Supplementary Figure S1: **High RF power, low white phase noise.** Main panel: Single-sideband phase noise of a high-power open-loop Brillouin oscillator. A record low white phase noise floor (-160 dBc/Hz) for a microcavity-based microwave source is achieved for offset frequencies >200 MHz. Inset: RF spectrum of an open-loop Brillouin oscillator measured directly from a highspeed photodetector without any amplification.
Supplementary Note 1: Cascade and phase noise in a Brillouin microwave oscillator

For cascaded Stimulated Brillouin lasers, the N-th Stokes wave is excited once the intra-cavity circulating power of the (N-1)-th Stokes wave reaches the threshold excitation level. It has been shown that the excitation of the N-th Stokes wave will cause the intracavity power of the (N-1)-th Stoke wave to be clamped [44] on account of gain clamping. For microwave generation by heterodyne mixing of the first and third Stokes waves, the operation described in the main text is to achieve maximum output power of the third Stokes wave just before threshold excitation of the fourth Stokes wave. It can be shown that the Schawlow-Townes frequency noise of a third Stokes wave at the threshold of excitation of the fourth Stokes wave is:

\[ S_\nu(f) = \frac{\hbar \omega g_c [n_T + N_T + 1]}{4\pi^2} \approx \frac{g_c k T \omega}{4\pi^2 \Omega_B} = \frac{g_c k T c}{4\pi^2 2n V_A} \]  

where \( n_T \) (\( N_T \)) is the number of thermal quanta in the mechanical (optical) field. At room temperature, \( N_T \) is negligible and \( n_T \gg 1 \), which gives the approximate form in the last two expressions in equation (S1). \( \omega \) (\( \Omega_B \)) is the angular frequency for the optical (mechanical) field, and \( \frac{c}{\Omega_B} = \frac{c}{2\pi n V_A} \). \( g_c \) is the microcavity Brillouin gain and is related to the bulk Brillouin gain parameter \( g_0 \) by \( g_c = \frac{c g_0}{n^2 V_{\text{eff}} (1 + 4\Delta \Omega^2 \Gamma^2)} \) (Lorentzian Brillouin gain spectrum). \( V_{\text{eff}} \) is the effective mode volume, \( \Gamma \) is the Brillouin phonon damping rate, and \( \Delta \Omega \) is the offset of the cavity FSR relative to the Brillouin phonon frequency \( \Omega_B \). It can be seen from equation (S1) that the noise limit is independent of both the pump frequency and the phonon frequency. Assuming that cascade is not inhibited, equation (S1) sets a limit of the 1/\( f^2 \) phase noise of the Brillouin oscillator upon photomixing the first and the third Stokes lines. From equation (S1) and the volume dependence of \( g_c \), it can be seen that a larger effective mode volume will help reduce the microcavity Brillouin gain parameter \( g_c \) and also reduce the Schawlow-Townes limit of the Brillouin oscillator. Moreover, operation at cryogenic temperatures would lower this noise limit as well through reduction of the phonon thermal quanta. The incident pump power at which the cascade transitions from the third to fourth Stokes oscillation is given by the following form:

\[ P_{\text{th}} = \frac{27\pi^2 n^2 V_{\text{eff}}}{\tilde{g}_0 Q_T^2 \lambda^2} \]  

where \( \tilde{g}_0 \equiv \frac{g_0}{1 + 4\Delta \Omega^2 \Gamma^2} \), and \( Q_T \) is the loaded Q factor of the cavity at critical coupling. Therefore, in systems wherein the phase noise is limited by cascade the high optical Q enables attainment of this noise limit at low pump power levels.
Supplementary Note 2: High-RF-power, low white-phase-noise of the Brillouin Microwave Oscillator

High incident power to the fast photodetector means high RF power without amplification, and also a lower shot-noise-limited white phase noise floor. Higher RF power directly from the photodetector also makes the interconnection of the microwave Brillouin oscillator to the other RF components (e.g. the frequency divider) more convenient since it avoids the requirement of a microwave amplifier. Considering the heterodyne beating of two lasers with power $P_1$ and $P_2$, onto a photodetector with responsivity $R_s$ and load impedance $R_L$, the RF power is given as $P_{RF} = 2R_s^2P_1P_2R_L$. The shot-noise-limited phase noise floor is given as $L(f) = 10\log \left( \frac{2R_s^2(P_1 + P_2)R_L}{P_{RF}} \right)$. In the experiment, power levels as high as 11.8 mW and 4.4 mW for the first and third Stoke lines were delivered to the fast photodetector (PD). The generated average photocurrent was 10.66 mA, with a photodetector responsivity of 0.65 A/W. The microwave power measured at the RF spectrum analyzer was -1.38 dBm as shown in the inset of figure S1. Taking into account about 4.3 dB loss of the RF coax cable and DC block, the microwave power generated directly at the PD is 2.9 dBm (1.9 mW). It is important to note that this number is over 20 dB larger than what has been possible using microcombs [31, 32, 35]. For comparison, the calculated RF power is 3.4 dBm and the calculated shot-noise limited white phase noise floor is -163 dBc/Hz. In the measurement, the white phase noise floor shown in figure S1 is -160 dBc/Hz for offsets above 200 MHz, which is close to the calculated value. (Note: the instrument white phase noise floor is lower than -164 dBc/Hz.) In summary, a record-high, RF power and low phase noise floor for a microresonator-based microwave source has been demonstrated.