SOLAR MODULATION OF COSMIC-RAY PROTONS AND He NUCLEI


Division of Physics, Mathematics, and Astronomy
California Institute of Technology
Pasadena, California (USA) 91109

We present the results of a detailed, quantitative study of the solar modulation of cosmic-ray protons and He nuclei. The study is based on; (a) observations over the 1966 to 1970 period with balloon- and satellite-borne instruments, (b) numerical solutions of the cosmic-ray transport equation, and (c) propagation parameters derived from studies of near-Earth and interstellar spectra of cosmic-ray electrons. We find that consistent, quantitatively satisfactory results may be obtained with a model of solar modulation which uses a diffusion coefficient separable into functions of heliocentric radius and particle rigidity and in which adiabatic deceleration has a dominant role in shaping the low-energy spectra of nuclei near 1 AU.

1. Introduction. New observations of the energy spectra of cosmic-ray protons and He nuclei at 1 AU over the period 1966 through 1970 have been used to study the long-term variations of galactic spectra due to solar modulation. The data were obtained from Caltech's Solar and Galactic Cosmic-Ray instrument (Althouse et al., 1967) and from an essentially identical instrument (Garrard, 1972) flown on high-altitude balloons from Fort Churchill, Canada.

We shall show that a relatively simple model of solar modulation satisfactorily explains the observed features of the cosmic-ray spectra, and that, in particular, a relatively good understanding of the behavior of low-energy nuclei ($E \lesssim 300$ MeV/nucleon) allows conclusions on other physical features of the interplanetary medium.

2. Solar Modulation Parameters. The observational data can be understood on the basis of a simple, spherically symmetric model of solar modulation wherein the propagation of galactic cosmic rays in the interplanetary medium is described by the transport equation:

$$\frac{V}{r^2} \frac{\partial}{\partial r} (r^2 U) - \frac{2V}{3r} \frac{\partial}{\partial T} (\alpha T U) - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa \frac{\partial U}{\partial r}) = 0$$

(1)

with a constant solar-wind velocity $V$, and where $U = 4\pi j/\beta c$ is the cosmic-ray particle density/unit energy, $j$ is the intensity, $\beta c$ the particle velocity, $T$ kinetic energy, $\kappa$ the cosmic-ray diffusion coefficient, and $\alpha = (T + 2m)/(T + m)$, where $m$ is the particle rest energy. We have solved this equation with a numerical technique first proposed by Fisk (1971), and by representing the diffusion coefficient as

$$\kappa = \beta g(r)f(R)$$

(2)
i.e., a separable function of heliocentric radius, \( r \), and particle rigidity \( R = p/cZ_e \), where \( p \) is momentum. Satisfactory agreement with observations can be achieved by representing \( g(r) \) independent of radius by the integral expression

\[
g(r) = \frac{V(D-1)}{\eta} \quad \text{for} \quad r < D, \tag{3}
\]

where \( D \) is the boundary distance of the solar modulation region and \( \eta \) is a parameter for the "strength" of the modulation [e.g., for negligible adiabatic deceleration, \( j(r = 1 \text{ AU}) = j(r = D) \exp(-\eta/\beta f(R)) \)]. The rigidity dependence of the diffusion coefficient may be approximated by a piecewise power-law representation,

\[
f(R) = \begin{cases} 
R & \text{for} \quad R \geq R_2 \\
(RR_2)^{0.5} & \text{for} \quad R_1 \leq R \leq R_2 \\
(R_1R_2)^{0.5} & \text{for} \quad R \leq R_1 
\end{cases} \tag{4}
\]

The parameters \( R_1, R_2, \) and \( \eta \) were determined for the specific observation epochs from a study of the local and the interstellar electron and positron spectra (Cummings, 1973), and are listed in Table I.

**Table I.** Modulation Parameters (+ data references for Fig. 1)

<table>
<thead>
<tr>
<th>Epoch</th>
<th>Entry #</th>
<th>( \eta ) (MV)</th>
<th>( R_1 ) (MV)</th>
<th>( R_2 ) (MV)</th>
<th>Data References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965-66</td>
<td>1</td>
<td>1350</td>
<td>62</td>
<td>800</td>
<td>• Fan et al., 1966</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+ Ormes and Webber, 1968</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>△ Fan et al., 1968</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• McDonald, 1958</td>
</tr>
<tr>
<td>1968 (p)</td>
<td>2</td>
<td>1950</td>
<td>160</td>
<td>750</td>
<td>□ LeBron and Webber, 1971</td>
</tr>
<tr>
<td></td>
<td>1967/68 (He)</td>
<td></td>
<td></td>
<td></td>
<td>• Caltech</td>
</tr>
<tr>
<td>1969</td>
<td>3</td>
<td>2860</td>
<td>172</td>
<td>1500</td>
<td>□ Hsieh et al., 1971</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3070</td>
<td>229</td>
<td>1300</td>
<td>• Mason, 1972</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• Caltech</td>
</tr>
<tr>
<td>1970</td>
<td>5</td>
<td>3300</td>
<td>286</td>
<td>1200</td>
<td>• Caltech</td>
</tr>
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</table>

The interstellar spectra \( j_D \) can be represented by

\[
j_D = j_0 (T + \mu)^{-2.65} \tag{4}
\]

with a single parameter \( \mu \) for both proton and He spectra.

3. Results. Figure 1 shows a comparison of the calculated spectra with measured proton and He spectra, for the parameters of Table I, and using \( D = 12 \text{ AU}, \mu = 0.75 \) (note: the calculated spectral shapes are relatively insensitive to the choices of \( D \) and \( \mu \), see below). The agreement, within the uncertainties of the observations from different instruments, is generally
Fig. 1. Calculated and measured spectra of cosmic-ray protons (left) and Ne nuclei (right) at 1 AU for epochs marked. The calculated spectra are referenced to Table I by entry number. Data references are also in Table I.
good, except at the lowest energies, where some spectra show turn-ups. This feature is most probably due to solar particle emission; if of galactic origin, it would imply incompleteness of our modulation model [for further discussion see, e.g., Garrard (1972), Webber and Lezniak (1973)]. In the following, we shall restrict our discussion to the spectral regions above the turn-up.

4. Discussion. It is evident from Figure 1 that solar modulation reduces the flux of low-energy (E ≤ 300 MeV/nucleon) interstellar nuclei by several orders of magnitude near 1 AU. More detailed analysis (Parker, 1965, and Goldstein et al., 1970), in fact, shows that low energy nuclei from the interstellar medium cannot penetrate the interplanetary medium to 1 AU; the observed low energy particles derive from the adiabatic deceleration of higher energy particles by the solar wind. Urch and Gleson (1973) have shown that low-energy protons originate, roughly, at energies between a few hundred and ~ 1000 MeV. Thus the low-energy proton spectra near Earth are determined by the intensity of interstellar protons in a broad energy region above a few hundred MeV and by the diffusion coefficients of corresponding rigidity. They are essentially independent of the low-energy interstellar spectra and the low-rigidity diffusion coefficient. Since the cosmic-ray diffusion coefficient near rigidities of ~ 1000 MeV lies in a range where it is least uncertain (Cummings et al., 1973), its use with the low energy proton spectra allows the calculation of other phenomena, e.g., radial intensity gradients, (Garrard et al., 1973) with relatively good accuracy.

The fact that low-energy proton spectra near Earth originate from the interstellar intensities at energies greater than a few hundred MeV also explains why the calculated spectral shapes are relatively insensitive to the parameter μ in equation (4); the interstellar proton spectrum below ~ 300 MeV has essentially no effect upon the 1 AU spectra.

The effect of the modulation-boundary distance, D, upon the 1 AU low energy proton spectra is illustrated in Figure 2. All spectra in Figure 2 are calculated from the same input spectrum (see Fig. 1), with fixed f(R) and η. The effects of boundary distance are significant only for a nearby (≤ 7 AU) boundary (see also Garrard et al., 1973), where the low-energy proton spectrum is steepest, for larger boundary distances, the spectrum approaches asymptotically an approximate j ∝ p^2 dependence.

Fig. 2. Calculated proton spectra near Earth for various values of the boundary distance, D, for fixed η. D values used follow in order: 2.7; 4.4; 6.1; 12.9; 18.0; and 35 AU.
5. Summary. We have shown that the low-energy proton and He-nuclei spectra observed near 1 AU over significant periods of the solar cycle (1965-1970) can be understood - within the present observational uncertainties - on the basis of simple, spherically symmetric solutions of the cosmic-ray transport equation, using modulation parameters derived from studies of cosmic-ray electron spectra at 1 AU and in the interstellar medium. Satisfactory agreement of calculated and observed spectra can be obtained with a cosmic-ray diffusion coefficient which is a separable function of heliocentric radius and rigidity for both proton and He-nuclei spectra. Agreement with 1 AU observations also is achieved for interstellar proton and He-nuclei spectra which have the same functional energy dependence. Except for close modulation-boundary distances (≤ 7 AU), the calculated low-energy spectra at 1 AU are essentially independent of boundary distance for a given modulation factor.

6. References.


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