As a mathematical economist, I work at the foundations level of risk/benefit analysis, group decision processes, polling, and related areas. The foundations, however, are frequently removed from real applications.

My situation among all of you practitioners reminds me of a joke about an engineer and a mathematician stranded on an island. The only food source on the island was a coconut tree with coconuts high at the top. A coin was flipped to see who would retrieve the first meal. The engineer lost. He climbed up the tree, which was a difficult task indeed, and leaning far out on a branch that was little more support than a leaf, he grabbed a coconut and returned with it. Within a few days it was the mathematician’s turn to get the food. Up the tree he went, following exactly the same path as the engineer. He managed to reach a coconut much higher than the first. With the coconut in hand he returned to the branch originally used for support by the engineer. He leaned out with the branch quivering under this weight and carefully placed his coconut exactly where the coconut retrieved by the engineer had been. He then slid back down the tree with no fruit. The engineer looked at him dumbfoundedly and asked, “Why on earth did you do that?” The tone of the mathematician’s reply was as though he had only done what was expected of him: “I reduced it to a problem that has already been solved.”

My goal here is like the mathematician’s. I want to show you a few very simple examples of the problems that many theorists think are keys to understanding the difficult practical problems you face. These are, simply stated, unsolved problems to which many of our more complicated problems can be reduced. They seem to be symptomatic of the problems that we run into when theorizing about cost/benefit analysis and when theorizing about societal decisions in general.

The overriding purpose of this conference is to uncover guidelines for social decisionmakers. Everyone seems to agree, or at least the thrust of the conversations at the conference has been, that the guidelines should include the people who are going to be affected by your decisions. Except in the most narrow philosophies this means that—indeed, the political process requires that—you have got to find out what the people want. This question is translated almost unconsciously into a problem of determining the “social preference” among the alternative courses of action that you face. A lawyer might use the term “public interest” and tell you to follow it. Presumably all this, regardless of the terminology, has something to do with what people think is good for themselves, and you are charged with the responsibility of giving it proper “weight” in your decisions.

It is here that the interesting and difficult problems start. Somehow you must take account of how individuals feel, and then, using some tool or another, you must make a general statement, find a summary statistic so to speak, about how the group feels. There are two types of approaches to the solution of this problem. One is represented by cost/benefit or its reincarnation, risk/benefit analysis. I shall refer to approaches in this class as attitude aggregation models. The other approach is much more political and is represented by public hearings, testimony, committee decisions, etc. This second approach I shall refer to as public participation models. Both have some disturbing aspects.

ATTITUDE AGGREGATION

The basic theory of attitude aggregation as found in cost/benefit or risk/benefit is simple. The essentials are captured by equation 1, but what the
explicitly or implicitly will have you find the values the numbers \( V_i(x) \) and \( V_i(y) \). The numbers are considered. Cost/benefit and risk/benefit analysis either explicitly or implicitly will have you find the values that individuals place on \( x \) and find the values that individuals place on \( y \). These values are represented by the numbers \( V_i(x) \) and \( V_i(y) \). The numbers are summed over all individuals. If the sum of values they place on \( x \) is greater than the sum of values they place on \( y \), then you are directed to take action \( x \).

The basic theory represented by (1) seems sufficiently simple to be above questioning, but there are two serious problems. The first problem is how to get the individuals' values; How are these to be measured? There are many names for it such as utility, benefits, attitude intensities, etc. From a formal point of view, at the individual level, they are all the same thing and have the same measurement concept of a loaded question comes immediately to mind. If you are in favor of urban renewal, ask people, "Are you in favor of eliminating urban blight?" How could anyone be against that? Or if you are in favor of urban renewal, make sure that those conducting the interviews also favor urban renewal. Subtle things, like repeating questions or smiling at the proper time, can be effective. The individual answering the questions sees no immediate consequences from his answers, so a pleasant conversation with the interviewer could be a sufficient reward to bias the answers given. These things, as it turns out, are very effective in polling techniques and suggest strongly that polls should be used with caution at best.

The problem of determining individual values for use in equation (1) is much deeper. What happens when you really try to measure the strength of peoples' convictions and attitudes so that the measures can be added—such as (1) suggests? What must be measured? There are many names for it such as utility, benefits, attitude intensities, etc. From a formal point of view, at the individual level, they are all the same thing and have the same measurement problems. There must be some type of origin and unit of measurement. For example, how high a cliff would I jump off to see one president versus another? You can measure my intensity between presidential candidates that way and, in fact, this is the type of unit chosen in risk/benefit analysis. How many times would I run around the block to see one candidate over another? This is another measurement even though I am unaware of its having been incorporated into any formal methodology. How much would I pay to see one candidate over another? Clearly, this is the measurement structure of cost/benefit analysis.

Now, all of these measurements, when put to a single individual, have a type of internal consistency. They are definitely related to one another in a systematic way. It is rather remarkable, but it can be demonstrated. But when you put these measurements in a sum like (1) across individuals, the answer is sensitive to the measurement used. It is possible to measure the amount that people would pay and determine that \( x \) is better than \( y \). Then with individual attitudes unchanged, measure those attitudes in terms of some risk dimension and get the opposite result—that \( y \) is better than \( x \).

What is the point? While the individual scales are not sensitive to the unit of measurement in the sense that certain ordering properties are independent of the measurement system, the additive scales are sensitive to it. This is a problem about which users should be aware. The full ramifications and what to do about it remain as part of the currently unknown. Recall, in my introduction, I admitted that I am a basic researcher and my focus is upon key problems and not necessarily solutions.

The next problem which hides behind equation (1) is a rather paradoxical aspect of individual attitudes—the fundamental thing that must be measured. Suppose you are asked your preference between the two lotteries represented as lottery A and lottery B on Figure 1.

If you prefer lottery A, a random dart is going to be thrown which will hit within the circle with probability 1. If it hits the line, you get zero, but if it hits anywhere else, you get $4.00. That is lottery A. If you choose lottery B, a random dart will be thrown hitting inside the circle with probability 1. If it hits anywhere in the pie-shaped area, you get $16, but if it hits anywhere else, you get zero.

When people are faced with the option of playing either of these lotteries once, some will choose B while others who are more risk averse will choose A. These represent legitimate differences of opinions and the method of elicitation (observing choice behavior) represents a legitimate method of measuring.

\[
\sum_{i \in S} V_i(x) \geq \sum_{i \in S} V_i(y) \text{ implies } x \text{ is "better" than } y.
\]
preferences.
Here is where the paradox occurs. When a different measuring system is used, those who indicated a preference for A end up placing a higher value on B—a complete switch.

It works like this. Having indicated their preferences between the two lotteries, individuals are then asked the maximum price they would pay for each (occasionally they are asked the minimum price you would take for each rather than play the lottery themselves). In this way, the dollar value of each lottery to the individual is measured. Theoretically, the lottery with the highest personal dollar value is the one most preferred. But that is not what is observed. People who prefer A tend to place a higher value on B. So here is the problem. If you measure attitudes in terms of preference, you find their attitude going one way, but if you measure their attitudes in terms of dollars, how much they would take, how much they would pay, etc., you find the absolute opposite result.

This is not a little statistical game. The experiment has been replicated many, many times. It has been done for many dollars. Mathematical statisticians, economists, etc., have been used as subjects. Not only is it a phenomenon we really do not understand, it is also inconsistent with all known theories of decision. It involves an immediate intransitivity. For the theories represented by (1), it means we face a very perplexing problem; namely, for two options (x, y), one can get at, on the individual level of analysis, either \( V_1(x) > V_1(y) \) or \( V_1(x) < V_1(y) \), absolutely inconsistent measurements, depending upon such a subtle and innocent-looking aspect of the measurement system.

The second class of problems associated with (1) are independent of those discussed above. Suppose satisfactory measurements at the individual level of analysis have been obtained. What should be done with them? Equation (1) says to simply add them up. But, why should the numbers be added as opposed to being combined in some other mathematical way? This is a natural and important question, since different ways of combining the numbers imply different ways of resolving the implicit conflicts among citizens’ preferences.

The second class of problems stems from attempts to justify any method of conflict resolution implicit in formulas such as (1). The best way to see the problems is to examine some explicit methods of resolving conflicts which are not masked by complicated measurement systems. They are very interesting phenomena.

Consider an example with three people and three options, x, y, z. Mr. 1 likes x first, then y, then z. Mr. 2 likes y first, then z, then x. Mr. 3 likes z first, then x, then y. These rankings are represented below.

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You are in charge of the agency which is to make a decision for this society of three people. What is it that the society wants? How do you choose, if you want to do what the people want?

If you put x against y first, x wins by virtue of the preferences of 1 and 3. Put x against z, then z gets the majority vote from Mr. 2 and Mr. 3. You may be tempted to say that the outcome is z.

However, if you put z against x first, you find z gets the vote, and when z then comes against y, it loses due to the preference of 1 and 2. So the winner is y.

Now put z against y, the winner is y. Continuing to place y against x we find the winner is x. The outcome depends entirely upon the sequence of contests.

Those of you who have not seen this might say, ‘‘Well, the good professor is pulling one out of his hat. What is the likelihood that such a cycle will occur?’’ The news is bad. If preferences occur at random, the limiting probability of the cycle is one, as the number of people and options grow. It goes to probability one very rapidly.

The cycle is the case. It is not the exception. All of our theories must take account of it. All methods of preference aggregation utilize some principle for resolving this difficulty one way or another, even though the method of resolution is often buried under a layer of concepts and factual materials.

With the problem so simply stated, how should it be resolved? ‘‘Well,’’ you might argue, ‘‘if you have a cycle, then the group is indifferent.’’ That is a good idea, but the argument causes problems. First, it means that with ‘‘large’’ problems, groups will almost always be indifferent due to the likelihood of a cycle. It also has another problem. Suppose we are given a group with preferences over options (x, y, z, w) as listed above.

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A brief check will demonstrate a cycle from w to x to y, to z to w. Thus, according to the suggestion above, the group is indifferent among all four options. The problem with this suggestion is that everyone prefers w to z. Not only does this example discredit the suggestion, it also demonstrates that majority rule can begin with a status quo (assume w) and end with an option everyone thinks is worse (z).

Before drawing any conclusions, another example should be considered. This is a process which is fre-
The example involves four candidate options w, x, y, and z and seven people. Their preferences are given below.

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**Total Points**

|       | 18   | 15   | 19   | 14   | 20   | 13   |

For each individual the top ranked option gets four points, the next option gets three points, the third gets two points, and the bottom option gets one point. These points are then summed and the candidate with the greatest total number of points wins. When this is done, the winner is y with 20 points. Then comes x and w with z trailing with only 13 points.

Suppose z is dropped before the balloting. After all, everyone prefers y to z. If it is eliminated, three options are left. That means each individual's top option gets three points, the next one gets two and the lowest gets one point. If the group's preference is now computed, an amazing result can be demonstrated. The social preference has been inverted with w getting 15 points, x 14, and y last with 13. By removing the loser, the social preference can be inverted, even though no individual's preference changed—all individuals' preferences remained constant.

By now the reader should be a little surprised and should be wondering about the implications. Two things are being suggested here. The first is that group attitudes and group choices do not follow the same laws as individual choices do. The second is going to be a statement about the sensitivity of group choice to decisionmaking procedures. This second implication will be explored after more examples.

The first thing suggested by the examples is that group choices do not have the same type of internal consistency or optimization character that individual preferences do. Examples such as these inspired researchers to investigate all conceivable ways to pass from a set of individual preferences to a social preference. The problem has been elegantly axiomatized, but the results have neither been what people expected nor wanted. The principal results are impossibility theorems which say that there is no "nice" way to solve the problem.

Much theory and philosophy evolving from these efforts suggests that the concept of group preference itself is not a good concept. It seems to involve the classic fallacy of composition by assuming that a property of the individual, a preference, is also a property of the group. For participants of this conference, the bottom line should be underlined. The application of risk/benefit analysis, depending upon some technical aspects of the application, rests squarely on the concept of group preference as a foundational property. Even though the practitioners may not say so, the formal properties of the social preference concept which cause all the problems are lurking beneath the surface of risk/benefit procedures.

**PUBLIC PARTICIPATION MODELS**

Not only do we have a problem with this concept "social preference," but we are beginning to develop a rather striking conclusion about the nature of social decision processes in general. The choices of groups are very sensitive to the processes or procedures they use. Very small changes in procedures can have radical consequences for the final decisions. In fact, if you are good at choosing your procedures and are well informed, you can get groups to do almost anything you want, even though they appear to be voting democratically.

Let us take two more examples. Suppose the preferences are as below.

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If we apply the Borda count introduced above, then x is the winner with 16 points. However, if majority rule is used, w is a clear winner, since it beats both x and y.

Here is another example. Let the preference be as given.

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First consider the process whereby the individuals simply vote for their first preference. The winner is w. However, had the group voted for top two, giving their top choice two points and their second choice one, the outcome would have been x. If the group had decided to use a process whereby each voted for
the top three (in Borda fashion), the choice would be \( y \).

So, the choice between the three processes (vote for your top one, vote for your top two, or vote for your top three) turns out to be a choice between \( w \), \( x \), and \( y \). The group’s choice in this example is completely determined by the process.

There are again two punch lines. One is that when you start with a concept of preference which makes sense at the individual level and extend it to the group level, you have real problems, not all of which we understand. The second thing is that it appears as though the processes the group uses to make decisions systematically influences the decisions they make.

This second conclusion needs emphasis here. Other speakers have rested much of their philosophy of public decisions upon public hearings. It is very easy to say, "Let’s get the citizens together and listen to them. We want only to do what they want.” The thesis here is that the procedures used when listening to the public’s pulse largely determine what you hear. The procedures are overwhelmingly important. Some behavioral data will help elaborate the nature of this hypothesis.

Again, when reviewing the evidence, I must plead guilty of playing the academic trick of using only simple cases. I hope this audience will understand the difficulties of doing otherwise.

The experiment involves a group of five people who must choose a point on the blackboard. That seems easy. There is an infinite number of points on the blackboard, but a coordinate system makes any one of them easy to locate. The group is to use majority rule, and in particular, must use Robert’s rules. A motion (point) is placed on the floor. The motion on the floor can be amended freely (displaced to another point). The process continues until someone calls the question and the motion on the floor is adopted.

Ordinarily, this task would seem silly and useless, but here the individuals are given financial incentives. Those incentives are used to induce substantial conflict.

Look at Figure 2. Each individual is assigned a point on the blackboard. For example, Individual 5 has been told that if you can get the group to choose the point indicated as \((38, 52)\) he will receive \$28.00. He knows he will be paid in cash, so the incentives are real. The further the group’s choice is away from his point, the less he will get. In fact, as you sweep through the point \((39, 68)\), moving in a direction away from \((30, 52)\), it costs Individual 5 about a dollar to a dollar fifty per unit on a 150 by 200 grid.

![Figure 2. Hypothetical behavioral case.](image-url)
Each individual has a different optimum point. The optimals for Individuals 1, 2, 3, 4, and 5 are indexed by the numbers 1, 2, 3, 4, and 5, respectively. Each has a single optimum. An individual's payoff is reduced as the group choice is moved away from his optimum. The group can choose only one point. There can be no side payments nor bribes. Individuals can complain, argue, and curse, but no mention of monetary magnitudes can be made. The final point must be chosen by majority vote.

What do you think the group would do? There are many competing theories which predict a great variety of points. As it turns out, however, there is now really no controversy about what they will do. Under these circumstances, the group will choose the core of the cooperative game model. Some data are shown in Figure 3. Each dot is the decision of a different group of people. As you can see, they are well contained around the point (39, 68), which is the core. There are three points right on it.

Figure 3. Behavioral case results.

Figure 4 demonstrates what happens if people have elliptical indifference curves. As can be seen, the data are again grouped close to the core (63, 59). Figure 5, where individuals have rhomboid indifference curves, again demonstrates the accuracy of the core/equilibrium model.

Are we to conclude that groups using any set of procedures will naturally choose the majority rule core? No! We can conclude that groups using these procedures will choose the core, but alternative procedures lead to different outcomes. These data show only that groups behave in a systematic fashion and can, in special circumstances, be modeled mathematically. If Robert's rules had been dropped, the outcome variance would increase. If unanimity replaced majority rule, the outcome distribution would have been shifted to the right. The effects of parliamentary chances regarding amendment processes, such as the closed rule, are known to systematically change the outcomes.

Perhaps the most interesting and dramatic way to demonstrate the point is with fixed agenda. First, it is necessary to see agendas from an abstract point of view. Certain types of agenda motions operate to sequentially eliminate options. Suppose, for example, the vote was on a banquet. Four different types of
Figure 4. Elliptical indifference curves.

Figure 5. Rhomboid indifference curves.
banquets are available (two cuisines—Mexican and French, and two attires—formal or informal). If the agenda first called for a majority vote on attire and secondly for a majority vote on cuisine, the decision tree would be:

![Decision Tree Diagram]

If the agenda called for a vote on cuisine first and then attire, the tree would be:

![Decision Tree Diagram]

As can be seen, different agendas induce different trees. The interesting point is this: these different trees induce different group decisions. In fact, if there is ample conflict among group members, this single parameter can be used to induce the group to choose anything you wish. In other words, the agenda can be used to systematically influence, if not dictate, the group’s decision, even though the group is voting and discussing issues openly and democratically.

Not only is the agenda such an important parameter, the agenda appears in very subtle ways. Consider Figure 6 where several different ways of wording motions are listed. Take the first example, where the choice is between three options, x, y, and z. If the question is: “Do we want x or do we not?”, a “yes” answer yields x and a “no” answer yields a choice between y and z. However, if the question is: “Do we want x?”, a “yes” answer yields x while a “no” answer returns us to consider all three, since the question was not worded to discharge x as a possibility. The figure continues with other wordings and indicates the trees they induce. Agenda theory suggests that under a wide set of circumstances, these trees determine the outcome. ³

Individuals who control the group’s decisionmak-
QUESTION  | EXAMPLE OPTION IN THE "BLANK"  | TREE DIAGRAM

Do we want ____  | X  |
| or do we not?   |    |

Do we want ____? | X  |

Can we eliminate ____? | X  |

Of the two, which shall we eliminate? \( \{X, Y\} \)

Of the two, which do we prefer? \( \{X, Y\} \)

Figure 6. Some possible ways of applying a binary process to a three element choice situation.
ing procedures are in a very powerful position. In fact, some researchers feel that group expressions reflect little more than the opinion of the vested interest which won the premeeting jockeying for control of the procedures. The most cynical opinions could be offset if there were a natural set of "best" procedures. If a "group preference" could be defined, one could design procedures which result in the choice of the "most preferred" outcome. Unfortunately, these ideas are exactly those that the impossibility theorems suggest will not work.

CONCLUSION

I am afraid I leave you, as I remain myself, on the horns of a dilemma regarding the systematic incorporation of public attitudes into administrative (bureaucratic) decisionmaking. From my value point of view, I would think it outrageous if individual attitudes were not consulted. Yet the measurement problems and aggregation problems implicit in risk/benefit analysis leave me skeptical of the advantages of complete adoption of these methods. Certainly, it would seem premature to build upon them legislation which called for anything other than calculating the numbers. On the other hand, the alternative routes—public hearings, polls, meetings with citizen groups, and voting have their own problems. The procedures are of overwhelming importance, and the Ames theories which tell us to be skeptical of formal risk/benefit calculations tell us that all procedures have similar complications. The bizarre properties we have seen in the examples above are characteristic. Perhaps at this point the best we can do is to warn decisionmakers about being overconfident that they can simply, in any formal sense, "follow the preference of the people."

REFERENCES

