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# Some Recent Developments in Propeller Theory [1)]

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## 1. Introduction

When Prof. H. W. Lerbs and Prof. G. Weinblum asked me ) prepare a general and broad survey talk on the subject Propellers and Propulsion" for this International Sympoium in celebrating the Fiftieth Anniversary of Hamburgi·hen Schiffbau-Versuchsanstalt, I was pleased by having this pportunity to extend my personal congratulations and to parcipate in this happy event. In view of the fact that this sub·ct has a vast scope containing many special problems which ave been under a rapid development, I am fully aware of the ıallenge to prepare a thorough survey, even with the previous xcellent review of the state-of-the-art by Prof. Lerbs (1955a, ee Reference). Undoutedly, my effort would be limited by the hysical access to the information and literatures not geneally available, so I would entitle my talk as "Some recent evelopments in propeller theory".

Use of propellers as a propulsive device has a history over 00 years. In all forms of transportation the main aim is to rice speed; and in achieving speed a primary consideration the propulsive efficiency. This has led to an unceasing effort ı theoretical and experimental studies to improve our know·dge and to perfect the technology. The interest has been ırther stimulated by many applications where the conven·onal propeller is not adequate.

The basis of modern propeller theory has been laid on the reatise of Betz (1919) and Goldstein (1929) for the imporınt case of optimum propellers. In this case the flow far beind the propeller can be regarded as that produced by rigid railing vortex sheets receding with a constant axial velocity, hus reducing the problem to a two-dimensional one. Goldtein's solution then enables one to relate the circulation distriution of a propeller with a finite number of blades to one aving an infinite number. These valuable works have proided a solid foundation for the lifting-line theory as there xists a definite relationship between the flow quantities at he lifting-line and those far downstream. Application of the ifting-line theory has subsequently been simplified by the ıduction factor method of Lerbs (1952).

Further improvements of the propeller theory are strongly ıarked by the exact three-dimensional effects, accounts of ·hich were considered either infeasible or impractical only decade ago. This picture, however, has been drastically hanged with the rising of the high-speed computer era. In he last few years a rigorous linearized lifting surface theory has been developed by several authors. In these treatments, the analysis has been carried out to a stage where the laborious computation of the final results can best be done by computers. Actually, the numerical analysis and computing schemes involved in making use of computers have opened up another dimension of technology. When well performed, the elegance in numerical computation can be just as appealing as a beautiful theory itself. The success of such attempts have exerted noticeable influence on other areas such as ducted propellers, vertical-axis propellers, unsteady propellers, cavitating propellers, and so forth.

1) *This work has been supported by the Office of Naval Reearch of the U.S. Navy under Contract Nonr-220 (35).*

## 2. Linearized Lifting-Surface Propeller Theory (General Formulation)

The lifting surface theories developed recently for calculating the steady flow past a propeller having a finite number of symmetrically spaced identical blades may be classified into two categories:

(I) Lattice representation — The blade is represented by a set of concentrated radial and helical vortex lines. This method has been developed by Strecheletzky (1950) (1955), Guilloton (1957), Kerwin (1961) and English (1962). As pointed out by Kerwin (1961), the accuracy of the results depends on the lattice spacing chosen for the computation.

(II) Continuous vortex sheet representation — The blade is represented by a system of continuous vortex sheet, consisting of both bound and free vortices, and the induced velocity field can be derived by the law of Biot-Savart. In an early work by Ludwieg and Ginzel (1944), the camber correction factor is based on the chordwise rate of change of induced down-wash at the mid-chord. Next came the era of high-speed electronic computers, which have made feasible much of the laborious computations previously regarded as the principal hindrance. Since then the lifting surface theory has been further developed by Sparenberg (1959), Pien (1961), van Manen and Bakker (1962), Kerwin (1963), Cox (1961), Nishiyama and Nakajima (1961), Yamazaki (1962), and Nelson (1964). The effect of the blade thickness has been incorporated into the lifting surface analysis by Kerwin and Leopold (1963) and also by Nelson (1964). In view of the numerous recent contributions on this subject, it appears highly desirable to have a brief review here on the lifting-surface theory. The presentation below follows the work of Sparenberg, Pien, and Kerwin.

The linearized lifting-surface theory for propellers having N identical, symmetrically spaced blades is generally based on a small perturbation approximation together with the following assumptions.

(i) The flow is inviscid, incompressible, free of cavitation, and infinite in extent. The free stream is steady, axially directed, and may be a function of the radius only.

(ii) The blades are thin, their camber and incidence both small, so that the points on each blade describe a common helicoidal surface as it moves through the fluid.

(iii) The lifting pressure field may be represented by a distribution of bound vortices (over the blade plan form) and a trailing vortex system over the helical surface downstream of the leading edge. The lift is required to vanish at both the outer tip of the blade and at the axis.

The blade thickness may be represented by a distribution of source-sink system (over the blade surface) whose strength is proportional to the slope of the thickness function.

(iv) The relative velocity (with respect to the blade) for the purpose of locating the flow boundary is determined by the free stream and the blade rotation only, the induced velocity field being neglected in the first order theory. (In usual applications, this assumption can be readily modified to be applicable to moderately loaded propellers, — see Section 5 below.)

The analysis under the above assumptions may proceed as follows.

The coordinate systems adopted for this problem are shown in Figure 1.

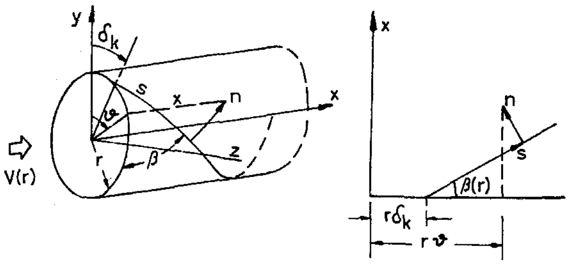


**Figure 1**

A Cartesian coordinate system is fixed on the propeller, with the x-axis lying along the axis of revolution, the y-axis passing through the tip of one blade, and the z-axis completing the right-handed system. A cylindrical system $(x, r, \vartheta)$ is defined by $y = r\cos\vartheta$, $z = r\sin\vartheta$. The tips of the N blades are located at $x = 0$, $r = R$, $\vartheta = \delta_k$, where

$$\delta_k = 2\pi(k-1)/N, \quad (k = 1, 2, \dots N.). \tag{1}$$

In the first order approximation, the streamtubes become $r = $ constant; and the relative velocity has an axial component $V(r)$, a function of r, and a tangential component $\Omega r$ at radius r, $\Omega$ being the angular velocity of the propeller, in radians per second. The helicoidal surfaces traversed by the blades are therefore characterized by the advance coefficient $\lambda(r)$, or the advance angle $\beta(r)$, as defined by

$$\lambda(r) = \frac{V(r)}{\Omega R}, \quad \beta(r) = \tan^{-1}\frac{V(r)}{\Omega r} = \tan^{-1}\left[\frac{R}{r}\lambda(r)\right]. \tag{2}$$

The pitch of the helicoidal surface is $P(r) = 2\pi\lambda(r)$. For simplicity the propeller disc radius R shall be normalized to unity. Consequently the helicoidal surfaces can be approximated as

$$H_k(x, r, \vartheta) = x - \lambda(r)(\vartheta - \delta_k) = 0, \quad (k = 1, 2, \dots N.). \tag{3}$$

It is also convenient to introduce an intrinsic coordinate system (s, n, r) on each of the surfaces $H_k = 0$, defined by

$$\begin{pmatrix} s \\ n \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} r(\vartheta - \delta_k) \\ x \end{pmatrix}. \tag{4}$$

so that s measures the arc length along the $H_k = 0$ surface at radius r, and the n-axis completes the right-hand system. For an arbitrary vector $\vec{q}$ having components $(q_x, q_y, q_z)$ in the (x, y, z) system, its components $(q_s, q_n, q_r)$ in the (s, n, r) system are given by the orthogonal transformation

$$\begin{pmatrix} q_s \\ q_n \\ q_r \end{pmatrix} = \tag{5}$$

$$= \begin{pmatrix} \sin\beta & -\cos\beta & 0 \\ \cos\beta & \sin\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin(\vartheta - \delta_k) & -\cos(\vartheta - \delta_k) \\ 0 & \cos(\vartheta - \delta_k) & \sin(\vartheta - \delta_k) \end{pmatrix} \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix}$$

This is obtained by first a rotation about the x-axis through an angle $-(\pi/2 - \vartheta + \delta_k)$ and then another rotation about the radail axis through an angle $-(\pi/2 - \beta)$ (see Figure 1).

Let the $\vartheta$-coordinates of the leading and trailing edges be $\vartheta_L(r)$ and $\vartheta_T(r)$. Then the corresponding s-coordinates are $s_L(r)$ and $s_T(r)$, where on the $H_k = 0$ surface the s and $\vartheta$ coordinates are related by

$$s = [r^2 + \lambda^2(r)]^{1/2}\,(\vartheta - \delta_k). \tag{6}$$

The chord of the blade is $\ell(r) = s_T(r) - s_L(r)$. The blade profile can be described by a mean camber line f (s, r), a thickness function t (s, r) and an angle of attack $\alpha(r)$.

### *2a. Induced Velocity due to a Distribution of Bound and Free Vortices*

Leaving the effect of blade thickness to be accounted for later by superposition, the effect of camber and incidence of each blade can be represented by a vortex sheet having a distribution of vorticity vector $\vec{\gamma}$ lying in the helicoidal surface behind the landing edge. Over the blade surface, $\vec{\gamma}$ has a radial component $\gamma_b$, which is taken to be the bound vortex, and a streamwise component $\gamma_s$ along the s-axis, which is the free vortex component. On the trailing helicoidal surface $(s > s_T(r))$, $\vec{\gamma}$ has only the $\gamma_s$ component. The induced velocity field depends on both $\gamma_b$ and $\gamma_s$, whereas the pressure difference across the blade surface, according to the linear theory, is

$$\begin{aligned} \Delta p(r, \vartheta) &= \varrho_0 V_0(r)\, \gamma_b(r, \vartheta) = \\ &= \varrho_0 V(r) \left\{[r^2 + \lambda^2(r)]^{1/2} / \lambda(r)\right\} \gamma_b(r, \vartheta) \end{aligned} \tag{7}$$

where $\varrho_0$ is the fluid density, $V_0$ the resultant relative velocity.

By the principle of conservation of vorticity, we have $\operatorname{div}\vec{\gamma} = 0$, or

$$\frac{\partial}{\partial r}\left[\gamma_b(r, \vartheta)\sqrt{r^2 + \lambda^2(r)}\right] + \frac{\partial}{\partial\vartheta}\gamma_s(r, \vartheta) = 0, \tag{8}$$

which yields, upon integration,

$$\gamma_s(r, \vartheta) = -\frac{\partial}{\partial r}\int_{\vartheta_L(r)}^{\vartheta} \gamma_b(r, \vartheta)\sqrt{r^2 + \lambda^2(r)}\, d\vartheta \quad (\vartheta > \vartheta_L(r)). \tag{9}$$

In general, $\gamma_s$ has a jump at the leading and trailing edges because of the dependence of $\vartheta_L$ and $\vartheta_T$ on r; these jumps have already been taken into account in (9).

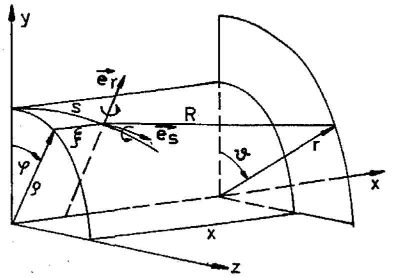


**Figure 2**

For a given distribution of bound vortices over the blade surfaces, the induced velocity field is given by the Biot-Savart's law (see Figure 2)

$$\vec{q}_b(x, r, \vartheta) = \int_{r_{hub}}^{1} d\varrho \int_{s_L(\varrho)}^{s_T(\varrho)} \gamma_b(\varrho, \varphi)\, \vec{G}_b(x, r, \vartheta; \varrho, \varphi)\, ds,$$

where

$$\vec{G}_b(x, r, \vartheta; \varrho, \varphi) = \sum_{k=1}^{N} \frac{\vec{e}_{rk} \times \vec{R}_k}{4\pi R_k^3} \tag{10}$$

$$\vec{e}_{rk} = (0; \cos\varphi_k, \sin\varphi_k), \qquad \varphi_k \equiv \varphi + \delta_k,$$

$$\vec{R}_k = (x - \xi_k, r\cos\vartheta - \varrho\cos\varphi_k, r\sin\vartheta - \varrho\sin\varphi_k),$$

$$\xi_k = \lambda(\varrho)\,\varphi, \quad ds = [\varrho^2 + \lambda^2(\varrho)]^{1/2}\, d\varphi.$$

When the velocity at the blade surface is evaluated, the above integral is interpreted by its Cauchy principal value.

Similarly, for a given distribution of free vortices over the blade surfaces and the trailing helicoidal surfaces the induced velocity fields is

$$\vec{q}_t(x, r, \vartheta) = \int_{r_h}^{1} d\varrho \int_{\vartheta_L(\varrho)}^{\vartheta_T(\varrho)} \frac{\partial\gamma_s(\varrho, \varphi')}{\partial\varphi'}\, \vec{G}_t(x, r, \vartheta; \varrho, \varphi')\, d\varphi',$$

$$\vec{G}_t(x, r, \vartheta; \varrho, \varphi) = \sum_{k=1}^{N} \int_{\varphi}^{\infty} \frac{\vec{e}_{sk} \times \vec{R}_k}{4\pi R_k^3}\, ds, \tag{11}$$

where

$$\vec{e}_{sk} = \frac{1}{\sqrt{\varrho^2 + \lambda^2(\varrho)}} (\lambda(\varrho), -\varrho\sin\varphi_k, \varrho\cos\varphi_k),$$

$$\varphi_k = \varphi + \delta_k,$$

and $\vec{R}_k$, s have the same expressions as before. The distribution of free vortices $\gamma_s$ can again be expressed in terms of the bound vortices $\gamma_b$ by using

$$\begin{aligned} \frac{\partial\gamma_s(\varrho, \varphi)}{\partial\varphi} = &-\frac{\partial}{\partial\varrho}\left[\gamma_b(\varrho, \varphi)\sqrt{\varrho^2 + \lambda^2(\varrho)}\right] \\ &+ \sqrt{\varrho^2 + \lambda^2(\varrho)}\left[\gamma_b(\varrho, \vartheta_L)\frac{d\vartheta_L}{d\varrho}\delta(\varphi - \vartheta_L)\right. \\ &\left. - \gamma_b(\varrho, \vartheta_T)\frac{d\vartheta_T}{d\varrho}\delta(\varphi - \vartheta_T)\right] \end{aligned} \tag{12}$$

in which $\delta(\varphi)$ denotes the Dirac delta function. The first term represents the contribution of the trailing vortices orginated from the blade surafce, while the second term represents the trailing vortices sprung from the leading and trailing edges.

## 2b. *Induced Velocity due to Blade Thickness*

The effect of blade thickness has been incorporated into the linear propeller theory by Kerwin and Leopold (1963), and Nelson (1964), by introducing a source-sink system distributed over the blade plan-form on the helicoidal surfaces $H_k = 0$, yielding the induced velocity field

$$\vec{q}_s(x, r, \vartheta) = \int_{r_h}^{1} d\varrho \int_{s_L(\varrho)}^{s_T(\varrho)} \sigma(\varrho, \varphi)\, \vec{G}_s(x, r, \vartheta; \varrho, \varphi)\, ds \tag{13}$$

where

$$\vec{G}_s(x, r, \vartheta; \varrho, \varphi) = \sum_{k=1}^{N} \operatorname{grad}_{(x,y,z)}\left(\frac{-1}{4\pi R_k}\right),$$

and the source strength $\sigma(\varrho, \varphi)$ is, by the usual linear approximation,

$$\begin{aligned} \sigma(\varrho, \varphi) &= V_0(\varrho)\frac{\partial t}{\partial s} = \frac{V(\sigma)}{\sin\beta(\varrho)}\frac{\partial t}{\partial s} \\ &= \frac{V(\varrho)}{\lambda(\varrho)}\sqrt{\varrho^2 + \lambda^2(\varrho)}\,\frac{\partial t}{\partial s}, \end{aligned} \tag{14}$$

t (s, r) being the thickness function of the blade.

## 2c. *Application of Boundary Conditions; the Integral Equation*

By superposition, the velocity component normal to the first blade surface is

$$q_n(r, \vartheta) = q_{nb}(r, \vartheta) + q_{nt}(r, \vartheta) + q_{ns}(r, \vartheta), \tag{15a}$$

with

$$q_{nb}(r, \vartheta) = \int_{r_h}^{1} d\varrho \int_{\vartheta_L(\varrho)}^{\vartheta_T(\varrho)} \gamma_b(\varrho, \varphi)\, B(r, \vartheta; \varrho, \varphi)\sqrt{\varrho^2 + \lambda^2(\varrho)}\, d\varphi, \tag{15b}$$

$$q_{nt}(r, \varrho) = \int_{r_h}^{1} d\varrho \int_{\vartheta_L(\varrho)}^{\vartheta_T(\varrho)} \frac{\partial\gamma_s(\varrho, \varphi)}{\partial\varphi}\, T(r, \vartheta; \varrho, \varphi)\, d\varphi, \tag{15c}$$

$$q_{ns}(r, \vartheta) = \int_{r_h}^{1} d\varrho \int_{\vartheta_L(\varrho)}^{\vartheta_T(\varrho)} \sigma(\varrho, \varphi)\, S(r, \vartheta; \varrho, \varphi)\sqrt{\varrho^2 + \lambda^2(\varrho)}\, d\varphi, \tag{15d}$$

in which $B(r, \vartheta; \varrho, \varphi)$, $T(r, \vartheta; \varrho, \varphi)$ and $S(r, \vartheta; \varrho, \varphi)$ are respectively the n-components of $\vec{G}_b(x, r, \vartheta; \varrho, \varphi)$, $\vec{G}_t(x, r, \vartheta; \varrho, \varphi)$ and $\vec{G}_s(x, r, \vartheta; \varrho, \varphi)$ evaluated on the first blade in question, i. e. $x = \lambda(r)\,\vartheta$. Resolution of $\vec{G}$ into the n-axis is readily achieved by using the orthogonal transformation (5).

Further application of this linear theory depends on the type of problems, which may arise in the following categories:

(i) Design problem — to determine the shape of the blade sections for prescribed plan form ($\vartheta_L(r), \vartheta_T(r)$), vorticity distribution $\gamma_b(r, \vartheta)$, and thickness function $t(r, \vartheta)$.

This problem is the most straightforward. Since the strengths of the singularities are all known, $q_n$ is readily obtained from (15) by integration; the section shape is then deduced by integration of the slope. Because of the helical geometry and neighboring blades, the section mean line is affected by vortices as well as the blade thickness — this is not true for a solitary planar lifting surface in an infinite fluid.

(ii) Inverse problem — to determine the load distribution for a given blade shape and plan-form.

This more complicated problem arises when a propeller operates at other than design condition. Since the initial *radial and chordwise load distributions are unknown,* (15) gives an integral equation for their determination. An alternative collocation method has been suggested by Kerwin (1963), which seems very efficient since it reduces the problem to a compound of problems of the first type. The essential idea is to decompose the unknown load distribution in a double summation of known chordwise and radial modes, the normal velocity induced at a set of points on the blade surface by each mode can be determined as before, and the unknown amplitudes of these modes then determined by collocation.

The above presentation of the theory facilities the subsequent discussions of various theories developed by different authors, they differ from one another by further simplifying assumptions and detailed methods of calculation. *With the* general view already evaluated, it becomes relatively clear to see as to *where each specific theory lies in the present state* of our knowledge.

## 3. Lifting-Line Theories

In order to deduce the lifting-line approximation from the above lifting-surface theory, we introduce the following de*composition of* $q_{nt}$

$$q_{nt}(r, \vartheta) = q_{nt}^{(0)}(r) + q_{nt}^{(1)}(r, \vartheta), \qquad (16a)$$

$$q_{nt}^{(0)}(r) = -\int_{r_h}^{1} \frac{d\Gamma(\varrho)}{d\varrho} T_o(r, \varrho)\, d\varrho,$$

$$T_o(r, \varrho) \equiv T(r, 0; \varrho, 0), \qquad (16b)$$

$$q_{nt}^{(1)}(r, \vartheta) = \int_{r_h}^{1} d\varrho \int_{\vartheta_L(\varrho)}^{\vartheta_T(\varrho)} [T(r, \vartheta; \varrho, \varphi) - T_o(r, \varrho)] \frac{\partial \gamma_s(\varrho, \varphi)}{\partial \varphi} d\varphi \qquad (16c)$$

in which $\Gamma(\varrho)$ arises from the integration

$$\int_{\vartheta_L(\varrho)}^{\vartheta_T(\varrho)} \frac{\partial \gamma_s(\varrho, \varphi)}{\partial \varphi} d\varphi = -\frac{d}{d\varrho} \int_{\vartheta_L(\varrho)}^{\vartheta_T(\varrho)} \gamma_b(\varrho, \varphi) \sqrt{\varrho^2 + \lambda^2(\varrho)}\, d\varphi$$

$$= -\frac{d}{d\varrho} \int_{s_L(\varrho)}^{s_T(\varrho)} \gamma_b(\varrho, \varphi)\, ds = -\frac{d\Gamma(\varrho)}{d\varrho},$$

upon using (12), *so that* $\Gamma(\varrho)$ *is the total circulation around* each blade at $r = \varrho$.

The usual lifting-line approximation for thin propellers is based on the assumption that $q_{nt}^{(1)}$ and $q_{ns}$ may both be neglected so that

$$q_n(r, \vartheta) \cong q_{nb}(r, \vartheta) + q_{nt}^{(0)}(r), \qquad (17)$$

and furthermore, $q_{nb}(r, \vartheta)$ may be approximated by the two-dimensional theory for the sections at the particular radius in question. The latter approximation is usually referred to as the "strip theory". These assumptions, however, are valid only for blades of large aspect ratio.

We now review some *of the lifting-line theories* which have played a significant role in the development of the propeller theory.

### Goldstein's theory

In its original version due to Goldstein (1929), this theory, applicable to a propeller having a zero hub diameter and with the optimum circulation distribution, determines the potential *flow past a set of rigid helical membranes. This important work laid the foundation in relating the circulation distribution* of a propeller with a finite number of blades to one having an infinite number. From this theory the relation between the optimum circulation $\Gamma(r)$ and the tangential component of the induced velocity $v_t$ can be expressed as

$$N\Gamma(r) = \varkappa\, 2\pi r\, (2 v_t) \qquad (18)$$

where N is the number of blades, the factor $\varkappa$ is a function of $\lambda$ and r only. The quantity $2\pi r\,(2 v_t)$ may be regarded as the optimum circulation for infinitely many blades which produces the same axial induced velocity as the propeller in question. The problem of the induced velocity by helical vortices has also been treated by Kawada (1936) and Reissner (1937).

Goldstein summed the infinite series representing the potential with approximations for the coefficients for the advance ratio up to $\lambda = 0.5$. These approximations introduces small errors for small $\lambda$ but the error increases with increasing $\lambda$. More accurate evaluations for larger values of $\lambda$ have been given by Kramer *(1938)* and *Tachmindji and Milam (1956)*.

Goldstein's theory has been subsequently generalized to moderately loaded optimum propellers by taking the induced velocity into account in approximating the hydrodynamic pitch angle. The factor $\varkappa$ and the induced pitch angle for such case has been calculated by Kramer (1938) and Tachmindji (1956) *based on Goldstein's theory. The effect of finite hub has been* treated by McCormick (1955, without requiring the circulation to vanish at the hub), by Tachmindji (1957, with the circulation vanishing at the hub), by Schultz (1957), and subsequently extended by Mercier (1962). This theory has often been used as a guiding calculation for non-optimum propellers.

### Lerbs' Induction-Factor Method

To facilitate the calculation, in particular of the Cauchy principal value of the integrals involved, Lerb (1952) has introduced the method of induction-factors, which amounts to factoring the singular part of $T_o(r, \varrho)$ in (16b), so that the tangential and axial component, $v_t$ and $v_a$, of the induced velocity evaluated at the lifting line can be expressed as

$$\frac{v_t}{V} = \frac{1}{2} \int_{r_h}^{1} \frac{d\tilde{\Gamma}}{d\varrho} \frac{i_t(r, \varrho)}{r - \varrho} d\varrho, \qquad (20)$$

and an analogous equation for $i_a$, where $\tilde{\Gamma}$ is the dimensionless circulation defined as $\tilde{\Gamma} = \Gamma / 2\pi RV$. The factors $i_t$, $i_a$ depend on r, $\varrho$, the number of blades, and the constant pitch angle $\beta$ only, and are independent of the load distribution. Therefore they represent the effect of a purely geometric nature, and can be calculated once and for all over *a useful range of* $\beta$. *These factors have been calculated by* Lerbs (1952, 1955) in an analytical manner, and later by Morgan (1957) and Wrench (1957). By using some highly accurate asymptotic approximations to several series involving the modified Bessel functions, which are expressed in terms of only elementary functions, Wrench has been able to greatly reduce the numerical computation involved.

The induction factors $i_a$, $i_t$, as well as the radial component $i_r$ have also been calculated by Streschеletzky (1950, 1955) by stepwise integration of the fundamental equations in the streamwise direction. These results can be used for estimating the contraction of the slipstream and the variation of the pitch of the helical surface, as used by Streschelezky for a lifting-surface approximation.

For further primary exploration and discussion of the method of calculating the induced velocity reference may be made to Moriya (1933, 1936), Schubert (1940).

## 4. Lifting-Surface Theories

The development of the lifting-surface theory is a rather recent endeavour, primarily because precise and extensive computations involved in applications of such theories appeared to be infeasible before the high-speed computer era. Even with a high-speed computer, the numerical calculation is by no means trivial and simple, as reported by various authors. In fact, use of high-speed computers often introduces another dimension of technology — i. e., to devise the most effective and simplified method of calculation to aid our purpose.

Before we discuss various specific theories, perhaps it is noteworthy to consider a particular case for which the basic features of the solution can be readily derived from the general theory. First, if $\lambda$ is a constant (generally a good assumption), then the fundamental solutions B, T and S can be seen to depend on $\varphi$ and $\vartheta$ only in the combination of $\mu = \varphi - \vartheta$. Furthermore, we note that

$$B(r, \varrho, \mu), \quad T_1(r, \varrho, \mu) = T(r, \varrho, \mu) - T_0(r, \varrho), \quad S(r, \varrho, \mu) \tag{21}$$

are all odd functions of $\mu$.

In addition, if the blade plan-form, chordwise load distribution, and blade thickness are all symmetric about the line $\vartheta = \delta_k$ through the blade tip, then it can be shown (see, e. g., Kerwin (1963)) that

$$q_{nb}(r, \vartheta), \quad q_{nt}^{(1)}(r, \vartheta) \text{ are odd in } \vartheta; \quad q_{ns}(r, \vartheta) \text{ even in } \vartheta. \tag{22}$$

Therefore it can be concluded that

(i) a propeller of zero thickness with symmetrical blade outline and load distribution needs no incidence correction to the lifting-line theory (which is determined by $q_{nt}^{(0)}$ alone), the lifting surface effect produces solely a camber correction in this case;

(ii) the effect of blade thickness may give rise to an additional pitch correction, but cannot induce a net camber.

Though these conclusions no longer hold without the assumed symmetry they can nevertheless be used for qualitative estimates.

Now we review below some of the recent development of lifting surface theories:

### The Theory of Ludwieg and Ginzel

In estimating the lifting-surface effects according to the theory developed by Ludwieg and Ginzel (1944), and extended by Ginzel (1955), both the induced downwash and its chordwise variation are regarded to be important since the camber in curved flow is less effective than in straight flow. The downwash variation gives rise to the camber correction according to the relation

$$f_{geom} = f_{eff} + f_{ind} = k\, f_{eff} \tag{23}$$

where $f_{geom}$ is the geometric camber at the mid-chord of the blade, $f_{eff}$ is the effective camber associated with a local two-dimensional profile characteristics (for the same load distribution), $f_{ind}$ is the induced camber, and k (or its inverse by some authors) is called the camber correction factor. Originally, the effective camber is determined for circular arc camber line with shock-free entry of flow at the leading edge (a design condition) so that the sectional lift is due to the local section camber only (the corresponding chordwise circulation distribution being then elliptical). With some approximations, the streamline curvature at the mid-chord is computed from the rate of change of induced downwash at that point, and is in turn related to the geometric camber. Thus, the factor k has been determined for a few plan forms and blade numbers with a prescribed spanwise load distribution. These results have been utilized in earlier pratical applications with a certain amount of interpolations for new cases. For sometime this was the only available means for estimating the flow curvature correction.

The case of optimum propellers with constant pitch and finite hub was subsequently treated by Cox (1961).

### Lerbs' Method for Pitch Correction

An approximate estimate of the pitch correction factor was proposed by Lerbs (1955b) assuming that the bound vortex is concentrated at the quarter-chord point and the boundary condition is satisfied at the $^3/_4$-chord point. Application of this Weissinger approximation for a lifting wing to the propeller problem thus offers a simple method for determining the pitch correction arising from the lifting-surface effect.

### Sparenberg's Lifting-Surface Theory

Sparenberg (1959) started from the basic equations of hydrodynamics and derived the integral equation for a distribution of bound vortices representing the propeller blades. His result is in complete agreement with the formulas previously presented here (aside from some change of notations). An equivalent expression, also in an integral equation form, has been derived by Sparenberg when the blades are represented by a distribution of pressure dipoles. These two expressions are equivalent, but the numerical computations involved in using them differ to some extent.

Both results of Sparenberg's theory have been adopted by van Manen and Bakker (1962) in systematic calculations, using the digital computer X 1, for symmetric blades and pressure distributions. The results give camber correction factors and pitch corrections for different pitch ratios, number of blades, chord-diameter ratios, and three types of radial load distributions. For the details of the computing program, the reader is referred to the original paper.

The numerical results are in conformity with the general features pertaining to the case of symmetric blade outline and chordwise distribution. A representative result is the radial distribution of the camber correction factor k arising from the lifting-surface effect. It is shown that the factor k is constant along the chord. In general, the factor k decreases with increasing number of blades and with decreasing chorddiameter ratio.

### Pien's Theory

In Pien's development (1961) of the lifting-surface theory, the same result as presented here was obtained. The induced mean line at any radius is derived from the down-wash at a sufficiently large number of points along the chord. Based on this theory a new propeller design method has been developed. The numerical work involved in this design method has been programed for the high-speed computer IBM 709 for a special case of uniform chordwise load distribution. Two design examples have been given, one with a symmetrical blade, the other a skewed blade. It is generally regarded that Pien's method produces reliable result for applications. The corresponding experimental verification will be presented in the future.

### Kerwin's Theory

Kerwin (1963) has successfully incorporated the thickness effect into the lifting-surface theory. For the propeller design problem a computer program has been furnished for calculating the velocity field at a sufficiently large number of points over the entire blade. Furthermore, with the help of this program, it becomes possible to design tandem propellers and a better determination of the velocities for counter-rotating propellers than what we have had in the past. (I have been told that a complete paper on these works will be presented by Kerwin at the SNAME meeting in November.)

Recently, both the work of Kerwin and of Pien on lifting-surface and thickness corrections have been applied to practical propeller designs. For some of the experimental checks already made at DTMB (private communication of W. B. Morgan), the results have shown that these corrections obtained for the case of load distribution corresponding to the NACA $a = 0.8$ meanline are satisfactory.

### Guilloton's Vortex-Lattice Theory

A vortex-lattice method, similar to that developed by Falkner for wings, has been employed by Guilloton (1957) to approximate the lifting-surface effect in propeller theory, the technique being simple enough for direct applications in particular cases. According to this method the bound vortex system is replaced by 5 concentrated radial vortex lines, at 20° apart, and each radial vortex line is divided into 5 steps. The circulation strength of the vortex line in each segment is assumed to be constant, proportional to the local distribution of bound vorticity, thus giving rise to six concentrated free vortices trailing on the helical surface. The induced velocities at these 30 points has been calculated and results tabulated by Guilloton for $0.416 < \lambda < 1.25$ for a three-bladed propeller. These tabulated results facilitate applications for individual cases. However, because of the fixed number of blades and fixed dividing angle between radial lines, other cases have to be calculated anew. Some results by using Guilloton's method have been compared with other theories by Johnsson (1962); a typical one is the camber correction factor.

### The Vortex-Lattice Theory of English

An improved, more flexible, vortex-lattice method has been formulated recently by English (1962). By this lattice pattern, the radial vortex distribution is divided into 20 steps and each radial strip into six equally spaced radial vortices. The radial steps narrow down towards the blade tip to achieve a higher accuracy. This vortex system is located on helical surfaces with the pitch corresponding to the final hydrodynamic pitch of the propeller. The latter is determined by the lifting-line consideration for optimum, moderately loaded propellers.

The camber correction factor has been calculated by English for two specific propellers.

## 5. Moderately Loaded Propellers

A higher accuracy in propeller calculation can be achieved if the sources and vortices representing the blades are distributed over the blade surfaces and if the trailing vortices follow the actual streamlines. However, both these two steps of improvement give rise to nonlinearity since the singularity strength depends on the local velocity, both this and the location of the trailing stream surface are not known in advance.

For not too heavily loaded cases, however, the so-called "moderately loaded" assumptions are particularly simple to apply and they keep the methods of solution for the lightly loaded case virtually intact. These assumptions, as discussed in detail by Lerbs (1952), are as follows:

(i) the effect of the radial velocity is negligible so that the streamlines remain on their own circular cylindrical surface;

(ii) the distortion of the streamline due to the axial and tangential perturbation velocities can be approximated by the final hydrodynamic pitch determined by the lifting-line consideration.

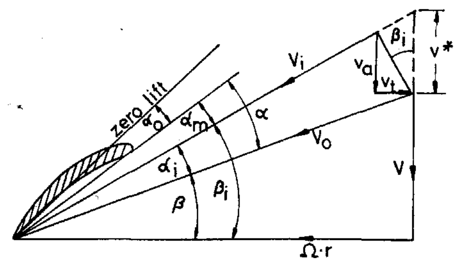


**Figure 3**

Thus, referring to Figure 3, we define the hydrodynamic pitch angle $\beta_i(r)$ and the hydrodynamic advance coefficient $\lambda_i(r)$ by

$$\tan\beta_i(r) = \frac{V(r) + v_a(r)}{\Omega r - v_t(r)}, \quad \lambda_i(r) = r\tan\beta_i(r) \tag{24}$$

where $v_a$ and $v_t$ are $v_a = q_x^{(0)}$ and $v_t = -q_\vartheta^{(0)}$ in the present notation, they are denoting the magnitude of the axial and tangential induced velocities at the lifting line in the framework of the lifting-line theory. They can be determined by using Kramer and Tachmindji charts or by using the induction-factor method of Lerbs. Thus, all the calculations for the lightly loaded case can be adopted for the moderately loaded operations, simply by replacing $\lambda$ by $\lambda_i$.

Further improvement of $v_a$ and $v_t$, such as by an iteration scheme, is perhaps not necessary, since the higher-order effects so obtained may not be more important than the heavily loaded effects which are still being neglected.

## 6. Heavily Loaded Propellers

When the loading on a propeller becomes sufficiently heavy, the problem is characterized by the following effects, which may no longer be negligible,

(i) slipstream contraction and distortion,
(ii) radial pressure gradient,
(iii) effect due to finite number of blades,
(iv) solidity, or the chord-diameter ratio.

The first two effects, so far neglected for lightly loaded, and even for moderately loaded propellers, become pertinent to the heavily loaded case. They may arise also in the problems of compressors and turbines when load is heavy. Though the last two effects are common features to propellers of all loadings, it is not certain if the evaluation of these effects made for the lightly load case will need further corrections.

In the previous investigations of heavily loaded propellers, the problem has mostly been formulated as a potential flow problem as a further extension of moderately loaded propellers. Betz and Helmbold (1932) first considered the effect of slipstream contraction and radial pressure gradient on the performance of an infinitely-bladed propeller. This theory has been further extended by Lerbs (1950) to heavily loaded prolers of a finite number of blades, using a modified lifting-theory.

\ different approach to determine the effect of arbitrary stream contraction has been developed by Wu (1962); method may serve as an independent means to supplement the existing theories. In order to obtain an axial symmetry of the flow, the propeller is approximated by an actuadisc. With respect to a cylindrical coordinate system $\vartheta$, z) fixed in the space (see Fig. 4), the flow velocity (u, v, w) possesses a stream function $\Psi$ (r, z) defined by

$$u = -\frac{1}{r}\frac{\partial \Psi}{\partial z}, \quad w = \frac{1}{r}\frac{\partial \Psi}{\partial r}. \tag{25}$$

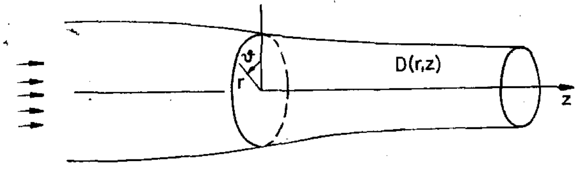


**Figure 4**

velocity $\vec{q}$ and coordinates r, z will be normalized with pect to free stream velocity V and disc radius R. The kinetics is then fully described by $\Psi$ and the tangential velo- v. By the conservation of angular momentum, (vr) is an itrary function of $\Psi$ inside the slipstream. Then the tantial component of vorticity yields a partial differential iation for $\Psi$:

$$\frac{\partial^2 \Psi}{\partial r^2} - \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{\partial^2 \Psi}{\partial z^2} = \varepsilon_s \left(\frac{r^2}{\lambda} - vr\right) \frac{d}{d\Psi}(vr)$$
$$= -g(r, z; \Psi(r, z)) \tag{26}$$

ere $\lambda = V/\Omega R$, the advance coefficient, and $\varepsilon_s = 1$ inside slipstream and $\varepsilon_s = 0$ otherwise. This nonlinear differen- equation can be converted into a nonlinear integral equa- for the perturbation stream function $\psi = \Psi - r^2/2$ as lows

$$\psi(r, z) = r \iint_{D(r,z)} G(r, \varrho; z - \zeta)\, g(\varrho, \zeta; \psi(\varrho, \zeta))\, d\varrho\, d\zeta \tag{27}$$

ere D (r, z) denotes the domain of the slipstream, which inknown à priori,

$$G(r, \varrho; z - \zeta) = \frac{1}{2}\int_0^\infty e^{-(z-\zeta)t} J_1(rt)\, J_1(\varrho t)\, dt =$$
$$= \frac{1}{2\pi\sqrt{r\varrho}}\, Q_{1/2}\left(\frac{r^2 + \varrho^2 + (z-\zeta)^2}{2 r \varrho}\right) \tag{28}$$

which $J_1(z)$ is the Bessel function of the first kind, $Q_\nu(z)$ Legendre function of the second kind.

The above integral equation has been solved by an iteration cess. Starting from the lightly loaded solution $\psi_0$ (r, z), and nce with a known domain $D_0$ (r, z), and substituting them the right side of (27), we obtain the first order solution (r, z). Using $\psi_1$ in the integral of (27) again yields $\psi_2$ (r, z), d so forth. The detailed numerical calculation has been proımed for the computer IBM-7094, and a few typical eases being carried out to exhibit the effect of heavy loading.

## 7. Ducted Propellers

The problem of ducted propellers has drawn recent interest both in aeronautics and in naval hydrodynamics. By adding a duct or shroud to a propeller one can produce a thrust on *the duct and increase the flow rate through the propeller,* especially in hovering flight. For the same thrust on the duct-propeller system the propulsive efficiency can be increased with the flow rate, particularly for heavily loaded propellers. Furthermore, the decrease in loading on the propeller blades can alleviate compressibility effects, cavitation inception, and noise generation.

At an incidence, the interaction between the duct and propeller can generate a larger lift and thrust than that on the seperate duct and open propeller combined. This significant feature makes it an attractive device for use in hovering flight and vertical take-off vehicles.

The interest in the above fields has led to various experimental and theoretical investigations. Experimentally the advantages of ducted propellers were demonstrated in 1931 by Stipa in Italy. Earlier theoretical investigations have made use of the ring airfoil theory developed by Dickmann (1940) and Weissinger (1955), and the representation of the propeller by an actuator disc. A rather recent review of these experimental and theoretical activities has been given by van Manen (1957), and by Sacks and Burnell (1962), the details of which will not be repeated here.

Because of the analytical complexity, accurate lifting-line theory has been developed only recently for ducted propellers (such a task would seem infeasible in pre-computer days). A three-dimensional theory of ducted propellers has been formulated by Ordway, Sluyter and Sonnerup (1960) based on the vortex theory for the propeller of finite blades and the thin ring-wing theory for the duct. The general harmonic solutions of this problem have been given subsequently by Ordway and Greenberg (1961). Another theory has been developed by Morgan (1961, 1962) based on the lifting-line propeller theory and a linearized ring-airfoil theory. Here the analysis of the ring-airfoil of an arbitrary shape (in camber, thickness and annular incidence) includes the effect of incidence of the axis. In the lifting-line calculation of the ducted propeller the method of Lerbs' induction factor has been adopted. Both the design and inverse problems are treated. The numerical program of this theory is being completed for future applications (Morgan, private communication).

Also recently, the static and dynamic stability derivatives of a ducted propeller have been evaluated theoretically by Kriebel, Sacks and Nielsen (1963). In this investigation the ducted propeller is represented by a uniformly loaded actuator disc ducted in a short, straight, thin ring airfoil. Experimental observations have shown that flow separation is generally present at the duct leading edge.

## 8. Unsteady Propeller Theory

It is hardly necessary to emphasize the importance of the unsteady flow effects on propeller operations as it is inevitable *to encounter various circumstances characterized by unsteadiness.* These circumstances may arise from unsteady, as well as non-uniform, free stream, such as in waves and in wakes behind obstacles, from the presence of asymmetric boundary, from dynamic vibrations, etc. The main purpose in applications is the prediction of the induced vibratory forces on the propeller itself as well as on nearby bodies. Such studies are pertinent to hull vibration and other hydroelastic instabilities, material failure, and underwater acoustics.

Earlier, theoretical investigations have been performed for unsteady rotating wings of an infinite number of blades by Timman and van der Vooren (1957), Loewy (1957), and Isay (1958). The effect of finite number of blades was evaluated by Ritger and Breslin (1958) by a strip theory, using Sear's two-dimensional response function for the sinusoidal gust. However, comparison with experimental results by Tsakonas and Jacobs (1961) shows that the strip theory is inadequate for unsteady marine propeller, despite the use of semi-empirical correction factors to account for the three-dimensional effects.

In search of more realistic approximations for the unsteady propeller problem, Shioiri and Tsakonas (1963) developed a three-dimensional theory with the application of the Weissinger's lifting surface approximation (originally for the steady case), use of which reduces the surface integrals into line integrals. As a modification of the Weissinger method, the chordwise boundary conditions are satisfied by a weighted average over the chord. Compared with the experimental measurements, the modified Weissinger method appears to be a considerable improvement over the strip theory. A similar treatment has been given by Yamazuki (1962).

An unsteady lifting-surface theory has been formulated by Hanaoka (1962) by means of Prandtl's acceleration potential from which the velocity potential can be derived. The boundary-value problem is expressed in terms of the singular integral equation relating a prescribed downwash distribution to an unknown lift distribution for oscillating blades. This development has been carried out to the stage for detailed numerical calculation of the kernel function.

More recently, Tsakonas and Jacobs (1964) have solved the surface integral equation for a mathematical model in which the chordwise loading is taken to be the flat-plate distribution (the first term of Birnbam's distribution) together with use of Glauert's integral operator, which amounts to satisfying the chordwise boundary conditions by a weighted average. This model has been shown to be a further improvement over the modified Weissinger model, especially for large reduced frequencies. From the results the authors conclude that the three-dimensional effects decrease with increase in frequency, in pitch, and in aspect ratio.

At the 1963 Annual Meeting of the Schiffbautechnische Gesellschaft in Hamburg, Schwanecke (1963) presented a paper on unsteady propeller motions caused by unsteady incoming flows as well as by vibrations of the propeller shaft.

## 9. Contra-rotating Propellers

In selecting the propulsion system to meet the ever-increasing demand for larger bulk and higher speed of ships, appropriate considerations must be given to the problem of cavitation and propeller-induced vibrations in addition to the requirement for an optimum efficiency. Recent studies have indicated the trend of increasing cavitation and propeller-induced vibrations and decreasing propulsive efficiency with higher and higher shaft horsepower absorbed by conventional propellers. The urgent need to meet the aforementioned requirements have stimulated interest in the contra-rotating (or counter-rotating) propellers and tandem propellers besides the conventional screw and shrouded propellers. In this respect contra-rotating propellers have several advantages: (i) reduction of rotational energy in the slipstream; (ii) lower loading per blade (or smaller optimum diameter); (iii) more stable torque balance. All these features are in favor of meeting the basic requirements.

In the early development of contra-rotating propellers, drastic simplification was introduced by assuming that the fore- and aft-propellers act as two single propellers. Later the method of induction-factors was developed by Lerbs (1955c) for contra-rotating propellers. Lerbs' theory has been subsequently extended to formulate a design method by Morgan (1960). This problem has also been treated by van Manen and Sentic (1956), who have made a comparison between the optimum efficiencies of conventional screws and contra-rotating propellers consisting of two three-bladed screws. This work has been continued to investigate a system of contra-rotating propellers having a four bladed fore-screw and a five bladed aft-screw — a configuration expected to be superior in mutual interaction and induced vibrations. It may be mentioned here that in these treatments the induced velocity evaluated at the fore- and aft-propeller blades are approximated by the time average value, whereas in reality, to be strictly speaking, it is an unsteady lifting surface flow problem. A recent treatment of this problem has been given by Zwick (1962) using the unsteady three-dimensional vortex theory.

## 10. Vertical-axis Propellers

Among the propellers having special advantages in manoeuvering, the vertical-axis propeller (Voith-Schneider type) plays an important role; it has found wide applications on river and lake vessels. The literature on this subject is quite extensive. Some earlier developments in the design and application of the vertical-axis propeller have been reviewed by Mueller (1955). More recent investigations of this type propellers have been carried out in Germany by Isay (1955, 1956, 1957, 1958), in Japan by Taniguchi (1944, 1950, 1960), in U. S. by Haberman (1961, 1962), Nakonechny (1961), and in Holland by Sparenberg (1960) and van Manen (1963).

The method proposed by Taniguchi for computing the performance characteristics of vertical-axis propellers is based on the approximation of the real motion by a quasi-steady state with furter simplifying assumptions (of a semi-empirical nature) for estimating the induced velocity. This method has been employed by Haberman to determine the performance characteristics of several propellers with semi-elliptic blades in cycloidal blade motion, yielding results in satisfactory agreement with the experiments. A series of experimental investigation has been carried out by Nakonechny (1961). The problem of minimum energy loss of a vertical axis propeller has been treated by Sparenberg, and the relevant experiments have been performed systematically by van Manen (1963) who also made an extensive studies on the cavitation characteristics of the blades, particularly in the high-pitch and high speed range.

An extensive theoretical investigation of the vertical-axis propellers has been developed by Isay in a series of papers. Some discussions of these papers have been given by Haberman and Caster (1961), and by Sparenberg (1960), and from an experimental view-point by Nakonechny (1961).

## 11. Supercavitating and Ventilated Propellers

As higher speeds are strived for, the cavitating flow regime eventually becomes unavoidable. It is well known that the efficiency of a conventional propeller, designed for noncavitating operations, decreases rather rapidly after the onset of cavitation, causing great loss of power. With a keen insight, Lerbs and Alef (1957) have observed an interesting camber effect for cavitating hydrofoils; this feature was extracted from the results of Tulin (1955) and Wu (1955). It is this consideration that has put the prospect of supercavitating propellers under a more favorable light, thus setting the stage for entirely new approach to the design of supercavitating propellers (henceforth abbreviated as SC propellers).

Since this problem involves an additional parameter, the cavitation number $\sigma$, which adds much complexity to experimental endeavors, the linearized cavity flow theory, due to Tulin (1953), as well as the nonlinear theories, played a significant role in the preliminary investigations, which led to initial success. These activities soon drew more attention and gained momentum at DTMB. Through a series of analytical and experimental studies, an early design method was given by Tachmindji, Morgan, et el. (1957). These developments are well covered in a review by Venning and Haberman (1962). Some of the important results include (i) the most favorable operating region, in terms of the advance ratio $\lambda$ and the cavitation number $\sigma$, has been determined, (ii) dependence of the thrust and torque coefficients and the efficiency on $\lambda$ and $\sigma$ has been systematically ascertained, (iii) agreement between theory and experiment is not uniformly satisfactory, (iv) the thin leading edges recommended by the design consideration suffer from weak material strength and flutter.

The interest in SC propellers soon spread to other laboratories, universities, and countries. As an outcome, there has been produced a literature so rich that it would call for a separate review.

In order to achieve or to maintain the design performance of a SC propeller at lower speeds, the idea of ventilated propellers (by ejecting a foreign gas, or a gas-water mixture, from the blades) has been introduced. This problem has been explored by Morgan (1959) and Hecker (1961) at DTMB. Important contributions on ventilated propellers have also been made by Hoyt (1962) and Roberts (1961) through a series of investigations at the Naval Ordnance Test Station (NOTS) at Pasadena, California. These research studies are closely related to a parallel effort made by Lang (1959), Lang and Daybell (1960) on base-vented hydrofoils, also at NOTS. It has been shown that when the cavity is fully developed in both SC and ventilated propellers, the two types virtually have the same performance, based on the same cavitation number. Furthermore, it is indeed possible to effectively extend the supercavitating operation to a lower speed range by ventilation.

In this connection a recent Soviet contribution by Bavin and Miniovich (1963) [2]) on the interaction between the hull and a SC propeller has attracted some attention. Experimental results there indicated a decrease of induced velocity (becoming negative, as was also observed by Posdunine in an earlier Soviet work) in front of a fully cavitating or ventilated propeller, and it was also indicated that the thrust decuction tends to zero when the cavity becomes sufficiently long. This problem has been recently investigated by Nelson (1964) using a theory of infinitely-bladed propeller. By studing theoretically a 3-bladed fully cavitating propeller and a set of parallel experiments, Beveridge (1964) has shown that the thrust deduction due to a SC propeller may indeed become zero.

## 12. Conclusion

Thus I have gone through a brief survey of the active and rich field of the propeller theory. It is gratifying to see numerours fruitful developments achieved in the last decade, after the solid foundation being laid somewhat forty years ago. Some of these recent advances have come from new ideas and concepts, such as the supercavitating and ventilated propellers. From this acquired knowledge it is not difficult to find the areas in which important contributions are still to come. I would like to venture to list here just a few topics which I think may reward further efforts:

(a) The dependence on scale of wake fraction, thrust deduction fraction, propeller efficiency in laboratory and in open water.
(b) Supercavitating and ventilated propellers still have quite a future, especially the ventilated, which offers many advantages. The important problems are the lifting-surface and cavity-thickness corrections.
(c) Wall effect in water tunnel experiments with propeller models, especially for the supercavitating case.
(d) Developments in non-uniform flow investigations, both theoretical and experimental, are very important in applications.
(e) Scaling of cavitation effects on ship and model propellers.
(f) Vertical axis propellers.
(g) Problems of propellers in unsteady motion.

In connection with the applications of propellers in various manners of transportation over land, sea and through air, I would like to make reference to the famous chart of Gabrielli-Karman (1950), in which the lift-drag ratio is plotted versus speed. In this chart (with horse and pedestrians included for comparison) you will see that many types of vehicles employ propellers of different kinds for propulsion. The effectiveness of various modes of travel depends not only on the efficiency of the propulsive device used, but also on the basic nature of the vehicle, such as sliding, rolling, floating, lifting, or jet-propulsion, etc. Improvements made for the propulsion unit should certainly make the vehicle more attractive. It is of significance to note that it is by improving the propulsion device and reducing the drag the original limiting line of 1950, which supposedly confined the known world of transportation, has been promoted to 1960; and extrapolated to 1970 line. The impressive performance in the past thus tend to promise more future success that is calling for our continued effort and dedication.

Finally, I wish to extend my hearty congratulations to HSVA on this memorable occasion of its 50th Anniversay, for its brilliant contributions in the past, and anticipated success in the future. Many happy returns!

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I wish also to thank Prof. W.-H. Isay of Universität Hamburg for showing me his recent book entitled "Propellertheorie", published by Springer-Verlag, 1964, which I did not have the pleasure of knowing during my preparation of this manuscript.

2) *I am indebted to Dr. J. W. Hoyt for bringing this problem to my attention.*

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# La Facilité de Manœuvre des Navires

J. Dieudonne

La manœuvre d'un navire consiste à l'amener à un emlacement déterminé avec un cap et une vitesse donnés. Dans cas du sous-marin on devrait ajouter avec une immersion une assiette données.

Je ne m'occuperai pas, ici, de la manœuvre des sous-marins ıalgré l'intérêt considérable que présentent les études relaves à cette question. Elle a, en effet, des analogies assez roites avec l'étude de la manœuvrabilité en surface, elle est lus simple en ce que le navire est placé dans un milieu connu sans surface libre, plus compliquée en ce qu'il faut condérer ce qui se passe dans le plan vertical et non pas seuleıent ce qui se passe dans le plan horizontal.

La facilité de manœuvre caractérise la commodité d'obtenir s résultats souhaités. Il s'agit donc d'une estimation de la ıleur du navire qui est, au départ, essentiellement subjecve. On couvre, en outre, sous cette même rubrique des caracristiques très variées et cela rend difficile une appréciation lobale. Un navire peut être facile à manœuvrer pour obtenir n résultat déterminé, difficile à manœuvrer pour obtenir un utre résultat. La substitution de critères objectifs à l'appréiation subjective globale est un des problèmes les plus imortants à résoudre dans l'étude de la facilité de manœuvre. e n'est qu'avec de tels critères judicieusement choisis qu'on ourra comparer des navires entre eux, étudier l'effet du choix es différentes caractéristiques de la carène sur la facilité de ıanœuvre et documenter l'architecte naval sur ce qu'il doit ıire pour réaliser un navire donnant pleine satisfaction à s utilisateurs à ce point de vue.

Thieme [14] a donné une énumération très complète des diverses phases de la manœuvre et des facteurs numériques qui peuvent être utilisés pour les caractériser, donnant ainsi un guide dans lequel on pourra choisir les critères essentiels à retenir.

D'un autre côté, Gertler et Gover [1] ont énuméré, de la façon suivante, les opérations que la manœuvre du navire de surface doit permettre de réaliser.

1° — Maintenir le cap avec une précision suffisante et une action réduite sur le gouvernail.

2° — Permettre d'amorcer rapidement un changement de cap.

3° — Permettre d'exécuter un changement de cap rapidement avec un faible dépassement de cap et un transfert latéral réduit.

4° — Permettre l'exécution d'une manœuvre de giration permanente efficace avec des valeurs réduites du diamètre tactique, de l'avance et du transfert.

5° — Permettre d'accélérer ou de ralentir rapidement tout en gardant un bon contrôle du navire.

6° — Permettre de manœuvrer dans les ports et à leur voisinage, en marche avant et en marche arrière, à vitesse lente sans l'aide de remorqueurs.

A cet ensemble de conditions répond un ensemble de moyens. Ceux-ci sont essentiellement:

— le gouvernail,

— les appareils de commande de la machine,

— les appareils de commande des propulseurs (lorsque ceux-ci sont des hélices à pas variable ou des propulseurs Voith Schneider),

— les appareils spéciaux dont l'usage se répand et qui sont destinés à faciliter la manœuvre dans les ports (hélice à axe transversal située dans l'avant du navire, gouvernail actif de Pleuger).

*

Nous examinerons, tout d'abord, les problèmes relatifs au gouvernail qui est, en fait, le seul appareil utilisé pour la commande directionnelle du navire à la mer. On restreindra encore le problème en s'en tenant pour l'instant au cas où le navire est en eau calme illimitée, tant en profondeur qu'en surface. L'expérience montre, en effet, que les conditions de manœuvre d'un même navire se présentent de façon très différente selon que celui-ci est en eau profonde illimitée ou en eau de faible profondeur ou en eau limitée, par exemple en canal.

Les fonctions qu'on demande au gouvernail d'assurer en haute mer sont les suivantes. Tout d'abord maintenir la route au cap choisi en rectifiant les embardées produites par l'action variable de la mer agitée et du vent: cette opération doit pouvoir être faite avec un angle de barre faible pour réduire au minimum l'augmentation de résistance de la carène et, par conséquent, la dépense de combustible. On doit d'autre part être en mesure d'éviter un obstacle c'est-à-dire pouvoir faire changer rapidement la route: dans ce cas le freinage produit par le gouvernail ne présente plus d'inconvénient, au contraire, et on peut envisager pour cette opération une forte orientation de la barre. Enfin, lorsqu'il s'agit de navires de guerre, il est nécessaire que ceux-ci puissent exécuter les évolutions qui sont nécessaires à l'exécution de leurs missions de combat et on est alors conduit à examiner leurs possibilités dans des conditions très variées.

Comme il a été dit plus haut, la première question à examiner est le choix de grandeurs qui permettent d'évaluer objectivement les qualités de manœuvre d'un navire donné et qui puissent être déterminées, expérimentalement, sur le navire et sur son modèle. On doit donc, en fait, déterminer à la fois ces grandeurs et les essais à prévoir pour permettre de les mesurer.

Il paraît utile d'introduire ici une distinction entre deux catégories d'éléments qui se rattachent l'une et l'autre à la notion générale de facilité de manœuvre du navire mais se réfèrent à deux aspects distincts de cette notion. Cette distinction a été faite en France par Maurice Roy dans ses études sur les qualités évolutives des avions. Elle a été d'autre part utilisée par Thieme [14].

Le premier aspect qu'on désigne, en français, par « manœuvrabilité » concerne les régimes de mouvement permanent; il est caractérisé par l'étendue du domaine des régimes permanents que le navire peut prendre.

Le second aspect, désigné en français par le mot « maniabilité » est caractérisé par la rapidité et la facilité avec lesquelles on peut faire passer le navire d'un régime permanent à un autre.

M. Thieme a bien voulu examiner pour moi les mots qui pourraient traduire en allemand ces deux termes français: il m'a indiqué qu'on traduirait, au mieux, manœuvrabilité par « Drehfähig und Kursstetigkeit » et manabilité par « Handigkeit ». Les équivalents anglais qui m'avaient été donnés par M. Newton sont « turning ability » pour manœuvrabilité et « handling qualities » pour maniabilité.

Ces deux caractéristiques correspondent à deux catégories différentes du problème de manœuvre mais elles correspondent aussi à deux aspects différents de l'étude théorique des mouvements du navire. Si on s'en tient aux régimes permanents, les efforts hydrodynamiques développés sur la carène sont entièrement définis par les conditions instantanées du mouvement du navire c'est-à-dire par sa vitesse de translation, son angle de dérive et sa vitesse de rotation. Ce que nous appelons mouvement permanent en giration c'est en effet un mouvement pour lequel l'écoulement dans l'eau autour du navire est permanent par rapport à des axes fixés au navire. Si cette hypothèse était toujours valable, l'étude du mouvement se ferait au moyen d'équations différentielles. Au contraire, si on considère la maniabilité, on a affaire à des situations successives dans lesquelles les écoulements autour du navire sont déterminés par l'histoire antérieure du mouvement. Les efforts hydrodynamiques développés sur la carène ne dépendent plus seulement des valeurs instantanées des vitesses du navire mais aussi de l'évolution antérieure de ces vitesses. Les équations du mouvement du navire lui-même, ne sont plus des équations différentielles mais des équations intégro-différentielles.

La manœuvrabilité, considérée comme l'ensemble des régimes permanents, est entièrement définie par les valeurs des rayons de giration (ou des vitesses angulaires) correspondant, en régime permanent, aux différents angles de barre et aux différentes vitesses. Un élément secondaire est la réduction de vitesse entre la marche en ligne droite et la giration permanente pour un même réglage de l'appareil propulsif.

Ces caractéristiques sont mesurées au cours de l'essai de giration. Cet essai est pratiqué depuis longtemps mais il n'est pas d'usage, et cela serait d'ailleurs peu pratique, de l'exécuter avec de très petits angles de barre: ainsi l'anomalie constituée sur certains navires par l'instabilité du régime de route en ligne droite n'était pas mise en évidence.

Ce phénomène est pourtant connu depuis longtemps: il a été signalé pour la première fois à ma connaissance par Dupuy de Lôme en 1866 [2] à propos d'une batterie flottante. Il se manifeste sur certains navires par le fait que, même avec la barre à zéro, le bâtiment tourne dans un sens ou dans l'autre; le sens de rotation est défini par les conditions antérieures. Il peut changer sous l'effet d'une vague ou de toute autre perturbation. Le fait qu'un flotteur symétrique se mette spontanément en rotation n'est pas, comme il semblerait a priori, en opposition avec le principe de symétrie de Curie. Le phénomène physique est que le régime de marche en ligne droite existe mais est instable et que tout effet secondaire, si petit qu'il soit, empêche ce régime de se maintenir. Par contre, le flotteur a deux régimes de marche stables correspondant à des rotations dans l'un et l'autre sens [21].

L'existence de deux régimes de marche stables correspondant à des sens de rotation opposés pour un même angle de barre s'étend sur une zone plus ou moins développée des angles de barre. Dans les essais de modèle sous plateforme on peut déterminer un troisième régime de rotation correspondant à des forces nulles sur les balances qui tiennent le modèle mais ce régime est instable. Il ne peut donc pas être obtenu pratiquement sur un modèle libre ou sur le réel. La courbe complète des vitesses de rotation en fonction des angles de barre a la forme d'un S; la partie comprise entre les deux tangentes verticales correspond à des régimes instables.

La condition de stabilité du régime de route a été donnée par Contensou dans son mémoire à l'A.T.M.A. 1938 [3]. Elle a été établie de nouveau par Davidson et Schiff dans leur mémoire à la S.N.A.M.E. en 1946 [4]. J'ai montré [5] que cette condition pouvait s'exprimer simplement en se référant à trois points qui sont: le centre de gravité, le centre

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