Submicron Deformation Field Measurements
II: Improved Digital Image Correlation

G. Vendroux and W.G. Knauss
Graduate Aeronautical Laboratories
California Institute of Technology
Pasadena, California, USA

Abstract

This is the second paper in a series of three devoted to the applicaiton of Scanning Tunneling Microscopy to mechanics problems. In this paper improvements to the Digital Image Correlation method are outlined, a technique that compares digital images of a specimen surface before and after deformation to deduce its (2-D) surface displacement field and strains. The necessity of using the framework of large deformation theory for accurately addressing rigid body rotations to reduce associated errors in the strain components is pointed out. In addition, the algorithm is extended to compute the three-dimensional surface displacement field from Scanning Tunneling Microscope data; also, significant improvements are achieved in the rate as well as the robustness of the convergence. For Scanning Tunneling Microscopy topographs the resolution yields *4.8 nm* for the in-plane and *1.5 nm* for the out-of-plane displacement components spanning an area of *10 μm x 10 μm.*

1. Introduction

Experimental methods in solid mechanics rely heavily on surface displacement measurements. Many optical methods have been developed for that purpose such as optical interferometry, high resolution moiré (interferometry) [1] or Coherent Gradient Sensing [2] to name only a few. Starting in 1983, Sutton *et al.* proposed a technique [3-5] by which two-dimensional correlation using white or laser speckles, may be applied to obtain the deformation of a body (surface) via comparison of digital video images of the undeformed and deformed configurations with a typical spatial resolution of 0.1 *mm*. This technique has demonstrated good flexibility and accuracy warranting pursuit of determining the three-dimensional displacement field through stereo imaging [6].

As the need for smaller scale investigations related to inhomogeneous materials (fiber-matrix composites) and micromechanics issues grew, displacement measurements based on Digital Image Correlation were performed on images acquired in a Scanning Electron Microscope with a spatial resolution on the order of about 1 μm [7]. However, in this latter case, the computations were limited to two-dimensional (in-plane) displacement field and the Fourier based correlation algorithm required strong smoothing of the results precluding the observation of eventual inhomogeneities in the displacement field at the submicron level.

Although the following developments apply equally to measurements on the macro scale applications (*mm and cm scales*), we extend the method here in connection with investigations at the submicron scale through use of Scanning Tunneling Microscopy (STM). Modifications to the two-dimensional Digital Image Correlation algorithm, as developed in refs. 3 - 6, are proposed to extract the three-dimensional displacement field from STM topographs of the same specimen area, but with increased robustness and speed. Moreover, data post-processing issues are discussed which facilitate the accurate determination of rigid body rotations and, accordingly, improve the accuracy of the in-plane strain determination.

2. The Digital Image Correlation Method.

Because the modifications to the original Digital Image Correlation code (DIC) sought here are closely tied to the inner workings of the algorithm for the two-dimensional correlation scheme, the latter is summarized briefly. The three-dimensional extension to the code is then presented and its numerical implementation is discussed subsequently.

2.1 Two-dimensional Digital Image Correlation

A surface profile, as obtained, for example, by a Scanning Tunneling Microscope, is a discrete record of the "height" of the surface at grid points assigned to a specimen surface. Let f(x,y) represent the surface profile of a specimen in an undeformed state at point G(x,y), and $g(\tilde{x}, \tilde{y})$ the surface profile after deformation at the corresponding point $\tilde{G}(\tilde{x}, \tilde{y})$. If the profile pattern before deformation is uniquely related to the profile pattern after deformation, a correlation of these two patterns exists to detect the profile difference which represents the object deformation. Let χ be the mapping from the undeformed to the deformed configuration

$$G \rightarrow \tilde{G} = \chi(G) \quad \text{such that} \quad g(\tilde{x}, \tilde{y}) = f(x,y) \tag{1}$$

or

$$\begin{aligned} \tilde{x} &= x + u(x,y) \\ \tilde{y} &= y + v(x,y) \end{aligned} \tag{2}$$

with u and v the in-plane displacements of G, and let $\tilde{G}_0(\tilde{x}_0, \tilde{y}_0)$ be the image of $G_0(x_0,y_0)$ through χ; further, let S be a (sub)set of points around G_0 and $\tilde{S}$ the corresponding (sub)set of point around $\tilde{G}_0$. Assuming that S is sufficiently small, eq (2) can be expanded into

$$\tilde{x} = x + u(x_0, y_0) + \frac{\partial u}{\partial x}\Big|_{x_0,y_0}(x - x_0) + \frac{\partial u}{\partial y}\Big|_{x_0,y_0}(y - y_0) \tag{3a}$$

$$\tilde{y} = y + v(x_0, y_0) + \frac{\partial v}{\partial x}\Big|_{x_0,y_0}(x - x_0) + \frac{\partial v}{\partial y}\Big|_{x_0,y_0}(y - y_0) \tag{3b}$$

as the linearization χ_l of χ around G_0. For a discrete set of data define the correlation coefficients

$$C = \frac{\sum_{G_s \in S}[f(G_S) - g(\chi_l(G_s))]^2}{\sum_{G_s \in S} f^2(G_S)} \tag{4a}$$

or

$$C = 1 - \frac{\sum_{G_s \in S} f(G_S) g(\chi_l(G_s))}{[\sum_{G_s \in S} f^2(G_S) \sum_{G_s \in S} g^2(\chi_l(G_s))]^{\frac{1}{2}}} \tag{4b}$$

It is clear that C will be zero when the coefficients of the mapping $\chi_l\{....\}$ are indeed the displacements and their derivatives at G_0 [4, 8]. The best estimate of these values are determined by minimizing C, which process can be viewed as a non-linear optimization scheme, some details of which will be discussed in section 3.2 under "Optimization Scheme".

2.2 *Three dimensional Digital Image Correlation:*

Since STM scans measure surface topographies, one should be able to extract the out-of-plane displacement in addition to the in-plane components. There are two potential ways of obtaining this information. The first involves invoking the two dimensional correlation method, as described above. Once the displacements u_0 and v_0 at G_0 are known, the out-of-plane displacement w_0 can be computed as

$$w_0 = g(\tilde{x} = x_0 + u_0, \tilde{y} = y_0 + u_0) - f(x_0, y_0) \tag{5}$$

While straight-forward in principle, this method is prone to produce sizable errors in the presence of large surface gradients. An alternate approach introduces w_0 into the correlation coefficient and minimizes it with respect to w_0 in addition to the six in-plane variables. While this goal might be accomplished for any definition of a correlation coefficient, because of programming considerations relative to the convergence rate (see section 3.2), this technique was used here only for the least square coefficient which can then be written as

$$C = \frac{\sum_{G_s \in S}\{f(G_S) - [g(\chi_l(G_s)) - w_0]\}^2}{\sum_{G_s \in S} f^2(G_S)} \tag{6}$$

In eq (6) w_0 appears now as a global offset in "height" between the two scans and assuming the dimension of subset S to be small, it approximates the out-of-plane displacement component at G_0. This minimization may fail in the event of large out-of-plane deformations, the bounds of which have not been studied here, because that situation has not yet arisen.

3 Implementation of the numerical scheme

Since G_S is a grid point in the undeformed configuration, $f(G_S)$ is defined. On the other hand, $\tilde{G} = \chi_I(G_S)$ may not be at a grid point in the deformed configuration so that interpolations over the deformed field are necessary to define $g(\tilde{G})$ for any $\tilde{G}$ in the deformed field. This interpolation scheme is discussed next along with a computationally efficient second order optimization method to minimize the least square correlation coefficient with respect to the seven deformation parameters.

3.1 Interpolation scheme:

The Digital Image Correlation code was initially implemented by Sutton and coworkers with a bilinear interpolation scheme. However, the associated lack of C^1 continuity was detrimental to the convergence properties of the overall displacement measurement technique [9], especially for rough topographic profiles. Consequently, it appeared essential that a third-order polynomial interpolation, often referred to as bicubic spline interpolation, be considered which could ensure the continuity of the topographic in-plane derivatives through the form

$$g(\tilde{x},\tilde{y}) = \alpha_{nm}\tilde{x}^n\tilde{y}^m \qquad \text{summed over n,m = 0,1,2,3.} \qquad (7)$$

The higher computational cost of this method *vis-à-vis* the bilinear formula is offset by the faster convergence rate of the subsequently outlined

3.2 Optimization scheme:

Let the seven unknowns u; v ; u_x ; u_y ; v_x ; v_y and w define an (unknown) seven-dimensional vector $P(u, v, u_x, u_y, v_x, v_y, w)$. Then C is a function of P through eq (6), *i.e.* $C=C(P)$, or

$$C = \frac{\sum_{G_s \in S}[f(G_S) - \tilde{g}(G_s,P)]^2}{\sum_{G_s \in S} f^2(G_S)} \qquad (8)$$

where

$$\tilde{g}(G_S,P) = g(\chi_l(G_S)) - w \tag{9}$$

With P_0 denoting an initial guess of the solution and P the actual solution of the minimization problem write C(P) as a truncated Taylor series around P_0 as

$$C(P) = C(P_0) + \nabla C(P_0)^T (P - P_0) + \frac{1}{2}(P - P_0)^T \nabla\nabla C(P_0)(P - P_0) \tag{10}$$

Taking the gradient of eq (10) renders, under consideration of $\nabla C(P_0) = 0$ (which expresses the fact that P_0 defines the minimum),

$$\nabla\nabla C(P_0)(P - P_0) = -\nabla C(P_0) \tag{11}$$

an iteratively deduced solution to which will converge in a Newton-Raphson process, provided the initial estimate is "sufficiently close" (see later). This scheme is likely to be very computing intensive because it requires the determination of the Hessian matrix

$$\nabla\nabla C(P) = \left(\frac{\partial^2 C}{\partial P_i \partial P_j} \right)_{i=1,7;j=1,7} . \tag{12}$$

It is possible, however, to circumvent the involvement of eq (12) in an excellent approximation [10]. To this end consider the components of eq (12) as derived from eqs (8) and (9) and given by

$$\begin{aligned} \frac{\partial^2 C}{\partial P_i \partial P_j} = & -\frac{2}{\sum_{G_s \in S} f^2(G_s)} \sum_{G_s \in S} \left[f(G_s) - \tilde{g}(G_s, P) \right] \frac{\partial^2 \tilde{g}(G_s, P)}{\partial P_i \partial P_j} \\ & + \frac{2}{\sum_{G_s \in S} f^2(G_s)} \sum_{G_s \in S} \frac{\partial \tilde{g}(G_s, P)}{\partial P_i} \frac{\partial \tilde{g}(G_s, P)}{\partial P_i} \end{aligned} \tag{13}$$

When P is close to the exact solution

$$\tilde{g}(G_s, P) \approx f(G_s) \tag{14}$$

so that for sufficiently small out-of-pane displacements one finds

$$\frac{\partial^2 C}{\partial P_i \partial P_i} \approx \frac{2}{\sum_{G_s \in S} f^2(G_s)} \sum_{G_s \in S} \frac{\partial \tilde{g}(G_s, P)}{\partial P_i} \frac{\partial \tilde{g}(G_s, P)}{\partial P_i} \tag{15}$$

This approximation for calculating the Hessian matrix makes the method easy to implement because eq (15) involves terms such as $\frac{\partial \tilde{g}(G_S,P)}{\partial P_i}$ which are now very simple to obtain. Through the bicubic spline interpolation, eq (7), one has

$$\tilde{g}(G_S,P) = g(\tilde{x},\tilde{y}) - w = \alpha_{mn}\tilde{x}^n\tilde{y}^m - w \qquad \text{(m,n=0,1,2,3, summed)} \tag{16}$$

and the relations (the counterparts of eq 3)

$$\begin{aligned} \tilde{x} &= x + P_1 + P_3(x-x_0) + P_5(y-y_0) \\ \tilde{y} &= y + P_2 + P_4(x-x_0) + P_6(y-y_0) \end{aligned} \tag{17}$$

lead to terms exemplified by

$$\begin{gathered}
\frac{\partial \tilde{g}(G_s,P)}{\partial P_1} = \frac{\partial \tilde{g}(G_s,P)}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial P_1} + \frac{\partial \tilde{g}(G_s,P)}{\partial \tilde{y}}\frac{\partial \tilde{y}}{\partial P_1} = \frac{\partial \tilde{g}(G_s,P)}{\partial \tilde{x}} \\
\frac{\partial \tilde{g}(G_s,P)}{\partial P_2} = \frac{\partial \tilde{g}(G_s,P)}{\partial \tilde{y}} \qquad \frac{\partial \tilde{g}(G_s,P)}{\partial P_7} = 1 \\
\frac{\partial \tilde{g}(G_s,P)}{\partial P_3} = (x-x_0)\frac{\partial \tilde{g}(G_s,P)}{\partial \tilde{x}} \qquad \frac{\partial \tilde{g}(G_s,P)}{\partial P_4} = (y-y_0)\frac{\partial \tilde{g}(G_s,P)}{\partial \tilde{y}} \\
\frac{\partial \tilde{g}(G_s,P)}{\partial P_6} = (x-x_o)\frac{\partial \tilde{g}(G_s,P)}{\partial \tilde{y}} \qquad \frac{\partial \tilde{g}(G_s,P)}{\partial P_5} = (y-y_o)\frac{\partial \tilde{g}(G_s,P)}{\partial \tilde{x}}
\end{gathered} \tag{18}$$

This minimization method depends on the initial value of P_0. For initialization the displacement derivatives and the out-of-plane displacement are set to zero and all possible integer values of u_0, v_0, within a given range are tried. The set (u_0, v_0, $0,0,0,0,0$) that produces the lowest correlation coefficient is then used as the starting guess for the optimization procedure.

4 Rigid body rotations

The work on this method by Sutton *et. al*. [4] has employed the framework of small deformation theory to compute in-plane strains and the local rigid body rotation angle θ by

$$\varepsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$$
$$\theta = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{18}$$

Test evaluations showed, however, that for two images which differed merely by a rigid body rotation θ, this angle could only be deduced accurately for $\theta \leq 10°$; in addition, and more importantly, apparent and erroneous rotation-induced "straining" appeared. This latter inconsistency is the consequence of the small deformation assumption implicit in the linearized kinematics and is not tolerable in the present context. A closer analysis shows that in the context of STM work the statement of "sufficiently small rotation θ" of small deformation theory requires that $\theta \ll 1°$, which cannot always be guaranteed. It becomes thus mandatory to draw on large deformation theory. Accordingly, the in-plane deformation tensor **F**, the Right Cauchy-Green tensor **C** and the Lagrangian strain tensor γ are evaluated at every computational point from u_x, u_y, v_x, and v_y by

$$\mathbf{F} = (1 + \nabla u), \quad \mathbf{C} = \mathbf{F}^T\mathbf{F}, \quad \mathbf{g} = \frac{1}{2}(\mathbf{C} - 1) \tag{19}$$

so that the rigid body rotation is determined through polar decomposition from

$$\mathrm{Q} = \mathrm{F}\left(\sqrt{\mathrm{C}}\right)^{-1} \qquad q_{ij} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{20}$$

Detailed analysis shows then that the error in either γ or $\tan\theta$ is on the order of the error in the displacement gradients. Thus, if the uncertainty in the latter is (typically) 10^{-3} it will result in an error of 0.1% in strains whereas the error in the rigid body rotation angle will be at most 10^{-3} radians or 5.7×10^{-2} °. The determination of θ is thus more accurate than that of strains. There is more to the accurate computation of θ: If the mapping between the undeformed and deformed image is a combination of a deformation superposed on some rigid body rotation, the in-plane displacement component will reflect both. It follows that, as long as the deformation induced displacements are small compared to the field under observation, the displacements induced by the rigid rotation can and will cloud the strain data. It is therefore important to accurately extract any rotation **Q**. To this end, the global rigid body rotation of a given deformation field must be determined by taking the average of the local rigid body rotation angle at every pixel location where DIC computations are carried out.

5. Experimental Code Evaluation

In order to explore the computational efficiency of the method, four different Digital Image Correlation codes were written around the same skeleton scheme. The first code, called the Exact Least Square DIC, implements the two dimensional correlation method with the exact Hessian matrix calculation for the Least Square Correlation coefficient eq (4a). The second, the Approximate DIC, is also a two dimensional scheme but uses the approximate Hessian matrix formulation for the (same) Least Square Correlation coefficient. The third code is the now well accepted DIC based on the Cross-Correlation coefficient developed by Sutton *et al.* [3, 4, 8]. Finally, the fourth program, referred to as the three dimensional DIC, is the extension of the Approximate DIC scheme to three dimensions (eq 6) discussed in section 3.

Four sets of measurements were carried out to evaluate these correlations. First, translation tests were performed on speckle patterns (not STM data) using a CCD camera-frame grabber combination. The subsequent images were compared through each of the four DIC schemes to check their accuracy and resolution. Second, rotation tests were performed on the same white speckle pattern to assess the efficiency of the data processing scheme presented in section 2.

5.1 *Translation evaluation*

Macroscopic Polymethylmethacrylate specimen surfaces coated with a layer of white paint and then splattered with black spots so as to create a speckle pattern were mounted on a translation stage driven by an Newport Research Corporation actuator (model 360-30) with a resolution of $0.1 \mu m$ and its surface was imaged by an 8-bit CCD camera possessing a rectangular 640 x 480 pixel viewing field. Calibration yielded a pixel size of 0.06647 *mm* ± *0.3%*[1]. Digital images of the specimen surface were acquired for various amounts of translation through a frame grabber board in a 486-50 MHz PC. Each image was then compared to the first image of the translation sequence by way of the four DIC codes. The displacement field was computed at *(11 x 11=)* 121 points with computations carried out for each point on a subset of 41 x 41 pixels surrounding that point to determine u, v, u_x, u_y, v_x, v_y, w and θ along with the average and standard deviations of these quantities at the 121 points. A straight line, least-square-fitted to the results of the Exact Least Square DIC scheme, as depicted in Fig. 1, renders an average error of *0.17%* which is well within the calibration error of the pixel size (0.3 %). These results show that the DIC algorithm *per se* operates satisfactorily. To compare the performance of the four codes one needs to assess

[1] The error on the pixel size was estimated by evaluating the error associated with both the distance measurement under the microscope, *±0.01 mm*, and determination of the corresponding pixel locations, *i.e.* ± 1 pixel.

the error pointwise in terms of the standard deviation $s(\sqrt{u^2+v^2})$ of the in-plane displacements as well as the other relevant parameters. These computations[2] are recorded in Table 1.

All four codes render very comparable performance. The size of the measured translation $\sqrt{u^2+v^2}$ differs maximally by only 2×10^{-3} pixel at each computational point. This value is also the indicator for the degree of uniformity in determining the displacements in the field of view and we take it as the resolution for the displacement, conservatively as 10^{-2} pixel. This is a value that compares very well with what Bruck *et al.* [9] reported.

Following the same rationale one estimates the strain resolution to be 8×10^{-4} pixel/pixel. This number is again consistent with the results from other correlation work. However, it is large enough to preclude computing strains to characterize the deformation field accurately pointwise. Nevertheless, the error in the rigid body rotation is very reasonable at 3.26×10^{-2} °.

The four DIC schemes perform similarly with respect to accuracy, yet the computational efficiency of each is measurably different. Of the three two-dimensional schemes the approximate least square formulation runs 12 % faster than the exact one and 25 % faster than the cross correlation at identical resolution. Because the computational speed is scaled by the complexity of the Hessian matrix at each iteration the simplest code, *i.e.* the approximate least square scheme, outperforms the other two. The three-dimensional DIC code adds one more degree of freedom to the problem and is thereby slower than the approximate two-dimensional scheme but still faster than the exact ones. These results are also summarized in Table 1.

The approximate DIC code has also an advantage over the exact ones in terms of convergence "robustness". This property may be established by executing the optimization schemes through starting the algorithm with a user-defined guess on u and v for a first approximation instead of using the coarse initial search routine described above, but still

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[2] It is of interest to note that the standard deviations for in-plane STM-DIC displacements [11,12] are higher than those recorded for these translation experiments under a CCD-camera. Two factors contribute to this difference, with the first deriving from the contrast: While speckle pattern images contain large amounts of medium to high spatial frequencies, STM profiles are much smoother. Digital Image Correlation requires high frequency information within the correlation subset to fine-tune its calculation; the lower the high frequency content, the lower the consistency of the measurements and therefore the higher the standard deviation (the lower the resolution). The second cause for the reduced precison in STM data rests with the functioning of the microscope itself: Non-linearities in the piezoceramic (hysteresis, creep) and/or electrical noise in the system affect the accuracy with which the scanning probe is positioned over the roster points. Such uncertainty is reflected in subsequent Digital Image Correlation computations by higher standard deviations in the measured displacements.

with the in-plane displacement derivatives being set systematically equal to zero. If u and v are these user-defined guesses and u_0, v_0 the actual translations, define the "radius of convergence"

$$R = \sqrt{(u-u_0)^2 + (v-v_0)^2} \tag{21}$$

as a measure of convergence equalities, large "R" denoting robustness. "R" was found to be consistent within one pixel from point to point and from one pair of images to another, thus offering a reliable convergence measure for a given DIC code.

Table 2 presents the average values of convergence radii "R" for the four DIC codes, and it is clear that the robustness of the approximate least square DIC schemes is distinctly better than that for the exact formulations. This robustness derives from the fact that the exact schemes require the computation of the second derivatives of the image intensity with respect to the six parameters of the deformation mapping (see eq 13) which calculation is likely to incur more error than it is supposed to correct. Differentiation of experimental data is typically error prone, second derivatives being clearly more troublesome. The approximate formulation, involving only the first derivatives of the image intensity makes the Hessian matrix less noise sensitive, which may explain the improved convergence properties. We see thus that the approximate formulation of the Hessian matrix based on the least square correlation coefficient provides the same resolution as the other schemes but at higher speed and with greater robustness. The same behavior holds for the three-dimensional DIC, with a slightly reduced computational efficiency due to the addition of one more degree of freedom. In the rest of this work, displacement field measurements are performed by way of the three dimensional approximate DIC code.

5.2 *Rotation Tests*

In order to confirm or evaluate the necessity for using the large-deformation theory framework to accurately characterize deformation fields, rigid body rotation experiments were also performed using the same speckle pattern as used in the translation tests. The specimen was mounted on a manual rotation stage with a resolution capability of 0.084° with angles being changed by monotonically increasing the rotation. Computations were then carried out at again 121 points as in the translation experiments and with the same subset size of 41×41 pixels. The correlation between the DIC-measured data and a straight line is similar to the relation for the translation and is shown in Fig. 2 with a maximum error in the slope of 0.1%, which amounts to 1/10 of the uncertainty associated with the use of the rotation stage leading to an error of $\pm 0.041°$. Even though the strains associated with the rotations are supposed to be identically zero, we "observed", even within the large deformation theory framework, strain amplitudes amounting to 0.01%, 0.07% and 3% for rigid body rotations of 1°, 10° and 90°, respectively. By contrast, small deformation theory yields for the same angles the considerably larger (artificial) strains of 0.015 %, 1.5 % and 100 %, respectively.

Sutton *et al.* have proposed three techniques to increase the DIC resolution in strain [5]. The first consists of acquiring several (20) digital images of the specimen surface in the same deformation state and averaging their intensity to reduce the digitization errors associated with CCD cameras. The second increases the subset size over which the correlation is performed at a given computational point. The last and most efficient of these involves smoothing the computed displacement field with subsequent differentiation to calculate the strains. With this latter technique the strain resolution can be improved to 2×10^{-4}. If such is the resolution desired, one needs to be aware that the small deformation theory will still translate a 1° rotation into an "apparent" (uniaxial) strain of 1.5×10^{-4}. It is therefore imperative that the large deformation theory be considered not only to determine the rigid body rotation closely but also to accurately compute a rotation-independent Lagrangian strain tensor.

6. Conclusion

The core of the (two-dimensional) Digital Image Correlation method lies in the optimization of a correlation coefficient between the two images over six parameters characterizing the in-plane deformation namely the displacement components u and v, and the displacement gradients $\partial u/\partial x, \partial u/\partial y, \partial v/\partial x, and\, \partial v/\partial y$. The original code [3, 9] was based on the cross-correlation coefficient minimized by a Newton-Raphson scheme and a small deformation theory framework to calculate the strains and rigid body rotation angle from the in-plane displacement derivatives.

The present work proposes an alternate formulation of the Digital Image Correlation algorithm as well as of its data processing. First, implementation in terms of a least square coefficient together with an approximate but highly accurate Hessian matrix computation greatly simplifies the Newton-Raphson scheme, leading not only to a 25% improvement in speed but also in the convergence robustness of the code. Further, the large deformation formulation for computing the strains and rigid body rotations provides a significant improvement in determining the deformation field in that large rigid body rotation angles can be accurately computed (up to 90°) and essential decoupling between rigid body rotation and strains. The latter feature is particularly important when one investigates deforming systems for which the rigid body rotation is expected to be locally larger than 1° as, *e.g.* in crack tip problems, since in these cases the small deformation theory formulation will yield "apparent" uniaxial strains which are likely to be larger than the claimed resolution of the method.

With a view towards application to Scanning Tunneling Microscopy, an extension of the present correlation algorithm computes the three dimensional displacement field from topographic profiles of a deforming specimen surface. This modification in the correlation scheme was also examined by translation and uniaxial tensile tests, and the resolution in displacement field measurements was found to be *4.8 nm* for the in-plane displacements and *1.5 nm* for the out-of-plane ones over a *10×10 μm* area. More detail on this development is documented in refs. 11 and 12.

The three dimensional correlation scheme is readily applicable to topographies obtained by means other than Probe Microscopy (STM, AFM,...). In particular Confocal Microscopy seems suited for such an application since it also yields surface profile data of a specimen but at a scale from about a few millimeters to *60 μm.* Therefore correlation based algorithms can provide a universal method to measure three dimensional surface displacement fields, by way of stereo imaging on a large scale, by way of Confocal Microscopy for intermediate dimension and finally by way of Probe Microscopy for scales ranging from tens of microns to atomistic dimensions.

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List of Figure Captions