Presented here are some notions which we hope will help researchers in their attempts to model various aspects of the complex situations which, currently, come under the heading of "externality problems." Roughly, the major theoretical idea is to exploit the advantages of separating into different structural models the consumer-based activities of consumption and acquisition and the producer-based activities of production, marketing, and revenue collection. The links between these activities can then be used to characterize types or classes of externality problems.

Before continuing, we offer the reader two serious disclaimers. First, we offer no hard results (theorems). In fact, we do not even argue to a specific set of conclusions. Of course, implicit, as well as explicit, in our discussion is our belief that the currently accepted views of how one might go about modeling externality problems are too narrow in scope. Second, we offer no detailed example which would indicate that our classification really works. We simply address some points of view which we find help us in our attempts to lay things out in a coherent manner. We have attempted to streamline the ideas to fit within standard mathematical economics constructions, so that it is possible to use standard tools in applications. The ideas allow attention to be focused on special aspects of externality modeling, thereby allowing, to some degree, specialization of research efforts.

Production is the only basic activity in the standard characterization of a public good [5]. Consumption is not a separate activity since it occurs automatically with production. If an irritant consumed by a person

The Technology of Public Goods, Externalities, and the Exclusion Principle

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reaches him by means of some chain of events, e.g. irritant consumed
depends upon the amount of smoke reaching him and thus upon the
amount of smoke emission from the source, \( c_i = c_i(s_i(x)) \), then the utility
function is simply relabeled. The intermediate consumption variables
are eliminated and utility is expressed as a function of the amount of
smoke produced at the source, e.g. \( \tilde{U}_i(c_i(s_i(x))) \equiv \tilde{U}_i(x) \). Rather than the
several variables "irritant 1 consumes, irritant 2 consumes, ..., smoke at
the source," we only have the latter, "smoke at the source."

The fact that there is only one variable involved in the standard model
has an immediate and important consequence. Within models that have
this attribute, a researcher is not free to interpret actions taken by a re-
ceptor as an expression of preference. Any given receptor must realize that
the effects on total supply and, thus on his utility, of any action taken by
him, depends critically upon the action taken by other receptors. Thus,
decisions made by him may reflect both his preference and his expecta-
tions about the decisions of others, rather than his preference alone. Such
expressions cannot be expected to properly guide output decisions. The
current efforts at finding a solution such as outlined in [3], [7], and [9],
involve some type of pseudo-price mechanism (Lindahl prices) which, by
clever means, extracts the true preference from the consumer. This
preference then serves as the basis for output decisions. These procedures
are most complicated and in view of the complexities may never have
applied manifestations. Of course, our speculations along these lines are
based on current theoretical discoveries.\(^2\)

This conception of a public good is so broad it comes dangerously
close to being empty in content. Almost any problem can be called a
public goods problem. One may then inquire about changes in institu-
tional arrangements which assure that the choices of magnitudes made by
the choosing agents are coordinated in a manner such that the system
equilibria are Pareto optimal. The lack of coordination resulting in the
non-Pareto optimal behavior of the system can always be identified as the
lack of institutions within the system which would assure Pareto optimal
equilibria. The failure of the necessary institution to be voluntarily forth-
coming can then be called a "public goods" problem in cases where the
system has the proper "convexity" properties. This last sentence is a
major point made by Arrow [1].

Let us carry this argument a little further. A monopoly equilibrium
is not Pareto optimal (under reasonable conditions). In fact, it can be
shown that under a wide class of conditions, consumers could collectively
bribe the monopolist to price according to marginal cost. The resulting
contract could make both the monopolist and the consumers better off.
Why do such contracts fail to arise naturally thus eliminating the monopoly (or replacing it by a cooperative)? Because the contract is a public good. All consumers benefit from the contract but it is to their strategic advantage to withhold, in hopes that the other consumers will finance the project or at least take on a bigger share.

The point is that the monopoly problem can actually be viewed as a public goods problem. It would appear, in fact, that any equilibrium which is not Pareto optimal can be viewed as a public goods problem by definition. The problem is that the public good, the institution that would yield Pareto optimal change, is not forthcoming. While this view may be useful in many cases, it would appear that some effort should be expended in developing concepts, or characteristics of models, which, independent of optimality properties, can be identified as involving externalities. It is upon this "technology" of externality and public goods relationships that we wish to focus.

Two Examples

Before continuing, we will present two examples that motivate almost everything we have to say about the exclusion principle. The examples indicate the delicate interaction between the outcome of a market-type process and the legal-institutional setting. They also serve as examples, within our framework, of how and why traditional solutions to externality problems, which come under the heading "internalization of costs," seem to work. The theme is that property rights can sometimes be used to repair social institutions (or lack of institutions) which are "inefficient."

Demsetz [6] claims that Canadian Indians learned that beavers' hutch's could be farmed. If the kill on a pond was too great, the stock of beavers would be exhausted; this, as it turns out, was actually beginning to happen due to an increased demand for their pelts. Consequently, ponds were marked according to the tribe that claimed farming rights. One could say that a title to a pond was established according to who discovered the pond. The one who owned the pond had available to him the beavers that were there. For a hunter, there are two alternatives:

\[ a_1 = \text{hunt only those ponds marked as his}; \]
\[ a_2 = \text{hunt any pond that he comes upon}. \]

We can represent this problem, with a little imagination, in terms of a two-person noncooperative game. The game is reminiscent of the well-
known prisoner's dilemma problem. The payoff matrix (see Figure 1) gives the number of beavers obtained per year by each hunter as beaver kill for I, beaver kill for II.

**Figure 1**

<table>
<thead>
<tr>
<th></th>
<th>a₁</th>
<th>a₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₁</td>
<td>(5,5)</td>
<td>(3,6)</td>
</tr>
<tr>
<td>a₂</td>
<td>(6,3)</td>
<td>(4,4)</td>
</tr>
</tbody>
</table>

If both hunters follow a₁, no hutch is hunted twice and the equilibrium stock of beavers is sufficiently large to allow an annual kill of 10. The hunters would receive 5 each. If either person hunts both types of ponds, the equilibrium stock is reduced to a level where the annual kill is 9. The hunter following a₂ gets 6 and the other receives 3. With both following a₂ the stock is reduced (due to overkill) to a level which only yields a harvest of 4 each. Because of the prisoner's dilemma attribute of the game, the solution if both follow a dominance principle is (a₂, a₂).

General agreement arose, however, that if I violated the territory of II, then II had the right to exercise certain claims to property possessed by I—he could seek remedy. This change in the institutional structure surrounding the exchange relationship results in a different game. The new game is outlined below.

Let

- a₁ = abstain from invading other's property; not,
- β₁ = if invaded, exercise rights;
- a₂ = invade;
- β₂ = if invaded, abstain from the exercise of rights.

The strategies and payoffs are displayed in Figure 2.

The structural payoff of the game dictated by the physical technology is the same as before but the institutional technology has changed. The
game has been expanded according to the following rules governing property rights: the invaded has the right to take from the other individual an amount equal to his own “loss” (5 minus the harvest he actually obtained) plus a penalty cost of one beaver.

Notice the only Nash equilibrium of the game is for both players to use the strategy \((a,\beta)\). Thus, if we can depend upon the outcome being a Nash equilibrium, then by constructing penalties for rights violations—altering the rules of the game—we can make sure the efficient outcome is automatically achieved by the system. In other words, an appropriate specification of property rights has provided a resolution of the prisoner’s dilemma.

The second example has features which are a little closer to those economists have dealt with under the heading of social costs and or externalities, even though some major features of the private goods case are present. We can, affectionately, call the games STEAL and PROPERTY. We first view a game the structure of which is outlined by the accompanying “tree” and normal form.

The first player is a seller, player I, who can make his goods available (act 0) or refrain from exposing them to the market (act N). The second player, II, is the customer. He has three alternatives in case the goods
are available. He can appropriate them for himself and pay for them (act $TP$, which stands for take and pay), he can appropriate them for himself and not pay for them (act $T\bar{P}$), or he can fail to act ($f$). Now, if act $N$ is chosen, or acts $0$ and $f$ are chosen, no "trade" takes place and player I keeps his goods (valued to him at $4$) and player II keeps the money (valued to him at $4$). If I plays $0$ and II plays $TP$, I gets the money (valued to him at $5$) and II gets the goods (valued to him at $5$). Suppose, however, that I plays $0$ and II plays $T\bar{P}$. Then I loses the goods (ending up with nothing, which he values at $0$) and II gets goods and money (total

\begin{figure}
\begin{center}
\begin{tabular}{c|ccc}
 & TP & $T\bar{P}$ & $f$
\hline
$O$ & 5,5 & 0,9 & 4,4 \\
$I$ & 4,4 & 4,4 & 4,4 \\
\end{tabular}
\end{center}
\end{figure}
value to him of 9) without worry, since, in this game, I cannot "seek remedy."

If we can be assured that individual II will always choose in accord with a dominance law and that I believes this also, then, from this design, we shall obtain a perfectly unambiguous outcome. Simply look at the game in normal form, as displayed. The dominant strategies are \((N, T\bar{P})\). Under this institutional arrangement the goods would never be offered. The structure of the institutions together with our behavioral law (the dominance principle) assures this. This system is "inefficient" [the Pareto optimals are \((5, 5)\) and \((0, 9)\)].

Notice we have a feature of particular interest. A social system organized along the lines modeled by the game STEAL will operate inefficiently—the outcomes are not Pareto optimal. Now, is there an "externality" involved? Is there a "divergence of social benefits from private benefits"? Yes. The social benefit of I's action is \(1(5 - 4 = 1)\), while the private benefit is \(-4\). In the traditional jargon, we would say that I would undertake a level of his activity below the "optimum amount" (he would offer nothing). Is there a divergence of social cost from private cost? The answer is again yes. The social cost of II's action is 4 (a benefit of \(-4\) to I), while the private cost is 0. In the traditional jargon we would say that II undertakes too much of his activity (of taking without paying).

Now, let us alter the game of STEAL a little and turn it into a game of PROPERTY. Suppose we give I the right to seek remedy in the form of damages, should he want to, in cases where he has made his goods available to II, and II in turn has appropriated them without paying I. The game represented by the tree below is the same as STEAL with the exception of a new move which, depending upon previous moves, may become open to player I. This new move involves his right to seek \((R)\), or not seek \((\bar{R})\), "remedy" for the previous actions of II. If II plays \(T\bar{P}\) and I, in turn, seeks remedy, the "courts" will return the goods to I and fine II an amount equal to the money he has. The payoff is thus \((4, 0)\). If \(\bar{R}\) is chosen by I, then II keeps both the goods (valued to him at 5), and the money (valued to him at 4), while I gets nothing. The payoff then is \((0, 9)\). See accompanying figures.

From the normal form we can see that \(OR\) is dominant for I. Player II, however, has no dominant strategy, so if the only admissible behavioral law is dominance, PROPERTY has no solution. It does have a solution if we can rely on II to follow a one-step conjectural variation. If he postulates that I will follow the dominant strategy, then II has a dominant strategy in this conjectured context. The solution is \((OR, TP)\) with the Pareto optimal payoff of \((5, 5)\). If we can rely on a simple behavioral law
then we are able to use legal institutions to transform an inefficient mode of organization, STEAL, into an efficient mode of organization, PROPERTY.

There are several morals to the story. First, if there is a principle called the exclusion principle, it is likely to involve some interrelationship between behavioral laws and institutional structures. In the case displayed in Figures 5 and 6, the interrelation is a connection between budget constraints. Secondly, calling the problem one of a “lack of markets” due to a lack of sufficient number of participants, as Arrow [1] does, seems to be a gross oversimplification. In the case presented here, there was no apparent “small number” problem as a reader of Arrow might be led to believe. We simply needed to alter the institutional technology.

Finally, representing the problem as one of “transaction costs,” as Demsetz tends to do, also seems to be glossing over some important features. There are several different ways by which one could introduce transactions costs into games such as PROPERTY so as to cause problems. For example, if the payments accruing to I as a result of remedy were reduced by a sufficiently large enforcement fee, the dominance principle could not be applied or used to guarantee an efficient outcome. Exactly how one goes about translating various altered organizational forms into something called transactions costs is not clear at all.

We do not intend to imply that there are no “costs” of reorganization.
There are likely to be costs which must be borne by someone and the covering of these costs does present a type of public goods problem. Both I and II benefit in the reorganization from STEAL to PROPERTY, but the pattern of resulting benefits is very sensitive to the organizational parameters. Consequently, it is not obvious that the altered form will be forthcoming. We are not addressing the problem of how the situation might thus evolve. We only note that lumping "organization" together with other things in a general category called "problems due to transactions costs" precludes a study of the independent features of each. Models that deal with the technology of externality relationships should be separated from models and theories that deal with the evolution of that technology.

Some Economic Quantities and Relationships

The examples above capture many aspects of our point of view. The idea is to take various aspects of "institutions," put them together in a manner which, cleverly taking advantage of a behavioral law, assures that the process outcome will lie in some class of outcomes which have been pre-designated as acceptable. We turn, now, to a consideration of institutions. In particular we consider those applicable to "market" or "economic"-related situations. We ignore situations which involve the problem of "publicness" of organization. That is, we do not address a theory of how organization evolves [14].
Now, as the reader probably suspected all along, we are not going to discuss real institutions. We shall study mathematical representations of institutions. Actually, we shall not even study the representation of any specific set of institutions at all; we shall study mathematical structures which (1) seem capable of capturing the "essence" of the major attributes of certain classes of institutions, (2) use forms typical of economic models, so that the concepts can be applied within standard models, and (3) appear to decompose the natural links among economic activities, allowing public goods to be viewed with respect to the link(s) where problems arise. It is at these links that we suspect the search for real "corrective measures" should concentrate.

Consumption and Appropriation

The links we seek first are those related to the consumer—the consumption and appropriation activities. It is convenient for us to depart from the standard model where consumption, ownership and use are not separated. In doing so, we identify a variable $c_i \geq 0, i = 1, 2, \ldots, n$, which we will call the consumed quantity commodity of $i$, and variables $x_i \geq 0, i = 1, 2, \ldots, n$, called the appropriated quantity of $i$. These variables are, at this point, scalars, and they should be thought of and labeled with respect to a particular individual.

Consumed quantities correspond to the idea of use or application while appropriated quantities correspond to the idea of ownership or claim. The distinction is important because many legal instruments are founded on such distinctions and because the relationship between the two concepts can serve as an interesting link in the externality chain.

In order to explore this distinction we postulate the existence of a consumption technology, $C(c, x) \leq 0$. The vectors $c$, $x$ are the consumed and appropriated amounts, respectively. The vector of consumed amounts affects utility, i.e., $U(c)$, and the consumer adjusts both $c$ and $x$ to maximize $U(c)$ subject to $C(c, x) \leq 0$ plus additional constraints (such as the budget constraint). The expression simply says that the limits of one's consumption activities are set by his activities of appropriation.

There are lots of different ways to think about the $c$'s and the $x$'s. A natural distinction might be a flow vs. stock distinction, where consumption is a flow.\(^3\) A service vs. goods distinction could also be used. The idea of attributes as discussed in [11] being represented by the $c$'s, while purchased commodities are represented by the $x$'s is also appealing. Notice that for purposes of modeling one may or may not want to identify
each of the c's with a particular x. For example, if c₁ is apple eating while x₁ is apple ownership, there seems to be some direct relationship between x₁ and c₁, perhaps independent from the other consumption activities and ownership activities. This closed system aspect is clearly not a necessary restriction. Suppose the consumer is interested in the beauty of his home as indexed by, say, the number of times per month that he receives compliments about it. Call this number c₁. The variables x₁, x₂, x₃ could reflect his contracts pertaining to the relative frequency of painting the house, washing the outside and supplementing any paint used with additives. The point of this example is that the seeming independence between the c and the x's is absent. A given level of c₁ may be attainable by several different combinations of the x's.

There are some potential advantages of dwelling upon the distinctions made here. First, the idea of ownership or appropriation can be connected with an act performed by someone. This connection will become important in the next section where the possibility of forced ownership is considered, and also later when we discuss the exclusion principle. Secondly, delineations found in legal institutions about the basis and extent of rights are hard to mirror within the standard public goods model unless one is willing to tolerate an endless proliferation of the dimensions of the commodity space. The distinction between consumption and appropriation might do a better job. Legal instruments sometimes base rights on theoretical distinctions about payment and possession as embodied in the concept of title. The act of acquiring ownership conveys certain rights to the owner. By virtue of being an owner, an individual has the right to take certain actions regardless of how others feel about it. Rights can also be founded on distinctions other than that of title. They can be based on the effects of actions. I may not have the right to take any action which causes you to experience certain types of discomforts regardless of your perhaps peculiar preferences and regardless of the pattern of ownership. The ownership of a gun does not carry with it the right to shoot people.

Rights which are defined in terms of the admissible actions afforded by ownership can be modeled as constraints on the consumption technology. These rights, when considered in conjunction with other institutions as well as the physical environment, would define, for every pattern of ownership (the amounts of the various goods appropriated), a consumption possibilities set. For example, if I own land I have the right to build any of the structures consistent with local building codes, regardless of how others feel about it. These form a consumption possibilities set. If I appropriate more land I expand alternative building possibilities accor-
ingly. A change in building codes would be a change in rights and can be captured in the model by a change in the set of building possibilities associated with any given land ownership.

In reality it would seem that this class of rights is important since some attitudes are bound to be ignored in actual policy applications. Any small thing we do might offend someone. Dales [4] claims to have searched hard for a purely private good before settling on a real example—slippers. Even this homely example is contained in a paragraph where a paper titled "Costume as a Means of Social Aggression" is cited.

Rights which are conceptually intertwined with ideas of "effects," such as "damage" and or "influence" are related to the "taxes" and "bribes" proposed as solutions to the externality problem. Payment by person A should offset at the margin any undesirable influence exerted on person B who has the right to be free from such influences. A problem with this approach is that "utilities," which in some sense would provide a basis for all concepts of influence, are not observed. More seriously, they might not, and probably do not, in any meaningful sense, even exist. However, if we have \( U(e) \) where \( e \) is observed—perhaps by convention where we, the court, or the law, can arbitrarily provide a definition—then, there exists something to work with. As we attempt to characterize the nature of problems and or model the effects of proposed remedies, the obvious magnitudes available for our use are the derivatives \( dc_1/dx_1 \) and \( dx_1/dx_2 \). We can seek ways, perhaps by following the leadership of the courts, to place values on magnitudes like \( dc_1/dx_1 \). In the case of \( dx_1/dx_1 \), we have the natural value of \( P_1(dx_1/dx_1) \) when \( P_1 \) exists, since it is a type of opportunity cost. Suppose I ruined some of your \( x_1 \), thereby causing you to adjust \( x_2 \); the damage, then, \( P_1(dx_1/dx_1) \), could serve as a natural definition for the extent of my liability. Of course, if this number were used as the magnitude of liability in a decentralized pricing system, it could induce inefficiencies, but analysis of that problem introduces the different sphere of welfare economics in general.

We would like to pursue a general theory of consumption technology as influenced by legal parameters. The reason we do not do so is because, at this point, we have nothing profound to say on the subject. We do offer, however, one general characterization: the consumption technology is characterized by total rejectability in which case \( \hat{C}(e, x) \leq 0 \) can be rewritten as \( \hat{C}(e) - x \leq 0 \). The idea is similar to free disposability. If you are stuck with more of the product, you can get rid of it without cost. In cases of total rejectability, a person would never make an effort to own less of a variable (unless he could sell it at a positive price). He can simply proceed to "dump" the "trash" without effort or cost. An increase in own-
ership does not automatically affect the consumption pattern. There cannot be too much of anything. Consequently, total rejectability is, for purposes of finding externalities, not at all interesting since it is the lack of rejectability which can cause problems. However, for purposes of correcting externalities the idea is very important.

Now, clearly the concept of total rejectability is rather gross. It could easily be refined into concepts which allow for rejectability or lack of rejectability of certain select quantities. Perhaps it is possible to say something specific about institutions which foster nonrejectability or even develop a theory along these lines. We have not done so. One thing is clear, however. If all consumption technologies were characterized by perfect rejectability, there could never be a problem with external dis-economies.

Before moving on, let us reflect on the concept of the appropriated quantity. This concept allows concepts pertaining to what one does at the consumption level to be separated from concepts and theories about how one appropriates or acquires ownership. Exactly what constitutes appropriation can be thought of as being determined by social conventions and or legal conventions. It sometimes involves overt actions. If I file a document proclaiming ownership of a property, then I have appropriated it. My signature on a contract may signify appropriation. If I attach a label, e.g., if I brand a cow, the appropriation is signified. On the other hand, the act of appropriation might be rather circumstantial. If I am in the theater during a show (whether or not I paid), it is assumed that I watched (appropriated). If I carry an item from a store (whether I paid or not), it is assumed that I appropriated it (claimed title)—even though I intended to bring it back.

Later, the concept of appropriation will play a fundamental role in our discussion of the exclusion principle. We will want to directly connect concepts of liability and or payment to the activity of appropriation. If I appropriate the pair of shoes or if I declare ownership over the pair of shoes, then I am automatically liable to the storekeeper for the amount posted on the price tag. Of course, in case I appropriated quietly, it might be difficult for the storekeeper to identify me as the one who is liable to him, but this is another story.

Appropriation and Availability

We now consider the activities of a producer or supplier. We will postulate two basic activities here. One is the act of production and to it is
attached a produced quantity, \( y_k \). The other is the activity of distribution or making available quantities, in terms of what is produced, to the other agents in the economy. The last concept, that of available quantities, \( q_k = (q_k^1, q_k^2, \ldots, q_k^m) \) where \( q_k^i \geq 0 \), are interpreted with respect to, and are unique to, each of the consumers (\( m \) in number) or other economic agents. Notice now that we have distinguished between appropriated quantity, which, in principle, is controlled by the consumer in question, and the quantity available to that consumer, which, in principle, is controlled by the producer. We have also identified a variable termed the produced quantity. This latter variable will be examined in detail in the next section.

Let us pursue the relationship between appropriated quantities and available quantities. In general, the relationship \( x_k^j \leq q_k^i \) must always hold. For example, if I am in a store, the amount of a commodity available to me is (presumably) the entire inventory. I can appropriate up to that amount, depending upon my own strategy in conducting my affairs. A park which is open 10 hours a day is available to me in that amount even though I might appropriate only one hour. The point is that appropriation and availability are, at points, independent concepts. The idea of access and opportunity, as opposed to ownership, can be found repeatedly in legal discussions. It is hoped that the distinction here will capture some of the complexities. We do not want to say that the available amount is necessarily what you pay for or is necessarily the amount you have appropriated. This would leave us no room within which we could characterize stealing, for example, or lack of rights enforcement or liability enforcement. Each of these involves the phenomenon of taking or appropriating without payment. Consequently, to the extent that these types of divergence between private and social costs are of interest, we need separate concepts.

Of particular interest is the case where a distinction between appropriation and availability cannot be made. The representation of this case is \( x_k^j = q_k^i \), and can be termed a situation of forced appropriation or lack of appropriation. This is the case where the concept of ownership begins to fade. If something is available to you, then it is yours. The example of smog seems relevant. We could also mention Mead's bees or Coase's rabbits. The point of both examples is that the courts decided that if the creatures are on your property then they are yours, even if you didn't want them. You acquired title through no act of your own. The ambiguity is underlined by the fact that we have not specified who controls \( x \) and who controls \( q \), in the case of forced ownership. The agent who controls one must necessarily control both. We chose to discuss the issue
on the presumption that the supplier, rather than the consumer, controls the variable. We could have just as easily called it forced availability, rather than forced appropriation, and postulated that the consumer controlled the variable; or we could always call it forced appropriation and label the agent who controls the variable the supplier.

As it turns out, the Mead and Coase examples are of interest because not only are they presented as though the relationship holds between the appropriated and the available amounts, but also because nonrejectability is present. If, for example, the receptor could avoid the effects of smog by some (costless) means, some of the complexities of the problem might be avoided. The usual public goods cases can be viewed as involving both nonrejectability and forced appropriation.

A convenient feature of the distinction is that on the one hand it allows a characterization of the exclusion principle which will be based on the variable $x_k$, and on the other hand it makes possible certain types of jointness in supply which are based upon the $q_k$. It is to these distinctions that we turn in the next section.

**Availability and Production**

We now consider the relationship between the vector of available amounts, $q_k$, and the scalar amount produced, $y_k$. In general, we will represent the relationship by the vector valued function $q_k = T(y_k, \alpha)$, where the vector $\alpha$ is a representation of some arbitrary, nonspecified set of controls (real or abstract). The producer can control the amounts which become available to the various individuals by altering $y_k$ and $\alpha$.

Just a glance at the general form $T(., .)$ indicates that a great number of forms of this mapping would make economic sense, and thus might be applicable for purposes of modeling. For example, one aspect of the usual private goods model where there are $m$ individuals, can be captured by postulating controls of the form

$$\sum_{i=1}^{m} \alpha_i = 1,$$

$$T(y_k, \alpha) \equiv (y_k)\alpha \equiv (\alpha y_k, \ldots, \alpha_m y_k) \equiv (q_k^1, \ldots, q_k^m).$$

This simply says that the supplier is physically able to make a given pro-
duced amount, \(y_k\), available at the individual level in any pattern \((q_k^1, \ldots, q_k^m)\) such that

\[
\sum_{i=1}^m q_k^i = y_k.
\]

Actually, the private goods case above is a special case of what can be called \textit{perfect selectivity of supply}. We say that \(T(\cdot, \cdot)\) allows perfect selectivity of supply in case the range of the function (it maps on to) is the nonnegative orthant of the Euclidian \(m\)-space (a type of invertability assumption). That means that any point in the shaded area of the accompanying figure could be achieved by a proper adjustment of the controls. The producer is able to discriminate between receptors even though some discrimination may be costly. For example, the distribution \((0, 5, 0, 1, 1)\) may take enormous amounts of \(y_k\), while all other points are achievable via the private goods form of \(T(\cdot, \cdot)\).

\textbf{Figure 7}

![Diagram](Figure 7)

We underline the note that perfect selectivity of supply can operate independently of the other concepts. The case of a private goods world with free disposability can be viewed as a case of perfect rejectability, nonforced appropriation and perfect selectivity of supply. We can take the same example and do away with the free disposal. For example, consider something like Limburger cheese which must be stored at home. If you own it, you cannot simply ignore the effects of the ownership on your consumption activities—it simply stinks. Yet there is perfect selectivity of supply (the supplier is able to remove it as you approach the counter, even though it would be available to any other customer).

Another example might be interesting. Consider a big, fast fellow who supplies handbills on the street. He is fast, so has perfect selectivity in
supply (he can reach anyone and or everyone). He is also big (with a slight whiff of ugliness), so there is forced appropriation. If he makes a handbill available to you, then you smile and take it. Now, if it is impossible to throw it away (without considerable effort), you also have nonrejectability. So, here is a seemingly private goods case which has many of the aspects of externalities. There are also many reasonable corrective measures for this problem which would not automatically evolve from our usual manner of modeling such processes: supply trash cans; make him stand in one spot; make him smile; give people tennis shoes so they can outrun him. This final suggestion, we are compelled to add, seems no less relevant to us than the several possible suggestions which involve the use of cost-benefit ratios.

Part of the existing models of public goods can be captured by another special form of $T(\cdot, \cdot)$. Suppose, where $\mathbf{1}$ is the vector of ones, we have

$$T(y_k, \alpha) = (y_k)\mathbf{1} = (y_k, \ldots, y_k) = (q^1_k, \ldots, q^m_k).$$

Here we have something that looks like the traditional public goods case of $q^1_k = q^2_k = \ldots, = q^m_k = y_k$. The range of $T(\cdot, \cdot)$ as shown is simply the diagonal. We can call this a case of no selectivity of supply.

**Figure 8**

A potential example falling within this class is the case of the park. Park service is necessarily available to everyone in equal amount and that amount is the total supply. The park service can clearly be supplied free of forced availability, the case of $x^i_k \leq q^i_k$ (we do not consider crowding). We have then the idea that appropriation by one person does not reduce the amount available to others. Of course, this verbal description is frequently taken as a definition of "pure public goods." We can see from the accompanying structure, however, that there is no forced availability.
and no nonrejectability. In fact, in this case, it is possible for the exclusion principle to be operative as well. Even though the park has aspects of publicness, there is no necessary market failure.

The usual case of national defense should also be considered here. Presumably, there is no selectivity of supply but there is forced availability and (perhaps) nonrejectability. Although, it would seem to be easy enough (things being what they are) to expose oneself to any degree of risk necessary to perfectly offset safety supplied by defense. If this is true then nonrejectability would not apply.

Of course, the examples of $T(.,.)$ discussed so far do not exhaust the possibilities. Consider, for example, the range shown on Figure 9. The idea is that availability to some types of individuals cannot be separated. Consider an airline with two close routes (there is only one control parameter, $a$, that takes only 2 values) between two mountains. The people situated between the two routes are going to have noise available to them as long as there is any supply and regardless of what the company does. However, if the company chooses route 1, some people are far enough away to receive no noise at all. The same is true with route 2.

**Figure 9**

![Figure 9 Diagram](image)

There appears to be no utility in pursuing these structures (which have been called cases of "parametric jointness" [16]) at this time. However, the alterations in the form of $T(.,.)$ could potentially provide vehicles for capturing the idea of transaction costs. There may be real costs (perhaps parameterized by $a$) in making a supply available to one person without making it available to others.

Before leaving the subject, we can offer two general concepts which can reduce speculation over admissible forms of $T(.,.)$. We say supply is depletable in case
and nondepletable in case for all $y_k$

$$T(y_k, \alpha) \leq (y_k)1$$

for some $\alpha^*$.  

The idea of depletability is simply that the amount produced must be no less than the sum of the amounts available to individuals. The traditional private goods model serves as an example. Nondepletability means that the amount available to each individual is limited only by the total supply. As an example, consider the case of one evening of TV production and broadcast. Suppose the station controlled scramblers on each individual's set. In this case, there would be perfect selectivity of supply and nondepleatability, since making the program available to an additional individual would not reduce the amount available to anyone else. The entire evening broadcast would be available to each individual.

### The Exclusion Principle

The exclusion principle is probably the most elusive of all the ideas presented here. In fact, from time to time, we have wondered whether it is a principle at all, since we were unable to obtain a really satisfactory characterization. The game examples given in the introduction do give some hints when taken in conjunction with the concepts thus far elaborated.

Here, the dominance principle can be used to considerable advantage. The structure we will propose involves the postulation of a liability function which depends directly upon an individual's act of appropriation. If this function is properly constructed then we will be assured by virtue of the dominance principle that a given individual will act in the proper manner.

Before making the idea more precise, let us return to the game of PROPERTY. We have complicated it in order to make clear the analogy between it and our abstract concept. The seller, player I, will have the first move. He has the alternative to offer, 0, or not to offer, N, the goods for sale. If 0 is played at the first move, the second player has a move with two alternatives. He can appropriate (take) the goods, $(T)$, and make a payment available to I, $(A)$; or, he can take the goods, $(T)$, and not make
the payment available to person I, \((\bar{A})\). We ignore here the option of not appropriating any of the goods, so he has two alternatives: \((T\bar{A})\) and \((T\bar{A})\). If he plays \((T\bar{A})\), then player I has the option of collecting the payment \((C)\) or not collecting \((\bar{C})\). The former induces a payoff of \((5,5)\), since I gets the money, valued to him at 5, and II gets the goods which are valued to him at 5. If \(\bar{C}\) is chosen, then I loses the goods and II gets both the goods and the money.

If player II plays \(T\bar{A}\) then I has the option of filing suit \((R)\) for seeking remedy or not filing suit \((\bar{R})\). The former leads to a payoff of \((5,0)\)—player I receives damages of the foregone value of the sale (to him), and player II pays the damages and a fine. If player I does not file suit, then II gets everything.

*Figure 10*

Player I has the dominant strategy of \((OCR)\)—he offers the goods, collects when possible and files suit when collecting is not possible. Player II has no dominant strategy unless he conjectures that player I will play a dominant strategy. In this case, he always plays \((T\bar{A})\). The outcome, of course, is Pareto optimal.

The key idea in Figures 10 and 11 is the interrelationship between the act of taking (appropriating) and the options that automatically become available to the other players as a result. In the traditional private goods model where \(P_e\) is the price of the good, the payment, \(L(x) = P_e x\), is simply a linear function of the amount consumed \((x)\). In our jargon, we would say, where \(x\) is the amount appropriated, that the amount \(P_e x\) is money made available to the seller. That is, the act of appropriating some amount will automatically induce a liability amounting to \(L(x) = P_e x\). Of course, if the seller "likes money" and also follows the dominance
principle, the individual can be assured that the seller will appropriate all of the money made available to him. If the individual can "steal" the commodity, we would say that \( L(x) = 0 \). That is, the individual's act of appropriation was not accompanied by an induced liability. The seller, in this case, would have no alternatives available as a result of the first person's act of appropriation.

In general, and more abstractly, we suggest the use of a function \( L(x_i^j) \) as a representation of the liability which results from appropriation. At a general level, \( L(x_i^j) \) would simply identify some set of actions. At a more specific level, we suggest that \( L(x_i^j) \) be interpreted in terms of a reduction in the amount of some given commodity (perhaps money) available to the appropriating party, individual \( i \). Furthermore, for our purposes, we can simultaneously view the absolute amount of \( L(x_i^j) \) as becoming available to the seller. We intend for this function, \( L(x_i^j) \), to capture what we call the *circumstances of exclusion*.

The liability function is intended to describe, in a sense, the potential retaliations that one can experience as a result of his act of appropriation. It could, in reality, take any number of interpretations, even though...
we have suggested that it is a reduction in the amount of something available to him. It could, however, represent a potential increase in the amount of something which he is forced to appropriate and unable to reject: e.g., I am able to park in my neighbor's driveway (appropriate this parking space), but as a result I am forced to tolerate more of his children's climbing on my car. The example is reminiscent of reciprocal externalities.

We have no general theory about exclusion, nor general forms characterizing the circumstances of exclusion, but we can make several observations about particulars. First, the failure of exclusion might imply absolutely nothing about forced availability, nonrejectability, nondepletability, etc. The case of "theft" described above is an excellent example of a private good where exclusion is absent. It is also a case which, within the traditional model, could not be separated. We can also see that the property of nondepletability, i.e., that everyone can appropriate up to the produced quantity, need not be accompanied by the failure of exclusion. Here, we need only consider a park, picnic area, or beach which is privately owned. By suitable organizational arrangement, e.g., a big guard authorized to collect a fee if possible, and backed by laws governing trespassing, the owner is assured that anyone who appropriates some of the amount available to him will incur the proper liability. In other words, from those who enter, the owner is assured of a proper fee. We can worry about crowding, etc., but there appear to be ways of capturing that phenomenon without destroying the nondepletability attribute we have used in the characterization here.

There are, of course, problems with this formulation of exclusion in the cases of forced availability. In these cases, there is really no act with which liability can be associated. As was mentioned above, it is here that even the concept of appropriation and or ownership becomes unclear. However, there is no need to follow the usual path and extend complications that arise in this case to all of the other cases. Things are complicated enough as they are.

We can make the concept, \( L(x) \), a little less abstract by discussing some features of processes which would be implied by its use. First, the supplier must be aware of \( L(x) \). This implies that when \( L(x) \) becomes available, it is accompanied by some type of signal. In the game theory jargon, the individual must be aware that it is his move. In some cases, the signal may occur almost naturally as part of the appropriation process. For example, large discount stores frequently place purchases in a bag, then staple the bag closed with the receipt of payment. Any customer who is observed carrying merchandise which is not so labeled serves as a signal to the supplier that certain options are available to him, e.g., col-
lecting for the purchase. Coin-operated machines are excellent examples of cases where the interrelationships between appropriation and liability are so automatic as to be almost unnoticed. The coins in the machine at the end of the day, in a very trivial sense, signal their availability to the vendor. In some cases the knowledge about \( L(x) \) may not be automatic. In fact, the knowledge may be acquired only after considerable search. For example, the actions available to you as a result of my appropriation of an idea on which you have a patent may never be known to you unless you “catch” me “using” it. In passing, we should observe that the knowledge of \( L(x) \) itself is of primary importance, as opposed to the knowledge of who appropriated what and how much was appropriated. The knowledge of these other aspects can be viewed as either secondary or indirectly implied by the nature of \( L(x) \).

The second feature of processes implied by the use of \( L(x) \) is that \( L(x) \) really represents some thing or some things which are actually available. After all, \( L(x) \) does represent some alternative form of retaliation, so it cannot be vacuous or token. It does little good for someone to be liable to you if it is impossible for some reason or another for you to collect. A supplier who requires the deposit of a driver’s license and/or credit card as part of a rental contract is attempting to assure himself that the agreed upon rate can actually be collected by him. In most cases discussed above, the thing represented by \( L(x) \) was some kind of monetary payment. This is a very natural instrument to use, because it can be readily transferred into something the seller desires. Other things could be involved. For example \( L(x) \) might involve the ability to initiate criminal proceedings. It could involve civil proceedings in pursuit of damages, injunctive relief or review of official decisions. It might be the case, as it is with most market models, that the terms of \( L(x) \) are set by the supplier himself in the form of a price.

The point of the above discussions, for those interested in market corrections, is that slight perturbations in institutions can be the means by which the correction takes place. A small change in procedure, such as requiring a signature, might make the seller aware of his rights, and thus correct an externality: simply providing an unambiguous set of retaliatory measures might do the trick.

Concluding Remarks

What is a purely public good? In our jargon, it is characterized by non-rejectability, forced availability and no selectivity in supply. What is a purely private good? In our jargon, it is depletable, it may or may not
involve nonrejectability, it may or may not have perfect selectivity of supply, and the exclusion principle applies. Do all permutations and combinations of our concept have a counterpart in reality? We have absolutely no idea.

There is no real divergence between what we have done and what has been developing in the literature. We have sought only to refine the concept of "cost internalization" and the elimination of "unnecessary" transactions costs. In doing so, we hope we have equipped the reader with a point of view which helps him recognize some modes of attack which may have otherwise gone unnoticed.

Perhaps we have been more preoccupied with noncooperative game formulations than some readers would like, but we have not found the characteristic function form of model useful when thinking about these problems, even when it is generalized as in [2] and [19]. Models using the characteristic function form of games collapse all variables, at the individual level, into one variable—the choice of coalition. The representation of institutions then must be introduced through the structure of the "possibilities set" or "payoff set" associated with a coalition and the closely associated description of an outcome of the game. Presumably, when one starts at the extensive form, such as we have done, the model can be collapsed to a characteristic function. But whether or not various standard institutions or technologies have a useful and straightforward natural representation as a characteristic function is simply not known. Some progress along these lines is evident. The interested reader should consult, for example, [15], [17], [21], [23] and, especially, [8], for examples where the technology is readily represented by a characteristic function.

Along with our tendency at the structural level, to avoid cooperative concepts, we have also avoided the use of solution concepts used typically in models of the cooperative form. This is somewhat implicit in what we have done, since the applicability of the concept we have used is limited to the operation of a dominance principle. It also placed limits on the type of formal apparatus we introduced. For example, we did not examine the type of institutions which would guarantee the existence of a core in any associated game model. We chose not to try to isolate institutions or technologies of this sort because we have serious reservations about whether or not the core can actually be depended upon as an operative behavioral proposition. We would aggressively pursue a theory of those institutions which would guarantee the existence of a core, if we were assured that existence of a core in the model was sufficient to guarantee that the outcomes would be restricted to it in reality.
We also diverge slightly from those, such as Drèze, Foley, Munch, Roberts, Samuelson, and others, who have been developing the structure of the Lindahl equilibria. We have made no attempt to systematically integrate our concepts with a set-theoretic general equilibrium model. Perhaps that can come later.

Footnotes

1. The research support for C. R. Plott provided by NSF grant No. GS-36214 is gratefully acknowledged. The word “technology” used in the title was adopted as a result of comments made by Robert Haveman.

2. The most recent direct attack at placing some structure on this problem of preference revelation can be found in Ledyard [12].

3. A model which capitalizes on distinctions along these lines can be found at [10]. They regard pollution as a stock which acts as a parameter on the flow of consumption—although they take “utility” directly to represent consumption rather than some intermediate variable like our variable. c.

4. This idea is also closely related to the type of jointness investigated by Smith [22].

References


COMMENT
Robert H. Haveman, University of Wisconsin

The paper by Plott and Meyer is designed to provide a new characterization of the standard concepts of private goods, public goods, and externalities. Although this characterization is based on the perspective of game theory and carries with it all of the complexities of that view, the
paper also has the flavor of a layman's guide to externality control. As such it is either deceivingly simple or deceivingly complex.

The new characterization grows out of the perspective of game theory in which the behavioral law, represented by the dominance principle, prevails. Because such a framework and principle, together with a multiperson game structure, is theoretically equivalent to the standard economic model, Plott and Meyer offer no new theorems regarding market failures. In this sense, the paper is but old wine in new bottles.

Having said this, however, I confess that Plott and Meyer do seem to get limited mileage out of their analysis of these phenomena. The categories and concepts which they develop tend to focus attention on some fundamental attributes of the market failure phenomena which may be obscured by the standard neo-classical model. However, by dissecting the public goods-externality phenomena, their paper becomes a discussion of the technology of externalities and public goods. Their justification for this detailed look at mechanics is that good policy toward externalities requires a perception and evaluation of the full range of institutional or policy correctives for a market failure.

To convey the flavor of the Plott-Meyer approach, I will examine a few examples of standard phenomena which, when looked at through their glasses, somehow seem a bit richer than before. If this examination does some minor violence to their framework, I am sure they will forgive it.

First, the notion of private goods. To Plott and Meyer, a private good is one in which the following characteristics are present:

- **depletability**—the sum of the amounts of a good which are made available are no larger than the amounts produced. This is to be set against nondepletability in which the amount of a good available to every individual is limited only by total supply.
- **perfect selectivity**—the provider of a good can target the good (or bad) on whomever he desires.
- **no forced appropriation**—the individual does not have to take any good made available to him—or alternatively, if the good is a bad and he is forced to take it, he can dispose of it at no real cost.
- **a liability function exists**—appropriation of the good results in an immediate liability (it is this which substitutes for the exclusion principle in the Plott-Meyer framework).

At the other extreme, consider the notion of public goods. In the Plott-Meyer framework, such goods have the following characteristics, whose meaning can be inferred from the above definitions:
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• non-depletability
• no selectivity
• forced appropriations, hence,
• no liability function exists
• no rejectability

Given these two polar cases, it is clear that the world of nonextreme externalities lies somewhere between them. Indeed, it appears that if any one of the characteristics of the private or public good cases are altered (or more than one appears in any combination or permutation), some phenomenon results which represents an externality-type market failure which may or may not have a counterpart in the real world. A couple of examples will illustrate this conclusion.

Consider first the phenomenon of theft as an external diseconomy. The producer-supplier in this case is the robber with the victim being the appropriator-consumer. In Plott and Meyer's jargon, there is no liability function present in this case or, in other words, exclusion is absent. Similarly, by the very nature of the case, there is no rejectability. Moreover, there is perfect selectivity by the supplier and forced appropriation. Finally, the good is depletable. Looking at the problem in this way, then, theft would be classified as a private good (or bad)—for example, there is selectivity—except that the liability function is absent. Correction of the market failure, then, requires an institutional change in which the opportunity for redress is imposed. Viewing the problem in this context, it is claimed, immediately focuses attention on the search for an institutional change to correct the absence of a liability function. It is the technology of this case which points directly to the corrective policy.

As a second example, consider the handbill distributor described by Plott and Meyer—a case similar to theft. Again there is no liability function even though there is forced appropriation. The pedestrian cannot make the distributor pay for the inconvenience of taking the handbill. Similarly, there is perfect selectivity, no rejectability, and depletability. In this case a number of policy remedies are available and the task is to choose the least costly of them (assuming equal effectiveness). A rule could be imposed denying the distributor the right to force appropriation. For example, he could be required to stand in one place. Or a rule could be imposed requiring him to incur a liability equal to the marginal inconvenience his action imposes. Or provision could be made for rejectability by placing a trash can just beyond him. All of these institutional changes would tend to turn an inefficient mode of social organization into an efficient one. In each case, the nature of the change falls directly out of
the specific characteristics of the case, as distinguished by the Plott-Meyer framework.

Having so characterized the Plott-Meyer framework, the question of the benefits of this view of the world must be confronted. The first benefit claimed by Plott-Meyer has already been alluded to: By focussing on the detailed characteristics of the case of suspected market failure, a deeper understanding of the externality problem and a clearer comprehension of the full range of policy alternatives or institutional changes are obtained. This statement, it should be noted, is both a plea for a comprehensive search and evaluation of available policy options (à la Baumol-Oates) and a belief that their "nuts and bolts" framework will be helpful in undertaking that search and evaluation.

With respect to their plea, I have absolutely no quibble. To insist on a full understanding of the institutions surrounding a market failure problem prior to taking corrective action is unexceptional. Few economists would advocate anything but the need for a comprehensive search for policy options and a careful examination of the benefits and costs of each. Such a procedure would seem to follow directly from standard welfare economic analysis and would be advocated by most careful public administrators. Plott and Meyer's framework would seem to add little to this proposition.

With respect to Plott and Meyer's belief regarding the efficacy of their framework, I do have a quibble. Surely, their perspective is helpful in illuminating the case of small numbers externality problems. Through their framework, insights are conveyed—for example, in the theft and handbill examples discussed above—which a more gross statement regarding externalities would not be likely to reveal.

However, I am not at all convinced that the Plott-Meyer framework provides illumination in the case of large numbers externality problems, such as those which dominate discussions regarding environmental pollution. Nor am I convinced that an economist or a policy maker confronted with a task of the framing, say, federal water pollution control policy would find the insights from their framework more helpful than those stemming from the standard welfare economics framework. Clearly the range of possible policies and policy mixes is very large—charges, subsidies, regulations, prohibitions, assignment of property rights, and combinations of any of these. Presumably, careful analysis within the standard welfare economic framework can point to the correct policy mix. The Plott-Meyer concepts of forced appropriation, rejectability, selectivity and so on add little insight if any to the design of optimal externality policy where large numbers interactions exist. It is revealing
that, in their paper, Plott and Meyer deal with no concrete environmental problem nor illustrate any large number externality problems for which their framework would be either a complement or a substitute for the standard analysis.

A second potential benefit of the Plott-Meyer framework is as a contribution to the formal externality literature and to the modeling of environmental problems. If I am correct regarding the lack of relevance of their framework to most large number externalities problems, it is not likely that it will contribute substantially to the success of modeling efforts. As a contribution to the formal analysis of externalities, their framework is enlightening. Their critique of Arrow's conclusion that the problem is a lack of markets due to too small a number of participants follows directly from their game theory framework. Similarly, their point regarding Demsetz' focus on the role of transaction costs as the prime cause of market failure is attributable to their framework and is helpful.

Finally, their framework does seem to illuminate some of the more subtle characteristics of externality problems which are not typically noted. In the terms of their framework, external diseconomies are a problem only if nonrejectability is present. Similarly, rejectability must be thought of as a continuous variable so that the closer the world is to the pole of nonrejectability, the more serious the problem. Can we then think of technological change to expand the possibilities for rejectability or to reduce the cost of rejectability? In much the same way, viewing the liability function as a continuous variable ranging from zero liability to full liability is helpful in understanding the meaning of the exclusion principle and in contemplating possibilities for "privatizing" goods.

In conclusion, then, I found the Plott-Meyer paper to be a mixed bag. The framework which it sets out does convey some insights regarding the process of externality generation. It is a frustrating paper, however, in several regards. The tie to the standard externalities literature is never made. For example, how do the notions of consumption and production externalities fit into the Plott-Meyer framework? Is their framework helpful in distinguishing real from merely pecuniary externalities? Moreover, it is not clear that the framework can really enlighten some real world, real life externality cases—perhaps even in the environmental area. The authors have yet to demonstrate the effective relevance of the framework. As a final point, if various constellations of Plott-Meyer characteristics yield a large, yet finite, number of externality cases between pure public and pure private goods, it should be possible to map corrective institutional changes onto the set of cases. While this might be a worthwhile effort, it has not been undertaken.