INTERSTELLAR ELECTRON SPECTRUM FROM THE GALACTIC
NON-THERMAL RADIO EMISSION

A. C. Cummings, E. C. Stone, and R. E. Vogt

Division of Physics, Mathematics, and Astronomy
California Institute of Technology
Pasadena, California (USA) 91109

A range of interstellar electron spectra at energies between
100 MeV and 5 GeV has been derived from an analysis of the
observed galactic non-thermal radio spectrum and from con-
sideration of the existing uncertainties in the other relevant
physical parameters of the galaxy. We find that for energies
larger than ~ 300 MeV the electron spectrum is uncertain to a
factor of 4 due to uncertainties in the galactic magnetic field
strength and the total line-of-sight emission length. The un-
certainty in the electron spectrum increases towards lower
energies, exceeding a factor of 50 near 100 MeV, primarily due
to uncertainties in the galactic parameters affecting inter-
stellar radio absorption.

1. Introduction. A knowledge of the interstellar cosmic-ray electron
spectrum is important to studies of source regions and of the physical
properties of the interstellar and interplanetary media. A number of
authors have previously inferred the interstellar electron spectrum from
the observed galactic non-thermal radio emission. However, the galactic
parameters relevant for this analysis have large uncertainties, and
previous studies have not explicitly considered the effect of these un-
certainties upon the calculated interstellar electron spectrum.

We have re-estimated the interstellar electron spectrum and its range
of uncertainty from the non-thermal radio data, assuming the electrons to
be uniformly distributed in the galaxy, and considering explicitly the
range of uncertainty of all galactic parameters entering the analysis.

2. Non-Thermal Radio Emission of the Galaxy. Figure 1 shows the observed
non-thermal radio spectrum in direction of the galactic anticenter. The
data above 5 MHz are observations in the galactic disc from the compilation
by Webber (1968). The radio spectrum below 5 MHz was observed by Alexander
et al. (1970) on the RAE-1 satellite. Although the RAE-1 instrument had
broad angular resolution, the isotropy of the observed radiation is
interpreted in terms of a rather local origin, i.e. from the disc of the
galaxy.

The non-thermal radio spectrum is due to synchrotron radiation of
electrons spiraling in the galactic magnetic field. We have calculated the
radio spectrum resulting from a given electron spectrum according to the
equations of Ginzburg and Syrovatskii (1964). The total intensity of
radiation, I (ν), from a given line-of-sight distance, L, is derived from
the equation:
\[
\frac{dI(v)}{ds} = \varepsilon(v) - k(v)I(v)
\]

(1)

where \( \varepsilon(v) \) is the emissivity, \( k(v) \) is the free-free-absorption coefficient, and \( ds \) is an element of length along the line of sight. The solution of equation (1) is dependent on the physical properties of the interstellar medium. Observations indicate a medium of dense, cold clouds with diameters of a few parsecs, separated by hot, rarified inter-cloud regions with scales of hundreds of parsecs. We have used as a simplified model of the interstellar medium a uniform distribution of cold clouds with diameters \( l_C \) and separation distance \( l_i \). (This model is identical to that used by Goldstein et al. (1970) for \( l_C = 1 \) pc and \( l_i = 1 \) kpc). The subscripts "c" stand for "cold" clouds and "i" for intercloud. The first cold cloud is assumed to lie at a distance \( l_i \). For this model the solution to equation (1) is:

\[
I(v) = \frac{\varepsilon(v)(1-e^{-\tau_i})}{k_i} \left[ \frac{\frac{l_i}{l_c} - (\tau_i + \tau_c)}{1 - e^{-\frac{l_i}{l_c} - (\tau_i + \tau_c)}} \right]
\]

(2)

where the optical depth \( \tau_i \) is defined by

\[
\tau_i = \int_0^{l_i} \frac{k_i}{c} ds
\]

(3)

Fig. 1. Non-thermal radio spectrum in direction of the galactic anticenter.

Data points: Radio observations. Curves: Radio spectra derived from characteristic interstellar electron spectra.

We have made a critical review of the existing literature for the uncertainties, i.e. range of possible values, of galactic parameters entering our calculations. We summarize the result in Table I in terms of "nominal" values and a "range" of values reflecting existing uncertainties.
Table 1. Galactic Parameters

<table>
<thead>
<tr>
<th>Parameter*</th>
<th>$B_L$</th>
<th>$L$</th>
<th>$l_i$</th>
<th>$l_c$</th>
<th>$T_i$</th>
<th>$T_c$</th>
<th>$n_{i/c}$</th>
<th>$n_{c3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>(µG)</td>
<td>(kpc)</td>
<td>(pc)</td>
<td>(pc)</td>
<td>(K)</td>
<td>(K)</td>
<td>(cm$^{-3}$)</td>
<td>(cm$^{-3}$)</td>
</tr>
<tr>
<td>Nominal</td>
<td>5</td>
<td>4</td>
<td>50</td>
<td>1</td>
<td>10000</td>
<td>250</td>
<td>.03</td>
<td>.02</td>
</tr>
<tr>
<td>Range</td>
<td>3–5</td>
<td>2–6</td>
<td>13–1670</td>
<td>1–10</td>
<td>500–10000</td>
<td>15–250</td>
<td>.02–.05</td>
<td>.02–.06</td>
</tr>
</tbody>
</table>

*B$_L$ = mean value of magnetic field normal to the line of sight;

$T_{i/c}$ = electron temperature of the intercloud/cloud region;

$n_{i/c}$ = intercloud/cloud ambient electron density.

3. Derivation of the Interstellar Electron Spectrum. If the values of all galactic parameters - principally those of Table I - were accurately known, one could vary the energy dependence and the intensity of the sought for electron spectrum until the computed radio intensity matched the observations. However, the uncertainties of the galactic parameters (see Table I, "Range") are sizeable - particularly in the intercloud temperature $T_i$ - and will be reflected in uncertainties of the derived interstellar electron spectrum.

We have derived a "nominal" interstellar electron spectrum on the basis of the nominal values of galactic parameters in Table I which yields a radio spectrum in agreement with the observations (see Fig. 1, nominal-spectrum model). The nominal electron spectrum can be expressed in power law form,

$$j(W) = \begin{cases} 1.34 \times 10^4 W^{-1.8} & 70 \text{ MeV} \leq W \leq 2000 \text{ MeV} \\ 2.75 \times 10^6 W^{-2.5} & 2000 \leq W \leq 5000 \text{ MeV} \end{cases}$$

where $W$ is the total energy of an electron.

In order to evaluate the effect of other than nominal galactic parameters, we have used the following method: We derived an approximate solution, which, for the nominal electron spectrum defines the energy $W$ which dominates the emissivity $\epsilon(v)$ at a given frequency $v$. This relationship is

$$v (\text{MHz}) = 20 B_L W^2$$

where $B_L$ is measured in µG, $W$ in GeV. A variation of any galactic parameter from the nominal value will result in a new calculated radio spectrum $I'(v)$. If we determine the ratio of the observed radio spectrum $I(v)$ to the computed spectrum $I'(v)$, we obtain a set of factors $F(v) = I(v)/I'(v)$. Using Eq. 5, we can find a factor $F(W) = F(v=20B_L W^2)$ which can be applied to the nominal electron spectrum $j(W)$ to derive an "adjusted" spectrum $j'(W)$ which is consistent with the observed radio spectrum, i.e. $j'(W) = F(W)j(W)$. We show in Figure 2 the factor $F(W)$ for the assumed variations of different galactic parameters. The dashed lines demarc the approximate envelope of all variations considered. At energies above ~300 MeV, the envelope is essentially determined from the propagation of uncertainties in $B_L$ and $L$, with a range in $F(W)$ of a factor of ~4. At lower energies, where the (poorly understood) interdependence of parameters makes a simple statistical consideration unfeasible, we used the maximum variation of a single parameter.
Fig. 2. Correction factor \( F(W) \) vs. electron energy \( W \). The nominal set of galactic parameters \( (F(W)=1) \) is bracketed in the figure. Each labeled solid curve is calculated by changing only the value of the indicated parameter from the nominal set.

for the envelope. Near 100 MeV, the range of \( F(W) \) exceeds a factor of 50.

We have used the envelopes of Figure 2 to derive the approximate limits to the galactic electron spectrum shown in Figure 3. The limiting spectra (high and low spectrum) are derived by correcting the nominal spectrum (equation 4) with the \( F(W) \) values of the envelopes. The radio spectra derived from any electron spectrum within the limits shown in Figure 3, can, with the appropriate choice of galactic parameters, be made consistent with the observed radio spectrum. Examples of this agreement for three possible spectra are shown in Figure 1.

4. Summary. We have derived a range of interstellar electron spectra which are consistent with the observed galactic non-thermal radio emission and which reflect the present range of uncertainties in the relevant other physical parameters of the galaxy.

We find that for energies \( > 300 \) MeV the electron spectrum is uncertain to a factor of 4 because of uncertainties in the galactic magnetic field strength and the total line-of-sight emission length. At lower energies, the uncertainty in the electron spectrum increases, exceeding a factor of 50 near 100 MeV, primarily because of uncertainties in the galactic parameters governing absorption of the radio signals.
5. References.


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