Calculation of beta-decay half-lives of proton-rich nuclei of intermediate mass

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We present the results of a calculation of the beta-decay half-lives of several proton-rich even-even nuclei of intermediate mass: $^{84}$Sr, $^{78}$Sr, $^{72}$Zr, $^{82}$Zr, $^{84}$Mo, $^{85}$Mo, $^{86}$Ru, $^{87}$Ru, $^{92}$Pd, and $^{86}$Cd. The calculation is based upon the random phase approximation with the quasiparticle formalism and takes into account the residual particle-particle interaction.

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Ever since Takahashi et al. [1] calculated in 1973 estimates for the beta-decay half-lives of virtually all beta-unstable nuclei, there has been a large effort to improve these estimates for neutron-rich nuclei because of applications in r-process theory and in the fate of fission products [2]. The theoretical effort to improve the estimates of half-lives of proton-rich nuclei has not been commensurate, although Hirsch et al. [3] and Muto et al. [4] presented calculations for light nuclei ($Z \leq 30$). (See, however, also Refs. [5–7].) In this Brief Report we present the results of an effort to improve the estimates for half-lives of several proton-rich nuclei of intermediate mass. These half-lives play a role in rp-process theory [8], that is, the process in which protons are quickly added onto C, N, O, and other “metals” with intervening fast positron decays resulting in heavy proton-rich nuclei. This process occurs in certain astrophysical contexts in which the temperature is greater than about 10⁸ K. In particular, this process is predicted to occur in massive stars with degenerate neutron cores (if they exist) [9], and information about the longer-lived (≥ 1 s) beta-unstable nuclei would allow one to predict the nuclear abundances on the surfaces of these stars [10]. For this reason we undertook the calculation of half-lives of some proton-rich even-even nuclei of intermediate mass.

We are interested in even-even nuclei which have 0⁺ ground states, so that the calculation is relatively simple. The positron-decay half-life $t_{1/2}$ is given by the following formula:

$$\frac{1}{t_{1/2}} = \sum_m \frac{B(GT)_m g_A^2}{6160 \text{ s}} f(\Delta E_m, Z),$$

where $m$ labels the accessible 1⁺ states in the daughter nucleus, $B(GT)_m$ is the Gamow-Teller $\beta^+$ strength (equivalent to $|\langle m | \sigma^+ | i \rangle|^2$ in this case), $g_A$ is the axial-vector-current coupling constant (which we set to 1.25), and $f(\Delta E_m, Z)$ is the Fermi function (including Coulomb and relativistic corrections), which describes the size of phase space.

We obtain energy levels of the daughter nucleus and evaluate $B(GT)$ using the random phase approximation based on the quasiparticle formalism (QRPA). (The generalization of the QRPA to charge-changing modes is due to Halbleib and Sorensen [11]. Particle-particle interactions were first included in the QRPA by Cha [12].) The formalism is described in detail in Vogel and Zirnbauer [13] and in Engel et al. [14]. In these papers the authors use the $\delta$ force as the residual interaction and describe the following four parameters: $\alpha_0$, $\alpha_1$ (the particle-hole interaction constants in the $S = 0$ and $S = 1$ channels, respectively), $\alpha_0'$, and $\alpha_1'$ (the particle-particle interaction constants). Although these constants are theoretically related, the authors present an argument that they can be treated independently in this calculation. Using the values given in Ref. [14], we set $g_{\text{pair}} = -270$ MeV fm³ when we solve the BCS equations, and we set $\alpha_0 = -890$ MeV fm³ and $\alpha_1 = -1010$ MeV fm³ for the RPA portion of our calculations. Because we are looking at positron decay of proton-rich nuclei, our results do not depend on $\alpha_0$ in the RPA calculations. Our results do, however, depend strongly on the value of $\alpha_1'$, so we must take care to choose it carefully.

We divide the nuclei into two categories, those with $74 \leq A \leq 80$ and those with $80 < A \leq 96$. For the heavier nuclei in our study, we calibrated $\alpha_1'$ using the known decay half-lives of $^{86}$Mo, $^{89}$Mo, $^{90}$Ru, and $^{90}$Pd. In order to calculate these half-lives, we identified the lowest-lying 1⁺ state in the daughter nucleus with the ground state given by the QRPA calculation. (This determines the values of $\Delta E_m$, used in the phase space integrals.) Our calculation is for positron decay only, i.e., no electron capture. In three of the calibration nuclei positron decay dominates over electron capture; however, 75% of the decay of $^{90}$Mo is due to electron capture. In that case we, therefore, use the proper partial decay rate. In our calculation, almost all (≥ 90%) of the predicted decays occur into the lowest-lying 1⁺ state. Figure 1 shows the log₁₀ of the ratio of calculated positron-decay half-life to experimental half-life versus $\alpha_1'$. From this figure we see that $\alpha_1'$ may be anywhere within a window from $-324$ to $-333$ MeV fm³ and yield values of half-lives correct to within a factor of 3. A value of $\alpha_1' = -329$ MeV fm³ yields a least $\chi^2_{\text{red}}$ equal to 0.22, where

$$\chi^2_{\text{red}} = \left[ \frac{1}{3} \sum \log_{10}(T_{\text{calc}}/T_{\text{exp}}) \right]^{1/2}.$$

Thus we predict that our results in Table I are accurate to about a factor of $10^{0.22} = 1.7$. By comparison, the $\chi^2_{\text{red}}$
for these four nuclei using results from Takahashi et al. [1] is 0.59, yielding an estimated accuracy of a factor of $10^{0.59} = 4$.

In order to calculate half-lives of the nuclei listed in Table I, we need to know the positron-decay energies.

Since the masses of the positron-decay parents (and often those of the daughters as well) are not known, we use the predicted masses of Jänecke and Masson [15]. (These seem to reproduce best the known masses of proton-rich nuclei.) We set $\Delta E_m = 0$, that is, the maximum total energy of the positron, to the difference of parent and daughter masses less 0.2 MeV. The 0.2 MeV represents a typical value for the energy difference between the ground state and the lowest-lying $1^+$ state of the daughter nucleus. (For these decays, however, $\Delta E_m = 0$ is large enough that the correction is trivial.) The results are shown in Table I. As stated in the previous paragraph, these values are accurate to within a factor of about 2. Electron capture is negligible in these nuclei, contributing less than 3% because of the large decay energies involved. (See Ref. [16].)

Similarly we use the known half-lives of $^{70}$Se, $^{72}$Kr, $^{74}$Kr, and $^{80}$Sr to calibrate $\alpha'$ and calculate half-lives for several nuclei with $A \leq 80$. In this case we obtain $\alpha' = -327$ MeV fm$^3$ for the best fit, yielding a least $\chi^2$ equal to 0.32. The results are also shown in Table I. We estimate that the results are accurate to within a factor of about $10^{0.32} = 2$, and again electron capture is negligible.

Also shown in Table I are the predicted half-lives of

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**TABLE I. Predicted beta-decay half-lives.**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\Delta E_m = 0^a$ (MeV)</th>
<th>Half-life$^b$ (s)</th>
<th>Takahashi et al. [1] half-life (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{74}$Sr</td>
<td>9.6</td>
<td>0.5</td>
<td>0.03</td>
</tr>
<tr>
<td>$^{76}$Sr</td>
<td>4.5</td>
<td>8.</td>
<td>3.</td>
</tr>
<tr>
<td>$^{78}$Zr</td>
<td>10.5</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>$^{80}$Zr</td>
<td>5.0</td>
<td>7.</td>
<td>3.</td>
</tr>
<tr>
<td>$^{84}$Mo</td>
<td>5.2</td>
<td>6.</td>
<td>0.8</td>
</tr>
<tr>
<td>$^{86}$Mo</td>
<td>3.9</td>
<td>90.</td>
<td>16.</td>
</tr>
<tr>
<td>$^{88}$Ru</td>
<td>5.8</td>
<td>1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>$^{92}$Ru</td>
<td>4.7</td>
<td>16.</td>
<td>5.</td>
</tr>
<tr>
<td>$^{92}$Pd</td>
<td>6.8</td>
<td>0.9</td>
<td>0.4</td>
</tr>
<tr>
<td>$^{99}$Cd</td>
<td>8.0</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

$^a$ This is the maximum total energy of the positron for a transition to the lowest $1^+$ daughter state.

$^b$ The estimated accuracy is a factor of 2. See the explanation in the text.
Takahashi et al. It is encouraging that our results are consistent with theirs, which are calculated by a different method; most of the difference is due to different $Q$ values (i.e., $\Delta E_{m=0}$), especially in the case of $^{74}\text{Sr}$.

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