-noise mode of operation as explained above are compared in Figs. 5, 6 and 7 respectively. The acoustic spectral levels (the SPL's and PWL's) of the coaxial jets with coplanar-exit are lower than those for the staggered-exit. Such comparison of SPL at \( \theta = 90^\circ \) (Fig. 5b) shows that for the coplanar
exit-configuration operated at the minimum-noise condition, at higher frequency bands (10^5 to 0.5 x 10^5) the SPL's for the staggered-exit l/w = 2.7, on the average, are 7 dB higher than those for the coplanar configuration. At θ = 30° (Fig. 5a), the SPL's for the staggered exit operation are higher at lower frequency bands (2 x 10^2 to 10^4). There is greater evidence of sharp fluctuations in the acoustic spectra for the staggered-exit configuration, indicating stronger shock structure in such coaxial flows. The comparison of OASPL's also shows that the acoustic levels for the coplanar configuration are lower at all azimuthal angles between θ = 15 to 120° where at θ = 90°, this difference in OASPL is about 6 dB. Therefore the side-line noise of coaxial jets in the minimum noise mode with coplanar-exit configuration is relatively lower than that of the staggered-exit operation.

Annular Underexpanded Jet Operated Alone

The influence of the exit-stagger on the acoustic performance of the annular jet alone (i.e., no inner jet flow) operated at ξ_2 = 3.04 (jet flow Mach number M_2 = 1.37) is shown in Fig. 8 where the 1/3 octave PWL and OASPL are compared for the coplanar l/w = 0 and the staggered-exit l/w = 2.7 configurations.

When the outer (annular) jet is operated alone, OAPWL's for the configuration with coplanar exits are higher than those for staggered-exit configurations (Fig. 3) where the difference between the l/w = 0 and l/w = 2.7 configurations is about 2 dB. One-third octave PWL's spectra for the coplanar and staggered-exit configurations of the annular jets alone, show that over the frequency range (2 x 10^3 to .8 x 10^5), the radiated power watt levels for coplanar-exit configuration are 1 to 2 dB higher than those for the staggered-exit configuration with l/w = 2.7 (Fig. 8a). At lower frequencies (0.5 to 2 KHz), PWL's for the staggered-exits are higher. However, the decrease from the peak values is so sharp that this part of the spectrum does not contribute significantly to the calculated OAPWL.

From the directivity plots of OASPL's (Fig. 8b), it can be seen that the differences in OASPL's between the coplanar configuration l/w = 0 and l/w = 2.7 is small (1.5 dB). The OAPWL peak occurs at 30° for both configurations. OASPL's for the annular jet alone with staggered-exits at 75° is higher and at 90° is lower than the corresponding OASPL's for the coplanar configuration.

Keeping all other factors the same except for a slight reduction in annular nozzle width for staggered-exits as noted before, the staggered-exit operation of an annular jet alone yields somewhat lower noise levels than its operation with a coplanar exit.

The observed differences in the annular jet underexpanded jet flows from the annular nozzles with coplanar and staggered-exit configurations respectively are noted under "Flow Features". The extended surface of the inner nozzle downstream of the annular nozzle exit and the appearance of the additional lip-base oblique shock at the staggered-
Fig. 5(b) Comparison of the 1/3 Octave Sound Pressure Levels from Coaxial Jets with Coplanar and Staggered-Exit Configurations. \( \theta = 90^\circ \); Minimum-Noise Mode of Operation.

\[
\begin{align*}
\text{FREQUENCY HZ} & \quad 2 \times 10^3 \\
\text{SPL dB re: } 2 \times 10^4 \text{ pa (eq.))} & \quad 105.0 \\
\theta = 90^\circ & \quad 90.0 \\
\theta = 90^\circ & \quad 85.0 \\
\theta = 90^\circ & \quad 80.0 \\
\theta = 90^\circ & \quad 75.0 \\
\theta = 90^\circ & \quad 70.0 \\
\theta = 90^\circ & \quad 65.0 \\
\end{align*}
\]

Flow Features

As stated before under "Optical Records and Flow Measurements", spark shadowgraphs of the exit of the inner nozzle modifies the annular underexpanded annular free jet flows. The noise reductions in the annular jet flow with staggered-exits as compared to those from the annular jet flows with coplanar-exits by themselves are rather modest. The presence of the co-flowing inner jet flow when used at the minimum noise mode of operation of coaxial jets, however, plays relatively a more dominant role in affecting the substantial noise reductions observed from coaxial jets with coplanar exits (Fig. 3). If the stagger-length is relatively large, then the role of the inner jet flow in further reducing the noise levels from the annular jet flow alone becomes progressively less effective and OAPWL's of the coaxial jets essentially approach those of the annular jet alone.

Additional Studies on the Role of the Exit-Stagger

Similar noise studies were conducted with coaxial convergent nozzles with staggered-exits of an alternate arrangement shown in Fig. 2. The dimensions and specifications of the coaxial nozzles are listed in Table 2. Different exit-staggers \( k/w \) were achieved by moving the outer annular nozzle relative to the inner nozzle. Since the end portion of the outer surface of the inner nozzle was designed to be straight, i.e., the convergence angle \( \alpha = 0 \) as shown in Fig. 2, for different exit-staggers, both the individual nozzle exit areas \( (A_{22}, A_{21}) \) and the width \( w \) of the annulus at the exit of the outer annular nozzle were maintained the same.

The detailed experimental results of this investigation are available in Ref. 12. Here only the comparison of OAPWL's, 1/3 octave PWL's and OASPL's for the coplanar \( (k/w = 0) \) and the staggered-exit configuration \( k/w = 1.9 \) is shown in Figs. 9, 10 and 11, respectively. The overall acoustic behavior of the annular jet alone and the coaxial jets at their minimum-noise mode of operation with exit-staggers \( (k/w = 0 \text{ to } k/w = 1.9) \) of coaxial nozzle arrangement shown in Fig. 2 is very similar to that discussed earlier for coaxial nozzle arrangement shown in Fig. 1. However, the OAPWL's for the larger exit-stagger \( (k/w > 1.26) \) vary less sharply with increasing inner jet pressure ratios \( \xi \) for this alternate arrangement with straight extended inner nozzle surface than those for the staggered-exit configurations with converging extension for the nozzle arrangement shown in Fig. 1.

FIG. 6 Comparison of 1/3 Octave Power Watt Levels from Coaxial Jets with Coplanar and Staggered-Exit Configurations. Minimum-Noise Mode of Operation

\[
\begin{align*}
\text{FREQUENCY HZ} & \quad 2 \times 10^3 \\
\text{PWL dB re: } 2 \times 10^4 \text{ W} & \quad 160.0 \\
\theta = 90^\circ & \quad 150.0 \\
\theta = 90^\circ & \quad 140.0 \\
\theta = 90^\circ & \quad 130.0 \\
\theta = 90^\circ & \quad 120.0 \\
\theta = 90^\circ & \quad 110.0 \\
\theta = 90^\circ & \quad 100.0 \\
\end{align*}
\]
Fig. 7 Comparison of 1/3 Octave Overall Sound Pressure Levels from Coaxial Jets with Coplanar and Staggered-Exit Configurations. Minimum-Noise Mode of Operation

\[
\begin{align*}
\frac{\ell}{w} &= 0; \quad \xi_2 = 3.04; \quad \xi_1 = 1.82; \quad \frac{V_{j2}}{V_{j1}} = 1.31 \\
\frac{\ell}{w} &= 2.7; \quad \xi_2 = 3.04; \quad \xi_1 = 1.82; \quad \frac{V_{j2}}{V_{j1}} = 1.31
\end{align*}
\]

Fig. 8(a) Comparison of 1/3 Octave Power Watt Levels from the Annular Jet Alone with Coplanar and Staggered-Exit Configurations.

\[
\begin{align*}
\xi_2 &= 3.04; \quad M_{j2} = 1.37; \quad \ell/\ell = 0; \quad \ell/\ell = 2.7
\end{align*}
\]

Fig. 8(b) Comparison of 1/3 Octave Overall Sound Pressure Levels from the Annular Jet Alone with Coplanar and Staggered-Exit Configurations.

\[
\begin{align*}
\xi_2 &= 3.04; \quad M_{j2} = 1.37; \quad \ell/\ell = 0; \quad \ell/\ell = 2.7
\end{align*}
\]

Fig. 9 Variation of Overall Power Watt Levels from Coaxial Jets of Different Exit-Staffers with Inner Jet Reservoir Pressure. Outer Jet Reservoir Pressure Ratio \(\xi_2 = 3.08\) (Fixed), Alternate Nozzle Arrangement (Fig. 2): \(t/r = 0\)

\[
\begin{align*}
\ell/\ell &= 0; \quad \xi_1 = 1.82; \quad \xi_2 = 3.08; \quad \frac{V_{j2}}{V_{j1}} = 1.31 \\
\ell/\ell &= 1.25; \quad \xi_1 = 2.02; \quad \xi_2 = 3.08; \quad \frac{V_{j2}}{V_{j1}} = 1.23 \\
\ell/\ell &= 1.92; \quad \xi_1 = 2.22; \quad \xi_2 = 3.08; \quad \frac{V_{j2}}{V_{j1}} = 1.15
\end{align*}
\]
Fig. 10 Comparison of 1/3 Octave Power Watt Levels from Coaxial Jets with Coplanar and Staggered-Exit Configurations. Alternate Nozzle Arrangement (Fig. 2); t/r = 0. Minimum-Noise Mode of Operation.

\[ \frac{P_{v/w}}{P_{v/w}} = 0; \quad \frac{v_2}{v_1} = 1.25 \]

Fig. 11 Comparison of 1/3 Octave Overall Sound Pressure Levels from Coaxial Jets with Coplanar and Staggered-Exit Configurations. Alternate Coaxial Nozzle Arrangements (Fig. 2); t/r = 0. Minimum-Noise Mode of Operation.

\[ \frac{P_{v/w}}{P_{v/w}} = 0; \quad \frac{v_2}{v_1} = 1.25 \]

Coaxial jet flows issuing from the coaxial nozzle configuration shown in Fig. 2 were recorded in conjunction with the measurements of local total pressures with a pitot-tube. A typical set of shadowgraphs of the coaxial jets with exit-staggers \( \frac{l}{w} = 0 \); \( \frac{l}{w} = 1.25 \) and \( \frac{l}{w} = 1.92 \) (nozzle configurations A, B and C, respectively) at their respective minimum-noise conditions with the outer jet pressure ratio \( \frac{v_2}{v_1} = 3.08 \) is shown in Fig. 12. Shadowgraphs of the individual annular and round jet flows operated respectively at the annular jet pressure ratio \( \frac{v_2}{v_1} = 3.08 \) and the corresponding inner jet pressure ratio \( \frac{v_2}{v_1} \) required for the minimum-noise mode of operation of the coaxial jets are also reproduced in Fig. 12. From these optical flow records and the total pressure measurements, an attempt was made to establish a qualitative correspondence between the salient features of the flows and the observed noise variations with the coplanar (\( \frac{l}{w} = 0 \)) and the staggered-exit configurations of coaxial jets operated at their respective minimum-noise conditions. Additional optical data and flow measurements for the off-minimum noise conditions of coaxial jets from the nozzle configuration shown in Fig. 2 are available in Ref. 12. These studies also include results from staggered-exit nozzle configuration where instead either the lip thickness \( t/r \) = 0.13 and/or the inner conical nozzle was convergent-divergent. Similar optical and flow investigations of coaxial jets issuing from staggered-exit nozzle configuration where the extended surface of the inner nozzle is converging as shown in Fig. 1, are currently being conducted. In the typical spark shadowgraphs reproduced here (Fig. 12), the following flow features of the individual and the coaxial jets with coplanar and staggered-exits are noteworthy.

(a) For the annular jet operated alone, the flow bends inward towards the axis because of the inner jet cavity region (Fig. 12: nozzle configuration A). The measured angle \( \Theta_j = 5^\circ \) of the annular jet flow boundary just downstream of the coplanar-exit is smaller than the expected Prandtl-Meyer expansion angle \( \Theta_j = 8.0^\circ \) (see flow illustration (a). For the exit-stagger \( l/w = 1.25 \), the inward bending of the annular jet flow reduces and the measured \( \Theta_j = 6^\circ \). For the exit-stagger \( l/w = 1.9 \), the annular jet flow boundary angle \( \Theta_j = 11^\circ \) and therefore it diverges more than expected from the coplanar-exit configuration operated at \( \frac{v_2}{v_1} = 3.08 \) (\( M_{j2} = 1.37 \)). By the same token, as noted below the divergence of the free boundary of the combined coaxial jets also increases with the increasing exit-staggers.
(b) In the coplanar-exit operation of coaxial jets, owing to the co-flowing inner jet flow, the shear of the inner flow boundary of the annular jet flow just downstream of the nozzle exit is reduced. Along the interface of the two co-flowing jet flows, both the pressure and flow direction mutually re-adjust. For the staggered-exit configuration, however, the inner boundary of the annular underexpanded jet flow just downstream of the annular nozzle exit is controlled by the straight section of the solid surface of the inner-nozzle extension. As shown in the flow illustrations below, the annular underexpanded jet flow expands to flow Mach number $M_{32}$ and bends outwards. The Prandtl-Meyer (P-M) expansion fan from the annular nozzle exit reflects as an expansion from the solid surface. This reflected expansion reflects back as compression waves from the outer annular flow boundary. The flow through the reflected expansion speeds up to flow Mach number $M_{42}$ ($M_{42} > M_{32}$) along the extended solid boundary and its local pressure reduces. Therefore, at the exit of the inner extended nozzle and the initial interface between the inner nozzle flow and the twice expanded annular jet flow, both the flow pressure and flow direction of each of the flows are different. To match the flow pressures and flow directions just downstream of the inner nozzle exit, an oblique shock front is generated at the sharp lip of the inner nozzle. Therefore the coaxial jet flow behavior at the minimum-noise mode of operation from the coplanar-exit configuration is different from that of the staggered-exit configuration.

Moreover, for a given operating pressure ratio of the underexpanded convergent annular jet $\xi_2$ ($\leq 3.0$), annular width $w$ $\leq 0.25$ cm and the zero divergence angle $\alpha$ of the solid extended surface of the face of the inner nozzle as shown in Fig. 2 and Table 2, the annular flow expands to flow Mach number $M_3$ = 1.37 with the P-M expansion function $\nu_2$ = 8.13° and the Mach angle $\nu_3$ = 46.88°. Therefore the angle $\beta$ of the last wave of the P-M fan with the solid surface of the inner nozzle 38.75° and the intersection of the expansion fan extends to $L_3/w$ = cot $\beta$ $\geq$ 1.25. For experimental confirmation of this, see Fig. 12.

The acoustic performance of the coaxial jets at the minimum-noise conditions with $\xi_2 = 3.08$, for staggered-exit configurations differs substantially from that of the coaxial jets from the coplanar-exit configuration for $l/w > 1.25$. Therefore, both from flow considerations and the acoustic performance, it seems that as compared with the coplanar-exit configuration, increasing the exit stagger beyond $l/w$ = 1.25 (for the operating pressure ratio of the annular jet $\xi_2 = 3.08$) has an increasingly adverse effect on the acoustic performance of the coaxial jets.

**Pitot-Tube Measurements**

To locate the sonic line in the coaxial jet flows at the minimum noise condition for the coplanar ($l/w = 0$) configuration A, the pitot-tube was traversed for total pressure measurements along the base line ($r_1/D_{01} = 1$) where $r_1$ is the radial position of the pitot-tube and set of spark shadowgraphs was also recorded at $L_1/D_{01} = 2.46$. At the same point (i.e., $r_2/D_{01} = 1$, $L_2/D_{01} = 2.46$), the coaxial flow from staggered-exit ($l/w = 1.25$) configuration B when operated at its minimum-noise condition was surveyed. From the measured stagnation pressure $P_2$ by the pitot-tube and the static pressure assumed to equal to the ambient pressure, the local flow Mach number was calculated by applying the Rayleigh supersonic (pitot-tube) relation. For subsonic flow, Mach number was calculated using the isentropic pressure Mach number relation. Pitot-tube readings (psig) and the corresponding calculated Mach numbers are on the following page. This table shows that for the staggered-exit ($l/w = 1.25$) configuration B at minimum-noise condition, the pitot-tube is inside the supersonic turbulent flow, at the same point, it was on the subsonic side of the sonic line of the supersonic turbulent flow for the coplanar configuration A (Fig. 12). This means that compared to that of coplanar-exit configuration, the staggered-exit configuration ($l/w = 1.25$) at minimum-noise condition has a wider and more extended supersonic turbulent flow region. Also, with the increasing exit-stagger, the decay of the coaxial jet flows along the cen-
Coaxial jets with different exit-staggers in their respective minimum-noise mode of operation can be assessed by normalizing the OAPWL's. Using the Lighthill relation for the intensity of the radiated noise from subsonic turbulent jet flows, the overall power watt level can be expressed as

\[ \text{OAPWL} = 10 \log \left( \frac{\rho_j \omega a}{a_0} \right) + \text{constant} \]

This relation has also been experimentally observed to be quite adequate even for supersonic shock-free jet flows where the density exponent \( \omega = 2 \). For the coaxial jets at their minimum-noise mode of operation discussed here, the individual annular and round jets are operated at relatively low pressure ratios (typically \( \xi_2 \approx 3.04 \); \( M_{j2} = 1.37 \) and \( \xi_1 \approx 2.02 \); \( M_{j1} = 1.05 \)). Therefore, the shock structure in the respective jet flows is rather weak and which is further weakened because the co-flowing jet flows result in modified boundary conditions at their flow interface. In the acoustic spectra the presence of only the mildly sharp spectral peaks (especially for the coplanar-exit configuration) indicates that shock structure in the coaxial jet flows at the minimum-noise mode of operation is rather weak. Therefore, to assess the radiated noise, the use of the normalized Lighthill relation is extended here to such coaxial flows.

To use this relation for fully-expanded single jet,

\[ T = \rho_j V_j^2 A_j \]

and

\[ \dot{m} = \rho_j V_j A_j \]

and

\[ U/a_0 = \frac{T}{a_0} \]

Also,

\[ A_j = A_e / V_j = V_e / \rho_j = \rho_e \]

However, for an underexpanded single jet,

\[ T = \rho V^2 A_e + (\rho_e - \rho_a) A_e \]

\[ \dot{m} = \rho V A_e \]

\[ U = \frac{T}{a_0} \]

If the outer annular jet is considered to be the dominant noise generator, then

\[ T = \rho_{e2} V_{e2} A_e + (\rho_{e2} - \rho_{a1}) A_{e2} \]

\[ \dot{m} = \rho_{e2} V_{e2} A_{e2} \]

and

\[ \rho_{j2} = \rho_{j} \]

\[ A_j = A_{e1} + A_{e2} \]

The normalized OAPWL's vs. \( \log_{10} T/a_0 \) are plotted in Fig. 13 for the following:

(a) Two-nozzle coaxial jets with different exit-staggers shown in Fig. 1 which are operated at their respective minimum noise conditions (see legend of Fig. 3). For a similar presentation of normalized OAPWL's for coaxial jets from coaxial nozzle arrangement shown in Fig. 2, see Ref. 12.

(b) Single convergent round underexpanded jets including the "equivalent" single round jet4,13.
(c) Single fully-expanded jet\textsuperscript{14,15}.

For coaxial jets at the minimum-noise condition, the outer annular jet is operated at a reservoir gauge pressure which is normally about twice that for the inner convergent jet. Thus for the cold convergent nozzles used here with \( \frac{V_2}{V_1} > 1 \) and \( \frac{\rho_2}{\rho_1} > 1 \), the outer annular jet is noise-wise dominant. Therefore, to predict the radiated noise from the coaxial jets operated at the minimum-noise condition, the outer jet flow velocity \( (T_2/\rho_2 A_2) \) and \( \rho_2 = \rho_2^\prime \) may be taken as the representative flow velocity and density effective over the total exit area \( A_j = A_{e1} + A_{e2} \). The normalized noise levels for the coaxial jets (at minimum noise condition) are presented in Fig. 13. On this basis, the comparison of the acoustic performance of the coaxial jets with different exit-staggers at their respective minimum-noise conditions shows that:

a) The coaxial jets with coplanar-exits operated at the minimum-noise condition generate noise levels about 11 dB lower than an underexpanded convergent round jet operated at the same normalized specific thrust \( T_1/\dot{m}_0 \).

b) The coaxial jets with coplanar-exit operated at their minimum-noise condition radiate lower noise levels (by about 4 dB) than a fully-expanded jet with the same \( T_1/\dot{m}_0 \).

c) The comparison of the acoustic performance of coaxial jets at their respective minimum-noise conditions for different exit-staggers (all other factors kept the same) shows that the noise reductions increase with increasing exit-staggers and that the noise reductions are the highest for the coplanar-exit configuration. The normalized \( OAPWL \) for the coplanar-exit configuration is lower by 3 dB than that for the staggered-exit configuration with \( \ell/\omega = 2.7 \).

On the other hand, if the representative effective flow velocity, \( U \), of the two coaxial jets operated at the minimum-noise conditions were to be calculated from \( T_1 + T_2/\dot{m}_1 + \dot{m}_2 \) and the measured noise levels from the coaxial jets were to be plotted as shown in Fig. 13 where the total exit area \( A_j = A_{e1} + A_{e2} \) is used as before but the density is taken now as \( \rho_1 \rho_2^\prime \) instead of \( \rho_2^\prime \) in the preceding scheme. Therefore on this basis of comparison, the observed noise reductions from coaxial jets are lower than when the outer jet flow velocity is taken as the representative effective velocity. Using the mean flow velocity \( T_1 + T_2/\dot{m}_1 + \dot{m}_2 \) as the effective flow velocity, the normalized \( OAPWL \) with the zero exit-stagger is about 1 dB lower than an equivalent fully-expanded single jet, and for the staggered-exit configurations, \( \ell/\omega = 2.7 \), the noise levels are higher than the fully-expanded single jet by about 2 dB.

For the assessment of the noise-suppression performance of the coaxial jets at their minimum-noise mode of operation when the flow velocity of the underexpanded annular jet, calculated from \( T_2/\dot{m}_2 \) is considered as the effective representation velocity over the total area of the coaxial jets, the normalized \( OAPWL \)'s and thus the noise reductions may have been overestimated. On the other hand, the calculated representative mean flow velocity \( T_1 + T_2/\dot{m}_1 + \dot{m}_2 \) is admittedly lower than the flow velocity of the outer jet alone. Considering the relative operating pressure ratios (or flow velocities) of the individual jet flows, it is apparent that noise-wise the outer annular jet is dominant. Therefore, in fixing the normalization factors, the use of the lower effective velocity results in an underestimation of the noise reductions by the coaxial jets at the minimum-noise condition. The true estimation of the noise reductions perhaps lies in between those given by these two schemes for normalizing and comparison. However, in assessing the relative role of the different exit-staggers by each of the two schemes, the observed difference between the noise reductions from the coplanar-exit and the staggered-exit configurations of the coaxial jets at their respective minimum noise conditions remains about the same (Fig. 13).

**Conclusions**

It has been shown that:

1. If the noise-suppression effectiveness, i.e., the noise reductions from the coaxial supersonic jets with the "inverted" operating conditions were to be optimized, then besides the selection of the appropriate operating conditions of the individual outer (annular) and the inner (round) jets for the minimum-noise mode of operation of coaxial jets issuing from coaxial convergent two-nozzle arrangements, the use of the coaxial nozzle with coplanar-exits should be preferred over the staggered-exit configurations;

2. Since the outer (annular) convergent jet is operated at a higher pressure ratio or flow Mach number and velocity than that of the inner jet and since the exit area of the annular jet is also somewhat higher than that of the inner jet, the annular underexpanded jet-flow is the dominant source of noise of the two component flows of the coaxial jets.

![Fig. 13 Comparison of Normalized Overall Power Watt Levels vs. Normalized Specific Thrust of Coaxial Jets with Coplanar and Staggered-Exits. For Operating Condition see Legend, Fig. 3.](image-url)
(3) The overall power watt level of the radiated noise from annular jets alone, keeping all other factors the same, reduce with the increasing exit stagger.

(4) For a given coaxial nozzle configuration and the outer (annular) jet operating pressure ratio, the observed noise reductions from the coaxial jets operated at their minimum-noise mode reduce with the increasing exit-stagger because the role of the inner jet flow in affecting the reduction in the radiated noise from high speed coaxial jets becomes less effective. Such effects become relatively more evident when the nozzle exits are staggered beyond the location of the intersection of the last wave of the Prandtl-Meyer expansion with the extended inner nozzle surface for the given operating pressure ratio (or the jet flow Mach number) of the annular jet.

(5) The sound pressure levels, acoustic power watt levels and overall sound pressure levels of coaxial jet flows increase with the increasing exit-staggers. Also, for the staggered-exit configurations, the coaxial jet flows at their respective minimum-noise mode of operation are shown to spread comparatively wider and spatially decay slower along the flow than the corresponding coaxial flows from the coplanar-exit configuration. The acoustic behavior of coaxial jets from coplanar and staggered-exit configurations operated in the "inverted" mode at the minimum-noise conditions.

References


