Measurements of Cosmic Ray Isotopes with a Super-Condacting Magnet Facility

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A method to resolve cosmic ray isotopes with a magnet-Cerenkov system is analyzed, with emphasis on applications to a Super-Condacting Magnet Facility for the Space Station.

1. Introduction - The cosmic ray community has recently been considering the possibility of a Super-Condacting Magnet Facility (SCMF) that could be flown on the Space Station (see, e.g., [1], [2], [3], and [4]). Such a facility would offer the exciting possibility of high-resolution measurements of a number of cosmic ray species over a wide range in energy, including antiprotons and antinuclei, electrons and positrons, and the elemental and isotopic composition of high energy nuclei. Perhaps the most rewarding (and also the most challenging!) of these prospects would be the extension of cosmic ray isotope measurements into the energy range from ~2 to as high as ~100 GeV/nucleon. Such measurements are particularly important to determine the source composition at energies where the interstellar pathlength is decreasing and secondary contributions are reduced, and to "read" radioactive clocks over a wide range of time-dilation factors.

A viable isotope experiment must achieve both sufficient mass resolution and collecting power. We have recently considered the energy coverage and collecting power that the SCMF and other isotope experiments might achieve with long-duration exposure in space [4]. In this paper we consider the mass resolution that can be achieved with a magnet-Cerenkov system. A reasonable mass resolution goal for isotope experiments is $\delta A \leq 0.25$ amu, sufficient in principle to resolve adjacent isotopes that differ in abundance by 100 to 1. Because the requirements for isotope studies may well drive the design of the magnet and its associated trajectory measuring devices, it is essential that parameters affecting the mass resolution be understood.

2. Isotope Resolution with a Magnet-Cerenkov System - To resolve isotopes we combine a measurement of the rigidity (momentum per charge) provided by a magnet and associated trajectory devices, with a measurement of the velocity (or momentum per nucleon, $p$), in this case provided by a Cerenkov counter. The Cerenkov signal from a nucleus with charge $Z$ and mass $A$ can be written:

$$L = L_0 Z^2 (1 - p_0^2/l_p^2)$$

where $L_0$ is the signal for $\beta = 1$, $Z = 1$ nuclei and where $p_0$ is the Cerenkov threshold in momentum/nucleon. The measured deflection ($X$) due to the magnetic field can be written $X = (kZ/pA)$, where $k$ (which is proportional to $\int B \cdot dl$ and trajectory dependent) can be treated as a constant for our purposes. Solving for the mass $A$ (in amu):

$$A = \frac{kZ}{Xp_0} \left[ 1 - \frac{L}{Z^2 L_0} \right]^{\frac{1}{2}}$$

Several contributions to the mass resolution have been considered. Among the most important is statistical fluctuations in the number of photoelectrons (PEs) produced in the photomultiplier tubes (PMTs) viewing the Cerenkov radiator, for which $\delta L = V L_0$, giving:

$$\delta A_{pe} = \frac{A}{2Z} \left[ \frac{p}{p_0} \right]^{\beta} \left[ 1 - \frac{1}{L_0} \right]^{\frac{1}{2}} \left[ 1 - \frac{p_0^2}{p^2} \right]^{\frac{1}{2}}.$$
For variations in the Cerenkov light-collection uniformity, we take \( \delta L = b L \), where \( b \) is the rms fractional non-uniformity (the residual non-uniformity after the counter "maps" have been applied). For this contribution we find:

\[
\delta A_u = b \left[ \frac{A}{2} \right] \left[ \frac{p^2}{p_0^2} - 1 \right].
\]

Because of a variety of effects there is usually additional light measured by a Cerenkov PMT that does not follow the Cerenkov relation for the main radiator. To account for this in an approximate manner the measured Cerenkov signal can be written: \( L = L_0 Z^{\beta} \left( 1 + s - \frac{p_0^2}{p^2} \right) \) where the constant \( s \) might represent residual scintillation (typically a few percent of the \( \beta=1 \) signal), here assumed to be independent of energy (or momentum/nuc). It might also represent a (saturated, and thus constant) contribution of Cerenkov light from material with a much lower index of refraction. In this case the formulas above are altered to give

\[
\delta A_{pe} = \left[ \frac{A}{2Z} \right] \left[ \frac{p}{p_0} \right]^2 \left[ \frac{1}{L_0} \right] \left[ 1 + s - \frac{p_0^2}{p^2} \right],
\]

for the PE contribution, and

\[
\delta A_u = b \left[ \frac{A}{2} \right] \left[ \frac{p}{p_0} \right]^2 \left[ 1 - \frac{p_0^2}{p^2} \right] + s^2 \left[ \frac{1}{L_0} \right],
\]

for the uniformity contribution.

The above relations can be solved to find the required number of PEs to achieve a given mass resolution, as shown in Figure 1, and the required uniformity, as shown in Figure 2. Because there will always be a number of separate contributions to the mass resolution, a reasonable goal is to limit the individual contributions to \( \leq 0.1 \) amu, so that when they get added in quadrature, the resulting total mass resolution is within the design goal of 0.25 amu. Note in Figure 1 that the principal effect of the PE contribution is to limit the momentum range that a given counter can cover. As an example, for \( L_0 = 50 \) PEs, \( \delta A \leq 0.2 \) amu for \( p/p_0 \leq 1.42 \).

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**Figure 1** - The number of photoelectrons required for 0.1 and 0.2 amu resolution, vs. \( p/p_0 \), where \( p_0 \) is the Cerenkov threshold in momentum/nucleon.

**Figure 2** - The required Cerenkov counter uniformity for \( \delta A \leq 0.1 \) amu vs. \( p/p_0 \). Figures 1 and 2 are independent of index of refraction.
For an index of refraction of 1.1, this corresponds to a kinetic energy of 1.3<s>≤</s>2.1 GeV/nucleon. To broaden this range there could be several such counters with different indices, which would also provide redundancy. The number and thickness of the counters must, of course, take into account nuclear interaction considerations. Although the number of PEs required does not seem to be unreasonable, at least for Cerenkov counters commonly used below a few GeV/nucleon (see, e.g., [5,6]), it is clear that the Cerenkov light collection efficiency must be optimized. In addition, as Figure 2 indicates, the counters must be mapped to very high accuracy (a few tenths of a per cent) to resolve isotopes as heavy as Fe or Ni very far above threshold. Although this mapping accuracy exceeds that of Cerenkov counters used to date, it should be possible. Note that the trajectory accuracy required to use such maps in flight will automatically be met in the SSMF.

Although it is beyond the scope of this paper to consider in detail the various contributions to the uncertainty in the rigidity determination, these will probably be dominated by the position uncertainty of the trajectory devices, with a resulting rigidity uncertainty that can be expressed as <s>δR</s>/R = R/R_m [3], where R_m is the "maximum detectable rigidity". This gives: <s>δR</s> = A(R/R_m). Note that the magnet will actually have a broad distribution of R_m values that depend on the allowed trajectories and that only a portion of the geometry factor will be available for studies requiring the maximum momentum resolution. If we limit <s>δA</s>_R, the maximum energy per nucleon to which isotopes can be resolved is:

\[ T_m = \left( \frac{\delta A_R R_m 2e / A}{m_p} \right)^2 + m_p^2 \]^{1/2} - m_p,

where m_p is the proton mass in GeV/nucleon. As an example, for R_m = 10 TV/c and <s>δA</s>_R = 0.1 amu T_m is 7.4 GeV/nucleon for 56Fe and 39 GeV/nucleon for 10Be. For <s>δA</s>_R = 0.2 amu these energy limits approximately double. It is clearly desirable to have even greater R_m (with a significant geometry factor) if the resulting requirements on the field strength and trajectory accuracy can be met [3]. At lower energies, note that in the 28.5° orbit planned for the Space Station, the geomagnetic cutoff limits the available energy range to greater than about 2 GeV/nucleon [4], and thus it will not be possible to achieve good mass resolution for Z>20 isotopes unless R_m > 1 TV/c.

![Figure 3](image1.png)  
**Figure 3** - 56Fe mass-resolution contributions vs. p/p_0 for the assumptions indicated.

![Figure 4](image2.png)  
**Figure 4** - Mass resolution vs. Z for the assumptions in Figure 4.
Discussion - Figure 3 illustrates the mass resolution contributions and shows the resulting total mass resolution for specific parameter choices which might be relevant to measuring Fe isotopes in a Space Station application of such a system. In Figure 4 the mass resolution is shown as a function of Z for the same set of parameters, with $p/p_0 = 1.3$. Although the contributions considered above probably dominate the mass resolution (at least for the Cerenkov counter), it should be remembered that there may also be other contributions, and thus these calculations represent the best that can be done for the given assumptions. In particular, fluctuations in the Cerenkov light produced by knock-on electrons will tend to degrade the resolution, especially for gas counters. It would clearly be useful to have accurate Bevalac calibrations of a particular counter with a variety of beams in order to measure these contributions, and the charge, mass, and energy dependence of any deviations from the predictions.

For a magnet-Cerenkov spectrometer to work there must be Cerenkov radiators with the appropriate indices of refraction. A 28.5° orbit requires $n \leq 1.1$, which includes the aerogel radiators. It appears that aerogel counters with $1.01 \leq n \leq 1.1$ would be adequate for the energy range from $\sim 1.3$ to $\sim 8$ GeV/nucleon. At higher energies gas Cerenkov counters might be used, and the possibility of other methods of velocity determination should also be investigated. At lower energies the demands on the rigidity measurement are less severe, and there are a variety of radiators available with $1.2 \leq n \leq 1.6$ that could cover the energy interval from $\sim 0.4$ to $\sim 1$ GeV/nucleon. Even a short high-latitude flight with a Shuttle or balloon-borne prototype of the SCMF would allow this approach and its required instrumentation to be tested, and it would contribute valuable measurements of many of the more abundant cosmic ray isotopes.

Summary - In this analysis we have derived some of the more important general requirements for a magnet-Cerenkov system to measure cosmic ray isotopes with good mass resolution. We have concentrated on the requirements for the Cerenkov detectors, although there are clearly also many challenges for the magnet itself and its associated trajectory-measuring devices. While this approach will require larger and more uniform Cerenkov counters than have been flown in the past, along with optimized light-collection efficiency, it does appear that this method could be successfully employed to measure high-energy cosmic-ray isotopes with a super-conducting magnet facility on the Space Station.

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References

4. R. A. Mewaldt, ibid.