PHYSICAL CONSTRAINTS ON FAST RADIO BURSTS
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ABSTRACT

Fast Radio Bursts (FRBs) are isolated, ms radio pulses with dispersion measure (DM) of order $10^{3}$ pc cm$^{-3}$. Galactic candidates for the DM of high latitude bursts detected at GHz frequencies are easily dismissed. DM from bursts emitted in stellar coronas are limited by free-free absorption and those from HII regions are bounded by the nondetection of associated free-free emission at radio wavelengths. Thus, if astronomical, FRBs are probably extra-galactic. FRB 110220 has a scattering tail of $\sim 5.6\pm 0.1$ ms. If the electron density fluctuations arise from a turbulent cascade, the scattering is unlikely to be due to propagation through the diffuse intergalactic plasma. A more plausible explanation is that this burst sits in the central region of its host galaxy. Pulse durations of order ms constrain the sizes of FRB sources implying high brightness temperatures that indicates coherent emission. Electric fields near FRBs at cosmological distances would be so strong that they could accelerate free electrons from rest to relativistic energies in a single wave period.

1. INTRODUCTION

FRBs are single, broad-band pulses with flux densities $S_{\nu} \sim Jy$ and durations $\Delta t \sim$ ms detected at $\nu \sim$ GHz (Lorimer et al. 2007; Thornton et al. 2013). They were discovered by de-dispersing data collected by the Parkes multi-beam radio telescope during pulsar searches. Thus far there are no reports of FRBs detected by other radio telescopes. The procedure followed in the detection of FRBs is similar to that which led to the discovery of rotating radio transients (RRATs, McLaughlin et al. (2006)) now firmly identified as sporadically active pulsars. Thornton et al. (2013) report the detection of four high-galactic-latitude ($>40^\circ$) FRBs with DM of several hundred pc cm$^{-3}$, well above the contribution expected from our Galaxy (Cordes & Lazio 2002). It has become popular to attribute these large DMs to propagation through the intergalactic plasma indicating source distances $d \sim$ Gpc.

Currently, it is unclear whether FRBs herald the discovery of a new type of astronomical source or merely that of an unidentified source of noise. The strongest argument supporting the astronomical origin of FRBs is the precise degree to which arrival times of individual pulses follow the $\nu^{-2}$ law that characterizes the propagation of radio waves through a cold plasma. Some pulses detected in searches for FRBs are clearly terrestrial although their origin is unknown. These have been named Perytons. It is notable that the classification of the Lorimer burst (Lorimer et al. 2007) remains controversial, although if it is a FRB it would be the first and brightest of those detected. For the remainder of this paper, we cast aside our doubts and proceed as through the Lorimer burst remains controversial.

In this section, we discuss two galactic candidates to produce DM for FRBs. One is a stellar corona, suggested by Loeb et al. (2014); the other is an HII region. We then demonstrate that neither can account for the large DM of FRBs. Thus, the sources of FRBs are probably extra-galactic.

2. SOURCES ARE PROBABLY EXTRA-GALACTIC

In this section, we discuss two galactic candidates to produce DM for FRBs. FRBs single wave period. We summarize our results in §7 and briefly comment on possible emission mechanisms for FRBs.

2.1. Free-free absorption in stellar coronas

Loeb et al. (2014) proposed that FRBs originate from flares on main-sequence stars and that the DM arises from propagation through the stellar corona. This proposal has the attractive feature of greatly reducing the source luminosity with respect to that required for an unspecified extragalactic source. Nevertheless, free-free absorption limits a stellar corona’s DM to be well below that of FRB’s.

In the Rayleigh-Jeans limit, $h\nu \ll k_{B}T$, the free-free absorption coefficient including stimulated emission reads (Spitzer 1978)

$$\alpha = \frac{4}{3} \left(\frac{2\pi}{3}\right)^{1/2} \frac{Z^{2}e^{6}n_{e}n_{i}\tilde{g}_{ff}}{c m_{e}^{3/2}(k_{B}T)^{3/2}\nu^{2}},$$

$$\tilde{g}_{ff} = \frac{\sqrt{3}}{\pi} \ln \left[\frac{(2k_{B}T)^{3/2}}{\pi e^{2}m_{e}^{1/2}\nu} - \frac{5\gamma}{2}\right],$$

where $\tilde{g}_{ff}$ is the Gaunt factor and $\gamma = 0.577$ is Euler’s constant. Other symbols are standard: $m_{e}$ is the electron mass, $e$ is the electron charge, and $n_{e}$ and $n_{i}$ are the...
number densities of electrons and ions. For cosmic abundances and in the temperature range of interest here, it suffices to evaluate α for a pure hydrogen plasma, i.e., \( Z = 1 \) and \( n_e = n_i \).

For a homogenous medium, the optical depth for free-free absorption is \( \tau \sim \alpha s \propto n_e^2 \), where \( s \) is the path length along the line of sight through the medium. Since \( \text{DM} = n_e s \), we express \( n_e \) in terms of \( s \) and \( \tau \). Then \( \tau \lesssim 1 \) sets an upper limit on DM,

\[
\text{DM} \sim \frac{3^{3/4}(n_e k_B T)^{3/4}(c s)^{1/2} \nu}{0^{5/4} \pi^{1/4} e^{3} q_{B}^{1/2}} \approx 3^{3/4}(n_e k_B T)^{3/4}(c s)^{1/2} \nu \left( \frac{R}{R_\odot} \right)^{2/3} \left( \frac{M_{\odot}}{M_*} \right)^{3/2} \left( \frac{R}{R_*} \right)^{-1/4} \text{ pc cm}^{-3}
\]

(3)

For \( k_B T \lesssim GMm_p/R \) the base of the corona would be in quasi-hydrostatic equilibrium. Since density drops rapidly with height in an isothermal atmosphere, we replace \( s \) in Eq. (3) by the scale height \( 2k_B T/(m_p g) \), and \( k_B T \) by \( GMm_p/R \) to obtain

\[
\text{DM} \sim \frac{3^{3/4}(Cn_e^2/\nu)^{3/4}(GMm_p)^{3/4}}{2^{3/4} \pi^{1/4} e^{3} q_{B}^{1/2}} \approx 50 \left( \frac{M_{\odot}}{M_*} \right)^{3/4} \left( \frac{R}{R_*} \right)^{-1/4} \text{ pc cm}^{-3}
\]

(4)

which is much smaller than the DMs of FRBs.

A hotter corona could provide a larger DM. If free to expand, it would essentially be a stellar wind even close to the photosphere. For simplicity, the wind is taken to have constant velocity and constant temperature. These approximations are not entirely consistent because a supersonic isothermal wind would slowly accelerate as its density declined. This inconsistency leads us to overestimate dispersion measure relative to free-free absorption because the former and latter are proportional to density and density squared. At constant radial velocity, \( n_e(r) \sim n_e(R)(R/r)^2 \).

\[
\text{DM} \sim \int_{s}^{R} n_e dr \sim n_e(R)R \quad \text{.}
\]

(6)

From Eq. (2), \( \alpha = Cn_e^2/(k_B T)^{3/2} \),

\[
\tau \sim \int_{R}^{s} \alpha dr \sim \int_{R}^{s} \frac{Cn_e^2}{(k_B T)^{3/2}} dr \sim \frac{Cn_e^2(R)}{3(k_B T)^{3/2}} \quad \text{.}
\]

(7)

The power carried by the wind would be

\[
P_w \sim 4\pi m_p n_e(R)R^2v_\text{th}^3 \sim \frac{2^{6}\pi^{3/2} e^{6} q_{B}^{1/2} \text{DM}^3}{3^{3/2} c m_e^{3/2} m_{H}^{1/2} \nu^2 \tau}
\]

(8)

\[
\gtrsim 40L_\odot \left( \frac{\text{DM}}{10^3 \text{ pc cm}^{-3}} \right)^3 \left( \frac{\nu}{1 \text{ GHz}} \right)^{-2} \quad \text{,}
\]

(9)

where we have expressed \( n_e \) and \( s \) in terms of DM and \( \tau \). The \( \gtrsim \) on the second line follows from setting \( \tau \gtrsim 1 \). Clearly a coronal wind cannot carry more energy than the luminosity of its star could provide. Thus even the lowest DM measured for the FRB’s reported by Thornton et al. (2013), \( \text{DM} \sim 553 \text{ pc cm}^{-3} \), could not arise from propagation through a coronal wind from the flare stars discussed by Loeb et al. (2014).

\footnote{In calculating the power needed to drive the wind, we neglect the heat that must be supplied in order to overcome the cooling effect of adiabatic expansion.}

A hotter corona might be confined by a strong magnetic field provided the magnetic stress is comparable to the gas pressure. Under this condition, the ratio of the electron cyclotron frequency to the plasma frequency would be

\[
\frac{\omega_{ce}}{\omega_p} \sim \left( \frac{k_B T}{m_e e^2} \right)^{1/2} \quad \text{.}
\]

(10)

Then the dispersion relation for radio waves would depend on \( \omega/\omega_{ce} \) in addition to \( \omega/\omega_p \) which might cause the frequency dependence of the pulse arrival times to deviate by more than the limits set by observations of FRBs.

Numerical results given above are scaled with respect to parameters pertaining to the sun. Typical flare stars are lower main sequence dwarfs for which \( R \propto M_0^{0.9} \) and \( L \propto M_0^{-4.4} \) (Demircan & Kahraman 1991). Application of these relations only strengthens our conclusion that the DMs of FRBs cannot be attributed to passage of radio waves through coronas.

Before moving on, we offer a few additional comments about radio emission from flare stars. This topic has been discussed for more than half a century, starting with the paper ”Radio Emission from Flare Stars” by Lovell (1963). To our knowledge, no bursts sharing the common properties of FRBs have been reported. Moreover, the most frequently studied radio flare stars are close by. For example, AD Leonis and YZ Canis Minoris are at distance of \( \sim 5 \text{ pc} \) and \( \sim 6 \text{ pc} \), respectively. These two stars figure prominently, and in most cases exclusively, in each of the papers on radio flares referenced in Loeb et al. (2014) and their strongest bursts barely reach the level of 1 Jy that is typical of FRBs. Dynamic spectra of radio bursts from AD Leonis observed with wide bandwidth and at high time resolution at Arecibo (Osten & Bastian 2006, 2008) do not resemble those of FRBs. Pulses suffering dispersion induced time delays should only show negative frequency drifts. But the histogram of these bursts (Figure 4a in Osten & Bastian (2006)) exhibits both positive and negative frequency drifts and is symmetric about zero drift with half width at half maximum of \( \sim 3 \times 10^{-4} \text{ s/MHz} \). Note that a DM \( \sim 20 \text{ pc cm}^{-3} \) produces a negative frequency drift rate of similar magnitude.

2.2. HII region

An HII region is another candidate to account for the DM of a galactic FRB. A lower limit on \( s \) is deducible from Eq. (3). With \( T \sim 10^{4} \text{ K} \), \( s \gtrsim 0.2 \text{ pc} \). The angular size of such an HII region at 500 pc is \( \theta_{\text{HII}} \sim 80 \text{ arcsec} \). At 1.4 GHz, the 64 m, Parkes telescope’s beam size is \( \theta_b \sim \lambda/\Delta \sim 20 \text{ cm}/64 \text{ m} \sim 650 \text{ arcsec} \). Thus the antenna temperature of such an HII region would be \( T_A = T_{\text{HII}} \times (\theta_{\text{HII}}/\Delta) \sim 150 \text{ K} \). The sensitivity of Parkes at 1.4 GHz for a 270 s integration time is 0.6 mK for 10σ detection of FRBs (cf. Parkes user guide). Thornton et al. would have recognized an HII region with these properties in the data they search for FRBs.

The bottom line from this section is that neither a stellar corona nor an HII region is a plausible candidate for the high DMs of FRBs. Thus FRBs are likely to be extragalactic.

3. TEMPORAL SCATTERING
We follow conventions developed in the investigation of angular and temporal scattering in the interstellar medium (Rickett 1990) and adopt the Kolmogorov spectrum, $\delta n_e/n_e \sim (\ell/L)^{1/3}$, for electron density fluctuations on scale $\ell$ where $\ell_{\text{min}} \leq \ell < L$. Moreover, we assume that this spectrum is associated with a turbulent cascade in which sonic velocity fluctuations are present at outer scale $L$.\(^2\) Finally, the scattering is described by projecting the phase differences that accumulate along the line of sight between source and observer onto a thin screen located midway between them. For a source at distance $d$, we obtain

$$\Delta \phi \sim \frac{n_e e^2 d^{1/2} \ell^{5/6} \lambda}{\pi m_e c^2 L^{1/3}}. \quad (11)$$

We are concerned with strong scattering which requires $\Delta \phi > 1$. Then the scattering angle

$$\Delta \theta \sim \frac{\lambda}{\ell} \Delta \phi \propto \ell^{-1/6} \quad (12)$$

dominated by the smallest scale for which $\Delta \phi > 1$. For sufficiently small $\ell_{\text{min}}$, this is the diffraction scale at which $\Delta \phi \sim 1$;

$$\ell_{\text{diff}} \sim \left(\frac{\pi m_e e^2}{e^2 n_e \lambda}\right)^{6/5} \frac{L^{2/5}}{d^{3/5}}. \quad (13)$$

Otherwise it is $\ell_{\text{min}}$. The temporal delay,

$$\Delta t_{\text{sc}} \sim \frac{d}{c} (\Delta \theta)^2 \quad (14)$$

is expressed as

$$\Delta t_{\text{sc}} \sim \begin{cases} \frac{d}{c} \left(\frac{\lambda}{\ell_{\text{diff}}}\right)^2 \propto \lambda^{4.4}, & \ell_{\text{min}} \leq \ell_{\text{diff}}; \\ \frac{d}{c} \left(\frac{\Delta \phi}{\ell_{\text{min}}}\right)^2 \propto \lambda^4, & \ell_{\text{min}} > \ell_{\text{diff}}. \end{cases} \quad (15)$$

FRB 110220 exhibits a well-resolved exponential tail with $\Delta t_{\text{sc}} \sim 5.6 \pm 0.1$ ms that has been attributed to plasma scattering (Thornton et al. 2013). Unfortunately, the data is not quite good enough to distinguish between the two cases given in Eq. (15) (Thornton et al. 2013). But both restrict the outer scale to be less than

$$L_{\text{max}} \sim \left(\frac{e^2 n_e}{\pi m_e c^2}\right)^3 \frac{\lambda^{11/2}}{(e \Delta t_{\text{sc}})^{5/4}} \sim 10^{13} \left(\frac{d}{\text{Gpc}}\right)^{11/4} \times \left(\frac{n_e}{10^{-7} \text{ cm}^{-3}}\right)^3 \left(\frac{\Delta t_{\text{sc}}}{\text{ms}}\right)^{-5/4} \text{ cm}. \quad (16)$$

$L_{\text{max}}$ is an impossibly small outer scale for extragalactic turbulence.\(^3\) Sonic velocity perturbations dissipate their bulk kinetic energy into heat on the timescale over which sound waves cross the outer scale. This would imply a doubling of the IGM temperature over several months since the cooling rate is comparable to the Hubble time.

Based on the argument given above, it seems unlikely that propagation through the diffuse IGM could make a measurable contribution to the scattering tail of a FRB. Indeed, an outer scale of order $10^{24}$ cm is required to reduce the turbulent heating rate to a level compatible with the cooling rate. With this value, $\Delta t_{\text{sc}} \lesssim 10^{-12}$ s for $d \sim \text{Gpc}$. Previously, Macquart & Koay (2013) expressed doubt that propagation through the diffuse IGM could produce discernible scattering tails for FRBs. However, they failed to recognize the incompatibility of a small $L_{\text{max}}$ with regulation of the IGM’s temperature.

4. CONTRIBUTION TO RADIO SKY

Thornton et al. (2013) estimate a FRB event rate of $\sim 10^4 \text{sky}^{-1} \text{day}^{-1} \sim 0.1 \text{s}^{-1}$. Given characteristic flux densities of a Jansky and durations of a few milliseconds, FRBs add about $10^{-9} \text{K}$ to the radio background at 1.4 GHz.\(^4\) This value is dwarfed by contributions of 2.7 K from the CMB and even by minor additions from the galactic halo, the galactic plane and extragalactic radio sources. According to Subrahmanyan & Cowsik (2013), these account for 0.79 K, 0.3 K and 0.14 K respectively at 1.4 GHz.

5. BRIGHTNESS TEMPERATURE

FRBs are not angularly resolved, and thus their brightness temperatures ($T_B$) are unknown. However, the duration of a pulse, $\Delta t$, constrains the linear size of the source and thus its angular size at a fixed distance. Relativistic beaming is a complication. Radiation emitted from a spherical shell expanding with Lorentz factor $\Gamma$ is beamed into a solid angle $\Delta \Omega \sim \Gamma^{-2}$. Arrival times of photons emitted simultaneously spread by $R/(c \Gamma^2)$ permitting a source size as large

$$R \lesssim c \Delta \Gamma^2. \quad (17)$$

Consequently, the brightness temperature in the observer’s frame is

$$T_B \sim \frac{S_v d^2}{k_B \Gamma^2 \nu^2 \Delta \Omega^2} \sim \frac{10^{36} \text{K}}{\Gamma^2} \left(\frac{S_v}{\text{Jy}}\right) \left(\frac{d}{\text{Gpc}}\right)^2 \left(\frac{\nu}{\text{GHz}}\right)^{-2} \left(\frac{\Delta t}{\text{ms}}\right)^{-2}.$$

Even at $d \sim \text{kpc}$, $T_B \sim 10^{23}/\Gamma^2 \text{K}$. Incoherent broadband radio emission from strong astronomical sources is usually synchrotron radiation. Upper limits on $T_B$ are typically no larger than a few times $10^{13}$ K (Kovalev et al. 2005). This is consistent with an upper limit on $T_B \sim 10^{12} \text{K}$ in the source frame set by the Compton catastrophe (Frank et al. 1992) with somewhat higher values due to beaming in AGN jets.

Terrestrial communications at radio wavelengths invariably involve coherent sources. Could FRBs be signals beamed at us from advanced civilizations? Might negatively chirped ms pulses be transmitted to facilitate their detection? Advanced civilizations would know the power of de-dispersing radio signals to investigate pulsars. They would also be aware of planets in their neighborhoods and have identified those with atmospheres suitable for,

\(^2\) Conclusions reached in this section depend on the assumption that the electron density fluctuations arise from a turbulent cascade.

\(^3\) In a clumpy IGM with volume filling factor $f$, $L_{\text{max}}$ would be larger by $f^{-3/2}$.

\(^4\) Unless FRBs are extragalactic, this is merely their contribution to the radio background near our position in the Galaxy.
or perhaps even modified by, biological life. After all, this information will be available to us before the end of this century.

How might advanced civilizations configure antennas to transmit narrow beams? Arrays of small telescopes are preferable to large filled apertures and also limit capital costs. With baseline, $b$, and transmitted power, $P_T$, the flux density of a broad-band signal received at distance $d$ would be

$$S_\nu \sim \left( \frac{b}{cd} \right)^2 \nu P_T. \quad (19)$$

Recasting the above equation with $S_\nu$ scaled by Jy as appropriate for a FRB yields

$$P_T \sim 10^9 \left( \frac{b}{10^3 \text{ km}} \right) \left( \frac{d}{1 \text{ kpc}} \right)^2 \left( \frac{\nu}{\text{GHz}} \right)^{-1} \left( \frac{S_\nu}{\text{Jy}} \right) \text{ watt}, \quad (20)$$

a modest power requirement even by current terrestrial standards.\(^5\)

Accounting for a burst arrival rate at Earth $\sim 0.1 \text{ s}^{-1}$ is the most challenging part of this exercise. With only a handful of detected FRBs, the fraction of planets hosting an advanced civilization might be quite modest. But then, the Earth must have been recognized as a particular object of interest to target. If this hypothesis has merit, the positions from which bursts arrive should eventually repeat. That would provide a lower limit to the number of our more advanced neighbors.

6. STRONG ELECTRIC FIELDS

The flux of energy carried by an electromagnetic wave is $F = eE^2/2\pi$. Thus the electric field at the observer associated with a broad-band pulse of flux density $S_\nu$ is

$$E_\nu \sim \left( \frac{4\pi S_\nu \nu}{c} \right)^{1/2} \sim 10^{-12} \left( \frac{S_\nu}{\text{Jy}} \right)^{1/2} \left( \frac{\nu}{\text{GHz}} \right)^{1/2} \text{ esu}. \quad (21)$$

At separation $r$ from a source at distance $d$, the electric field is larger, $E = (d/r)E_\nu$. For $r$ smaller than

$$r_{\text{rel}} \sim \frac{eE_\nu d}{2\pi m_e c \nu v} \sim 10^{13} \left( \frac{S_\nu}{\text{Jy}} \right)^{1/2} \left( \frac{\nu}{\text{GHz}} \right)^{-1/2} \left( \frac{d}{\text{Gpc}} \right) \text{ cm}, \quad (22)$$

the electric field is strong in the sense that it could accelerate an electron from rest up to relativistic energy on timescales comparable to $(2\pi \nu)^{-1}$. A free electron would maintain a position of nearly constant phase, in essence surfing on the wave (Gunn & Ostriker 1969). For $E$ given by Eq. (21) and $r \ll r_{\text{rel}}$, the electron would reach a Lorentz factor

$$\gamma \sim \left( \frac{r_{\text{rel}}}{r} \right)^{2/3}. \quad (23)$$

Acceleration of electrons in a thermal plasma by a strong broadband pulse would be more complicated. It is plausible that the electrons would drag the positive ions along with them to create an outgoing shock wave. Whether

\(^5\) Scattering by plasma density fluctuations in the interstellar medium of typical paths would not increase the angular width of these beams.

this might lead to the emission of coherent GHz radio waves is an open question that is best left for a separate investigation.

It is informative to compare the strength of the electric field near a cosmological FRB with that of giant pulses from the Crab pulsar. Sallmen et al. (1999) studied giant pulses in different frequency bands. At 0.6 GHz, $S_\nu \sim 7000$ Jy whereas at 1.4 GHz, $S_\nu \sim 3000$ Jy. Since the Crab is estimated to be at $d \sim 2.2$ kpc (Manchester et al. 2005), the corresponding values of $r_{\text{rel}}$ are a few times $10^7$ cm in both bands. These values of $r_{\text{rel}}$ are about 10 times larger than the radius of the Crab’s light cylinder (Manchester et al. 2005), but much smaller than $r_{\text{rel}}$ for FRB 110220.

7. DISCUSSION & CONCLUSIONS

We discuss several properties of FRBs. We conclude that their high DMs cannot be attributed to a stellar corona or a galactic HII region. Thus, if astronomical, they are extra-galactic sources. We also argue that the propagation through the IGM is unlikely to lead to measurable scatter broadening of GHz pulses. Thus if scatter broadening is confirmed, it would suggest that the sources are located in dense regions of external galaxies and raise the possibility that a substantial fraction of their DMs are produced there.

Few sources at Gpc distances are plausible candidates for producing ms pulses with Jy flux densities. Neutron stars and stellar mass black holes have dynamical timescales of the right order and their gravitational binding energies are more than sufficient. How the release of binding energy might power a FRB is not clear. Gravitational waves can be released on ms timescales, but their coupling to GHz radio waves is likely to be much slower. Neutrinos carry away most of the binding energy, but only over several seconds (Bionta et al. 1987). The sudden release of magnetic energy, perhaps in a giant magnetar flare (Lyubarsky 2014) or during the collapse of a magnetar into a BH (e.g., Falcke & Rezzolla (2013)) seems a better bet. An advantage of these proposals is that the initial energy is released in electromagnetic form. However, its rapid up-conversion to GHz frequencies poses a hurdle. Whether it can be overcome by the acceleration of dense plasma in strong EM fields is questionable. Moreover, it is doubtful whether these events occur with sufficient frequency to account for FRBs.

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