Scaling properties of gravity-driven sediments

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Received 28 October 1994 - Accepted 3 December 1994 - Communicated by D. Sornette

Abstract. Recent field observations of the statistical distribution of turbidite and debris flow deposits are discussed. In some cases one finds a good fit over 1.5–2 orders of magnitude to the scaling law \( N(h) \propto h^{-B} \), where \( N(h) \) is the number of layers thicker than \( h \). Observations show that the scaling exponent \( B \) varies widely from deposit to deposit, ranging from about 1/2 to 2. Moreover, one case is characterized by a sharp crossover in which \( B \) increases by a factor of two as \( h \) increases past a critical thickness. We propose that the variations in \( B \), either regional or within the same deposit, are indicative of the geometry of the sedimentary basin and the rheological properties of the original gravity-driven flow. The origin of the power-law distribution remains an open question.

1 Introduction

Gravity-driven sedimentation in oceanic basins occurs as the result of slumping, or avalanche, events at the edge of the continental shelf (Middleton and Hampton, 1976; Middleton, 1993). These slumping events originate on relatively steeply sloped submarine topography. They create subaqueous flows known as gravity currents (Simpson, 1987) that can flow for hundreds of kilometers or more. Once these flows finally lose energy and stop, they deposit the sediment that is no longer mobilized by the flow. The historical record of these gravity-driven sedimentation events is the sedimentary succession itself.

Gravity-driven sedimentation has been the subject of much study during the last half century (Kuenen and Migliorini, 1950; Middleton, 1966a, 1966b, 1966c), in part because its discovery helped explain the distribution of sediment in oceanic basins. One outcome of this work has been a classification of sediment types based roughly on the role that turbulence plays in the transport of the sediment (Middleton and Hampton, 1976).

At the turbulent end of the spectrum are turbidity currents, the deposits of which are called turbidites. At the laminar end of the spectrum are debris flows. Whereas in the former case the transport of sediment is supported by the upward motion of turbulent, turbid vortices, the latter type of sediment support is thought to be due to the intrinsic strength of a relatively thick mixture of sand and debris.

Recent studies have revealed some interesting statistical properties of turbidites (Hiscott et al., 1992; Rothman et al., 1994a; Hiscott and Firth, 1994; Rothman et al., 1994b). For each of the turbidite successions studied in these papers, if one measures the thickness \( h \) of all the layers, then the number of layers thicker than \( h \) scales like \( h^{-B} \) above a small thickness cutoff, with \( B \approx 1 \). Such a scaling property is significant for several reasons.

First, the appearance of a power law indicates that the dynamical mechanism responsible for turbidite deposition may be scale invariant. This in turn is suggestive (Rothman et al., 1994a) of the many recent studies on "self-organized criticality" (Bak et al., 1992), which is itself an attempt to formulate a theory for the ubiquitous occurrence of scale invariance in nature. The connection with self-organized criticality is due in large part to the fact that Bak et al. used avalanches as the archetypal example of their theory.

Second, one would like to know whether power-law scaling for turbidites represents an intrinsic dynamical property of turbidite systems themselves, or whether it is just the signature of power-law scaling for an external, causal, mechanism. Causal factors worth considering are floods (Turcotte, 1994) and earthquakes (Hiscott and Firth, 1994; Beattie and Dade, 1994), each of which exhibit power-law size distributions. However, the power-law scaling in all these systems suggests that there may be a generic physics describing them all. Such a generalization is indeed the objective of the proponents of self-organized criticality.
Last but not least is the importance of such a scaling law for geology. Specifically, the power law may be viewed merely as a statistical distribution with a parameter $B$. One is then led to the following questions:

- Is the power law generic?
- If so, is there a typical or generic value for $B$?
- If there is no generic value for $B$, what geologic factors determine its value?

It is the objective of this paper to provide preliminary answers to these questions. To achieve such a goal, we look at debris flows in addition to turbidite deposits. Our results indicate that the power law is not generic; moreover, there is no generic value for $B$ in the cases in which power laws exist. Like most other real-world phenomena, the problem is plainly more complex than idealized theories would lead one to presume. Nevertheless, we believe that there remains some order to extract from this complexity. Specifically, for the cases in which power laws exist, we propose that the value of $B$ is determined by factors related to both the geometry of sediment deposition and the rheology of the gravity-driven flow. By doing so, we leave open the possibility that generic mechanisms govern the dynamics; the available data, however, can neither support nor deny such a hypothesis. Moreover, the factors that determine whether one finds a power law remain a matter of speculation.

In what follows we first provide a brief qualitative introduction to gravity-driven sedimentation. We then discuss the statistical distributions of turbidite and debris flow sediments in three distinct geological settings.
At the turbulent and Newtonian end of the spectrum are turbidity currents. Here the density of the flow is not much greater than that of the sea water, and the sediment remains mobilized due to the action of powerful vortices at the head of the flow. The deposits are characterized by a graded sequence in which the largest clasts (i.e., pebble-size sediment) fall to the bottom whereas the finest-grade sediment settles last, and is thus found on top. A schematic view of such an ideal turbidite layer is shown in Figure 2a.

At the laminar and non-Newtonian end of the spectrum are debris flows. In this case, the flowing mixture of sediment and water can be an extremely dense mud. The salient property of the mud is that it has a finite yield strength; i.e., it flows only if subjected to a sufficiently large shear stress. Middleton and Hampton say that “debris flow essentially resembles flow of wet concrete” (Middleton and Hampton, 1976). Both the density of the mud and its finite yield strength give debris flows distinct characteristics that may be identified by a field geologist. Specifically, and in contrast to turbidites, graded sequences of clasts are infrequent, and the clasts are distributed widely in space. Indeed, large boulders can appear to have been levitated by the flow, because the carrying fluid 1) is nearly as dense as the boulder itself, and 2) acts like a solid if insufficiently stressed. A schematic view of an ideal debris flow layer is shown in Figure 2b.

3 Field observations

Below we describe three gravity-driven deposits that we have studied by direct observation in the field. The first, from southeast California, allows us to directly compare the statistical distribution of turbidites and debris flows in the same deposit. The second, from turbidites in Karoo Basin, South Africa, displays an intriguing crossover from one power law to another. The third, from Barberton, South Africa, does not conform well to a power-law distribution.

3.1 Kingston Peak Formation, southeast California

Turbidites and debris flows of the Kingston Peak Formation in SE California are approximately 700 million years old and accumulated in a narrow fault-bounded trough (Rothman et al., 1994a). Turbidites are well expressed as even beds up to 2 m thick which show minimal evidence of erosion along their bases. Also, amalgamation of individual turbidites is rare. Debris flows are much thicker (up to 10 m) and commonly show matrix support of clasts which range up to 0.48 m in diameter. In some cases debris flows are amalgamated.

In a previous study, we examined only turbidites from this area (Rothman et al., 1994a). We found a reasonably good fit to the power law $N(h) \propto h^{-B}$, where
Fig. 3. Logarithm (base 10) of the number of layers thicker than h as a function of the logarithm of layer thickness h, for 1235 turbidites (circles) and 24 debris flows (diamonds) observed in the Kingston Peak Formation, SE California. The turbidite data are compared to a straight line with slope $-B = -1.39$, the best fit to a linear regression computed from all points except the ones for the smallest and largest values of $h$. The debris flow data are compared to a straight line with slope $-B = -0.49$, the best fit to a linear regression computed from all points except those corresponding to the three thickest layers.

Fig. 4. Logarithm (base 10) of the number of layers thicker than $h$ as a function of the logarithm of layer thickness $h$, for 878 turbidites from Laingsburg Formation, Karoo, South Africa. The thin beds fall roughly on a line with slope $-B = -0.70$, the best fit to a linear regression computed from all points corresponding to $h < 10^{1.5} \approx 30$ cm. The thick beds fall roughly on a line with slope $-B = -1.47$, the best fit to a linear regression computed from all points corresponding to $h > 10^{1.5}$ except for the two largest values of $h$.

$N(h)$ is the number of layers thicker than $h$, and $B = 1.39 \pm 0.02$. The total number of turbidite layers was 1235, and the fit was over approximately two orders of magnitude in $h$, ranging from centimeters to meters.

Interspersed among those turbidite layers are also 24 debris flows. These debris flows were qualitatively distinguished from turbidites according to the schematic diagrams of Figure 2. Figure 3 is a log-log plot of $N(h)$ for the 1235 turbidites compared to $N(h)$ for the 24 debris flows. Three qualitative differences between the two plots are evident. First, there are more than 50 times as many turbidite as debris flows. Second, debris flows range in thickness from $10^1$ to $10^3$ cm, whereas turbidites are scattered from $10^0$ to $10^2$ cm. The third difference is equally unsuble, but is the most interesting: the debris flows scale with $B = 0.49 \pm 0.01$, which is roughly one-third of the $B$-value found for the turbidites. Why such a dramatic difference in $B$-values could exist within the same sedimentary succession is considered further below.

3.2 Laingsburg Formation, Karoo Basin, South Africa

Turbidites of the upper Laingsburg Formation (Karoo Basin) were measured along the Buffels River, north of the town of Laingsburg, South Africa. The Laingsburg Formation is about 275 million years old and its turbidites have been described by Bouma and Wickens (1991). Most turbidites are even bedded and only the thicker beds show frequent amalgamation. Most turbidites are laterally continuous for distances greater than 100 m. Deposition occurred within the confines of a tectonically active foreland basin developed in front of an advancing mountain belt.

Figure 4 is a plot of $N(h)$ for 878 turbidites from this formation. One finds that, for $10^0 \leq h \leq 10^{1.5} \approx 30$ cm, there is a good fit to the power-law scaling with $B = 0.70 \pm 0.01$. However, for $10^{1.5} \leq h \leq 10^{2.5}$ cm, the fit is for a value of $B$ more than twice as large; specifically, $B = 1.47 \pm 0.02$. Further investigation of the data shows that the thick layers are apparently randomly situated among the thin layers. For example, if the data set is divided into two halves, in which the first half has the first 439 layers and the second half the rest, then $N(h)$ for each set still looks qualitatively similar to Figure 4. Thus, unlike the turbidites of southeast California, we find two distinct scaling regimes rather than just one.

3.3 Fig Tree Group, Barberton, South Africa

Turbidites of the Fig Tree Group were measured along a road cut south of the Sheba Mine within the Barberton Mountain Land, South Africa. The Fig Tree Group is approximately 3.2 billion years old and contains some of the oldest turbidites on earth. These have been well-
Fig. 5. Logarithm (base 10) of the number of layers thicker than \( h \) as a function of the logarithm of layer thickness \( h \), for 962 turbidites from Fig Tree Group, Barberton, South Africa. The data do not conform to either a simple power law or a clean crossover as in Figure 4. One finds however approximate power-law scaling for the thicker turbidite layers; the straight line has slope \(-B = -1.58\), which is the best fit to a linear regression computed from the points corresponding to \( h > 10^{1.4} \approx 25 \) cm, except for the two largest values of \( h \).

4 Discussion

From the three datasets of Figures 3, 4, and 5, two conclusions may be drawn. First, simple one-parameter scaling behavior is not always observed. Second, if there is power-law scaling, the exponent \( B \) appears to depend on flow type—that is, whether the deposit is a turbidite or debris flow—or, in the case of Figure 4, the magnitude of the layer thickness. Whereas the origin of the power-law scaling is difficult to pin down, the dependence of the exponent on the type or size of the flow appears considerably easier to address. Below we consider some possible scaling laws. Our analysis is similar but simpler and more physical than our previous work (Rothman et al., 1994a, 1994b). A related study has also been recently reported by Malinverno (1994).

4.1 Scaling laws

Because our datasets are small, we have displayed only the (smoother) cumulative distributions \( N(h) \). They may be related to a frequency distribution \( \rho_h(h) \) by

\[
\rho_h(h) = -\frac{1}{\Delta t} \frac{d}{dh} N(h). \tag{1}
\]

Here \( \rho_h(h) \) is the number of layers of thickness \( h \) deposited per unit time and \( \Delta t \) is the duration of geologic time from the bottom to the top of the section. For the cases in which \( N(h) \propto h^{-B} \), we have

\[
\rho_h(h) \propto h^{-B-1}. \tag{2}
\]

Although we can measure only the thickness distribution of sedimentation events, we would like to know the volume distribution so that we may distinguish aspects of basin and flow geometry from intrinsic dynamical processes on the slope. We assume that the thickness \( h \) is approximately uniform throughout a deposit covering an area \( S \); thus the volume \( V = Sh \). Additionally, we assume that \( h \) scales with \( V \) according to

\[
h \propto V^\alpha. \tag{3}
\]

We call \( \alpha \) the spreading exponent and expect \( 0 \leq \alpha \leq 1 \). Consideration of some special cases gives some insight into the spreading:

- \( \alpha = 0 \). Perfect spreading. \( S \propto V \) and all layers have the same thickness.
- \( \alpha = 1/3 \). Self-similar areal spreading. If spreading is in two dimensions, then \( S \propto V^{2/3} \), and all three linear dimensions (length, width, and height) respond roughly equally to changes in \( V \).
- \( \alpha = 1/2 \). Self-similar channelized spreading. If spreading is confined to a channel, and thus only one dimension, then both the height and the length depend equally on \( V \).
- \( \alpha = 1 \). No spreading. Each sedimentation event spreads over the same area \( S \).

Interestingly, an empirical study of sturzstroms (rock slides) appears to have found self-similar areal spreading with \( \alpha \approx 1/3 \) (Davies, 1982). There is no reason to expect self-similar spreading in either one or two dimensions, however. In a study of channelized turbidity flows, Dade and Huppert (1994b) predict \( \alpha = 2/5 \). Here, however, we are considering deposits ranging from viscous non-Newtonian debris flows to turbulent turbidity currents in unknown geometries. In this continuum of
rheologies and geometries, we simply expect that as the flow becomes less like a debris flow and less confined, the tendency to spread increases, and thus the value of $\alpha$ decreases. A detailed analysis for non-Newtonian rheologies, à la Dade and Huppert, would nevertheless be necessary to prove this point and make it precise.

Given equation (3), a change of variables from $h$ to $V$ in equation (2) gives the number of events of volume $V$ per unit time:

$$p_V(V) = \rho[h(V)] \frac{dh}{dV} \quad (4)$$

$$= AV^{-\alpha B - 1}. \quad (5)$$

The prefactor $A$ is related to the volume flux $Q$ of sediment to the continental slope (see Figure 1). Specifically, we define $Q$ to be the volume of sediment delivered to the shelf per unit time. Clearly, $A$ depends on $Q$, and, naively, one would expect $A \propto Q$. The dependence of $A$ on the size of the system should also be considered, however. For example, although sediment may be delivered uniformly to the continental slope at rate $Q$, only a subset of the slope may be “active” in the sense that it would channel sediment into a particular part of the basin plain. Typically, these active regions would correspond to submarine canyons, and they may not fill the shelf-break uniformly. More specifically, as in analogous continental drainage networks (Tarboton et al., 1988; Rodriguez-Iturbe et al., 1992), the active drainage areas on the continental slope may form a fractal set. (Indeed, analysis of high-resolution digital bathymetric maps of submarine canyon networks supports such a conclusion (Rothman and Grotzinger, 1994).) In this case one would expect the more general relation

$$A \propto QV_{\text{max}}^{-\nu}, \quad (6)$$

where $V_{\text{max}}$ is the largest possible event (i.e., the system size) and $\nu$ is related to the fractal dimension of the active region. In particular, on a two-dimensional map, the fractal dimension of the active region would be $2 - \nu$, with $0 \leq \nu \leq 1$. We return to a discussion of $\nu$ below.

An assumption of a statistically stationary state—specifically, that $Q$ is roughly constant in a coarse-grained sense over the time period $\Delta t$—allows one to relate the flux to the volume frequency distribution:

$$Q = \int_{V_{\text{min}}}^{V_{\text{max}}} V p_V(V) dV \quad (7)$$

Here $V_{\text{min}}$ is the smallest possible sedimentation event. Substitution of equation (5) gives

$$Q \propto A \cdot (V_{\text{max}}^{1-\alpha B} - V_{\text{min}}^{1-\alpha B}). \quad (8)$$

By letting $V_{\text{min}} \to 0$ and substituting equation (6) for $A$ we can eliminate $Q$ to obtain

$$V_{\text{max}}^{1-\nu - \alpha B} = \text{constant}, \quad \alpha B < 1. \quad (9)$$

The inequality $\alpha B < 1$ is required for $Q$ to be finite. For equation (9) to hold, the dependence of the left-hand side on $V_{\text{max}}$ must vanish. Thus we find that $\alpha$, $\nu$, and $B$ are related by

$$B = \frac{1 - \nu}{\alpha}. \quad (10)$$

Note that the rigorous bound $\alpha B < 1$ requires $\nu > 0$. If the power law were not applicable in the limit $V_{\text{min}} \to 0$, the bound $\alpha B < 1$ would not be required but equation (10) would still be valid if $V_{\text{max}}^{1-\alpha B} \gg V_{\text{min}}^{1-\alpha B}$.

4.2 Interpretation

Since $B$ is measured, then if $\nu$ is known, equation (10) may be used to infer the value of the spreading exponent $\alpha$—thus providing quantitative insight into qualitative characteristics of the original flow. However, as we show below, knowledge of $\nu$ is not necessary to make purely qualitative conclusions.

Table 1 gives the results that would follow for the turbidites and debris flows of Figures 3 and the crossover behavior in Figure 4. Since $\nu$ is not known, we express $\alpha$ in terms of the reduced spreading exponent $\alpha/(1 - \nu) = B^{-1}$. The principal conclusion, given in the rightmost column, concerns the relative spreading of the two types of flows found in each formation. The determination of the relative spreading does not depend on the value of $\nu$, but it does require that $\nu$ be constant within the same formation.

For the case of SE California, the scaling theory predicts that the debris flows spread less (i.e., that $\alpha$ is larger) compared to the turbidites. For example, if $\nu = 1/2$, then one finds that $\alpha_{\text{debris}} = 1$ and $\alpha_{\text{turbidite}} = 0.4$. The former value would lead to the conclusion that the debris flows did not spread at all. The latter value would coincide with the Dade-Huppert prediction for channelized turbidity flows (Dade and Huppert, 1994b).

The case of the Karoo turbidites (Figure 4) has a more subtle interpretation. The thin-layer part of the curve appears to represent flows that had much less of tendency to spread than the thick-layer part of the curve. The presence of two such spreading characteristics within the same formation could be explained by

<table>
<thead>
<tr>
<th>Location</th>
<th>Deposit type</th>
<th>$B$</th>
<th>$\alpha/(1 - \nu)$</th>
<th>Spreading</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE Calif.</td>
<td>debris flow</td>
<td>0.5</td>
<td>2.0</td>
<td>little</td>
</tr>
<tr>
<td></td>
<td>turbidite</td>
<td>1.4</td>
<td>0.7</td>
<td>much</td>
</tr>
<tr>
<td>Karoo</td>
<td>thin layer</td>
<td>0.7</td>
<td>1.4</td>
<td>little</td>
</tr>
<tr>
<td></td>
<td>thick layer</td>
<td>1.5</td>
<td>0.7</td>
<td>much</td>
</tr>
</tbody>
</table>
two different sources of turbidity flows. The source of the thin layers would be relatively closer to the measured formation than the source of the thick layers. The absence of thin layers from the far (spreading) source would be due to the thin layers not having travelled sufficiently far to reach the measured outcrop. The absence of thick layers from the near source might indicate that the near source was fed by a smaller drainage system, with a smaller maximum event size.

In closing this section, it is worthwhile to comment on the role of $\nu$. If $\nu$ were approximately zero, as the simplest considerations would predict, then the value of $\alpha$ that would be inferred from the debris flows from SE California and the thin layers from Karoo would be physically implausible ($\alpha > 1$). Thus it appears that $\nu > 0$ may be a necessary component of the theory. However, the data may be too scarce to make such a conclusion or the scaling theory may be too simple to have any general validity.

5 Conclusions

We have two principal conclusions. First, power-law size distributions are not ubiquitous among gravity-driven sediments. Second, when power laws do exist, the scaling exponents can vary widely from formation to formation.

This paper has primarily addressed the second conclusion. The fact that the scaling exponents differ means that the size distributions are non-universal; i.e., that no generic size distribution exists when measured in terms of thicknesses. Here, we have exploited the differences among power-law thickness distributions to infer qualitative characteristics of the tendency for deposits to spread after slumping. However, it may still be possible that the product $\alpha B$ and the volume frequency distribution $\rho_V(V) \propto V^{-\alpha B-1}$ could be universal. Indeed, in our analysis, such would be the case if the exponent $\nu$ were everywhere the same.

Perhaps the most important question, however, concerns the origin of the power-law distributions, and why they are not always observed. If external mechanisms such as earthquakes or floods were the dominant cause of sedimentation events, the likelihood or lack thereof of these external factors would explain regional variations. However one would still need to ask why the size distribution of these external events were such that power-law sedimentation events were produced. As an indication of the complexity of all such questions, we note that if equation (3) did not hold—i.e., if thickness were not necessarily a power-law function of volume—then one could not observationally distinguish between power-law and non-power-law volume distributions. Thus, from the available data, it remains possible that volume distributions are generically distributed as a power law, but that geometric aspects of the depositional basin can move any indication of it. Only further work, in the form of theory, observation, and experiment, can help resolve these issues.

Acknowledgements. This work was partially supported by NSF Grants 9218819-EAR and 9058199-EAR. We thank Jen Carlson for her assistance in processing the South African turbidite data.

References


Dade, W. B. and Huppert, H. E., Predicting the geometry of deep-sea turbidites, Geology, in press, 1994b.


Eriksson, K. A., Transitional sedimentation styles in the Moodies and Fig Tree Groups, Barberton Mountain Land, South Africa: evidence favouring an Archean continental margin, Precambrian Research, 12, 141-160, 1980b.


Kuenen, P. and Migliorini, C. I., Turbidity currents as a cause of graded bedding, J. Geol., 58, 91-127, 1950.


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