Here we show that under the conditions considered in our work, the tube model predicts that
the mean-square radius of gyration and the mean-square end-to-end distance remain unchanged
from their equilibrium value, while their components are given by Eq. (1) of the main text.

The primitive chain at equilibrium can be considered as a freely jointed chain with \( n \) “bonds”,
each of length \( l_0 \); its contour length is \( L = nl_0 \) and its mean-square end-to-end distance is:

\[
\langle R_0^2 \rangle = l_0^2 \sum_{i=1}^{n} \sum_{j=1}^{n} \langle \vec{u}_i \cdot \vec{u}_j \rangle = nl_0^2 \tag{S1}
\]

where \( \vec{u}_i \) is the unit vector of the \( i \)-th bond and the second part of the equation follows from
the lack of correlation between the different bond vectors, i.e., \( \langle \vec{u}_i \cdot \vec{u}_j \rangle = 0 \) for \( i \neq j \). As envisioned by
the tube model, for deformation rate such that \( \dot{\gamma} \ll \tau_R^{-1} \), the chain remains relaxed in the tube. The
contour length, and hence the bond length, remain unchanged, while the unit vector \( \vec{u}_i \) undergoes
a uniform rotation by the strain tensor \( \mathbf{E} \) to the new unit vector

\[
\vec{u}_i' = (\mathbf{E} \cdot \vec{u}_i) / |\mathbf{E} \cdot \vec{u}_i| \tag{S2}
\]
where $\mathbf{E} = (1, \gamma, 0; 0, 1, 0; 0, 0, 1)$ for simple shear. Because a uniform rotation of all the bonds do not introduce new correlation between different bonds, the new bond vectors for different bonds remain uncorrelated; we immediately obtain

$$
\langle R^2 \rangle = l_0^2 \sum_{i=1}^{n} \sum_{j=1}^{n} \langle \vec{u}'_i \cdot \vec{u}'_j \rangle = nl_0^2 = \langle R^2_0 \rangle \quad \text{(S3)}
$$

i.e., the mean-square end-to-end distance remains unchanged from its equilibrium value.

For the $\alpha$-component of the mean-square end-to-end distance ($\alpha = x, y, z$), we write

$$
R^2_{\alpha} = l_0^2 \sum_{i} \sum_{j} \langle u'_{i\alpha} u'_{j\alpha} \rangle = l_0^2 \sum_{i} \langle u'_{i\alpha} u'_{i\alpha} \rangle = nl_0^2 \langle u'_{i\alpha} u'_{i\alpha} \rangle \quad \text{(S4)}
$$

where in the last part of the equation we have dropped the bond subscript $i$ since the average is the same for all the bonds. Making use of Eqs. (S1) and (S2), we obtain

$$
\langle R^2_{\alpha} \rangle = \langle R^2_0 \rangle \langle (\mathbf{E} \cdot \vec{u})_{\alpha} (\mathbf{E} \cdot \vec{u})_{\alpha} / |\mathbf{E} \cdot \vec{u}|^2 \rangle \quad \text{(S5)}
$$

which is the analogue of Eq. (1) in the main text for the $\alpha$-component of the mean-square end-to-end distance.

Since the mean-square radius of gyration is merely an average of the mean-square end-to-end distance between internal segments, the same reasoning applies to the mean-square radius of gyration and its components.