Text S1

Appendix A: Validity of the Bimaterial Approximation

In Section 2.2, we approximate the bimaterial crack as having an opening given by ξ times the opening for a crack in a homogeneous sample of the more compliant material. Here, we verify the validity of this approximation for an ice-rock interface. Following the analysis of *Rice and Sih* [1965] (see also *England* [1965] and *Erdogan* [1965]), we consider a crack of length $2L$ along the bimaterial interface within an infinite medium with upper medium characterized by shear modulus G_1 and Poisson's ratio ν_1 and lower medium characterized by G_2 and ν_2. For our ice-rock case, we take ice elastic parameters as in Section 3 ($E_1 = 6.2$ GPa, $\nu_1 = 0.3$ so that $G_1 = 2.4$ GPa) and rock elastic parameters from near-surface granite seismic velocities of *Lay and Wallace* [1995] (and $\rho_2 = 2750$ kg/m^3) which give $G_2 = 23$ GPa $\approx 9.6G_1$ and $\nu_2 = 0.3 \approx \nu_1$. With these choices, the bimaterial 'mismatch' constant

$$\epsilon \equiv \frac{1}{2\pi} \log \left[\left(\frac{\eta_1}{G_1} + \frac{1}{G_2} \right) / \left(\frac{\eta_2}{G_2} + \frac{1}{G_1} \right) \right], \tag{A1}$$

with $\eta \equiv 3 - 4\nu$, has a value of $\epsilon = 0.075124$. Given an arbitrary crack pressure loading $P(x)$ along $-L < x < L$, the complex displacements $u_k + iv_k$ (u_k in the horizontal direction and v_k in the vertical direction, throughout this appendix) on either side of the crack ($k = 1$ or 2) are given by Equations (14) and (15) of *Rice and Sih* [1965] (evaluated along $z = \bar{z}$ where $z = z_1 + iz_2$ is a complex variable, with z_1 horizontal and z_2 vertical coordinates) to be

$$2G_1(u_1 + iv_1) = \eta_1 \int^{z} g(s)F(s)ds - e^{2\pi\epsilon} \int^{\bar{z}} g(\bar{s})F(\bar{s})d\bar{s} \tag{A2}$$

on the upper side and

$$2G_2(u_2 + iv_2) = e^{2\pi\epsilon}\eta_2 \int^{z} g(s)F(s)ds - \int^{\bar{z}} g(\bar{s})F(\bar{s})d\bar{s} \tag{A3}$$

on the lower side. As also given in *Rice and Sih* [1965],

$$F(z) = (z^2 - L^2)^{-1/2} \left(\frac{z+L}{z-L} \right)^{i\epsilon}, \tag{A4}$$

with branch cut along the crack such that $zF(z) \to 1$ as $|z| \to \infty$, and

$$g(s) = \int_{-L}^{L} g(s,b)\, db, \tag{A5}$$

where

$$g(s,b) = \frac{P(b)}{2\pi} \frac{e^{-\pi\epsilon}}{s-b} (L^2 - b^2)^{1/2} \left(\frac{L-b}{L+b} \right)^{i\epsilon}. \tag{A6}$$

Along the crack face $-L < s < L$, $F(s)$ simplifies to

$$F(s_\pm) = -1 \cdot \pm ie^{\pm\pi\epsilon}(L^2 - s^2)^{-1/2} \cdot [\cos(\epsilon \log \frac{L+s}{L-s}) + i \sin(\epsilon \log \frac{L+s}{L-s})], \tag{A7}$$

where $+$ is used for s above the crack, $-$ is used for s below the crack.

Substituting Equations (A5) and (A7) into Equations (A2) and (A3) gives expressions for the complex displacements along the crack face. Expanding each of these expressions as a power series in the parameter ϵ and approximating the expressions to first order in ϵ (ignoring all higher-order terms, which is appropriate except extremely close to the ends, because of the logarithmic divergence), we find that we can express the complex displacements along the crack face as

$$u_1 + iv_1 = \frac{1}{E_1'}(\epsilon I_1 + iI_2) + O(\epsilon^2) \tag{A8}$$

and

$$u_2 + iv_2 = -\frac{1}{E_2'}(\epsilon I_1 + iI_2) + O(\epsilon^2). \tag{A9}$$

I_1 and I_2 are (complicated) expressions that involve only real integrals, and the full crack opening displacement in a homogeneous medium characterized by G_1 and ν_1 is given by

$$2(u_1 + iv_1) = 0 + \frac{1}{E_1'} iI_2. \tag{A10}$$

We then observe that to order ϵ, the displacement v_1 is unchanged from its value in the homogeneous case and that the displacement on the lower side, v_2 is given by

$$v_2 \approx -\frac{E_1'}{E_2'} v_1 \approx -\frac{v_1}{9.6}. \tag{A11}$$

Thus, the full opening in the bimaterial case $v_1 - v_2$ is approximately ξ of the full opening in the homogeneous case where ξ is given by

$$\xi \approx \frac{1 + E_1'/E_2'}{2} \approx 0.55. \tag{A12}$$

We therefore use the approximation $h = 0.55w$.

Appendix B: Stresses in the Bulk

Here, we describe the stresses in the elastic medium associated with the crack-tip solution of *Desroches et al.* [1994] that are used to obtain Equations (12) and (13). Following *Desroches et al.* [1994], we write the *Muskhelishvili* [1953] potential as

$$\phi(z) = \frac{A'}{2q} z^q, \tag{B1}$$

where $z = z_1 + iz_2$ is again a complex variable, and q is a constant. We follow *Desroches et al.* [1994] and take the other *Muskhelishvili* [1953] potential as $\psi(z) = \phi(z) - z\phi'(z)$ in order to maintain zero shear along the crack axis $y = 0$. We can then calculate the stresses in polar coordinates to be given by

$$\frac{\sigma_{\theta\theta} + \sigma_{rr}}{2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} = A' r^{q-1} \cos[(q-1)\theta] \tag{B2}$$

0148-0227/10/$9.00

and

$$\frac{\sigma_{\theta\theta}-\sigma_{rr}}{2}+i\sigma_{r\theta}=e^{2i\theta}\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}+i\sigma_{yx}\right)$$
$$=(1-q)A'r^{q-1}\sin(\theta)[-\sin(q\theta)+i\cos(q\theta)]. \quad \text{(B3)}$$

Solving for the stresses gives

$$\sigma_{rr}(r,\theta)=A'r^{q-1}\cdot\left[\frac{3-q}{2}\cos[(1-q)\theta]-\frac{1-q}{2}\cos[(1+q)\theta]\right], \quad \text{(B4)}$$

$$\sigma_{r\theta}(r,\theta)=A'r^{q-1}\cdot\left[\frac{1-q}{2}\sin[(1-q)\theta]-\frac{1-q}{2}\sin[(1+q)\theta]\right], \quad \text{(B5)}$$

and

$$\sigma_{\theta\theta}(r,\theta)=A'r^{q-1}\cdot\left[\frac{1+q}{2}\cos[(1-q)\theta]+\frac{1-q}{2}\cos[(1+q)\theta]\right]. \quad \text{(B6)}$$

These expressions give the stress components of the *Desroches et al.* [1994] solution except for a possible added uniform pressure, $\sigma_{\theta\theta}=-P$ and $\sigma_{rr}=-P$, and an additional added crack-parallel stress $\sigma_{xx}=constant$ (which will not enter our analysis). Equation (13) is then obtained by demanding that $p(x)$ and the crack opening gap satisfy the fluid equations (Equations (7), (9) and (10)) in the case of steady state growth, leading to $q=2/(2+m)=6/7$ and evaluating Equation (B6) along the crack opening to yield

$$\Delta p(x)-P=-\sigma_{\theta\theta}(R,\pi)$$
$$=-A'R^{-1/7}\cos\left(\frac{\pi}{7}\right)=-AR^{-1/7}, \quad \text{(B7)}$$

where $A=A'\cos(\pi/7)$ corresponds to the quantity introduced in Equation (12).

Appendix C: Displacement Calculations

The vertical surface displacements (uplift) due to both cracks are easily calculated using the reciprocal theorem and the Boussinesq-Flamant line-source solution (see e.g. *Timoshenko and Goodier* [1987]). The result, e.g. as in the Appendix of *Walsh and Rice* [1979], is that the vertical surface uplift h_s in a homogeneous half-space due to a vertical opening displacement $w^*=w^*(x)$ of a horizontal surface is

$$h_s(x_0,y_0)=\int_{surf}\sigma^*_{yy}(x-x_0,y-y_0)w^*(x)dx, \quad \text{(C1)}$$

where σ^*_{yy} is given by

$$\sigma^*_{yy}=\frac{2}{\pi}\cdot\frac{(y-y_0)^3}{[(x-x_0)^2+(y-y_0)^2]^2}, \quad \text{(C2)}$$

and (x_0,y_0) is the uplift location. Applying this to the basal crack, and utilizing the bimaterial approximation for the opening displacement of the crack, $w^*=h(x)\approx w(x)/2$, but ignoring bimaterial effects on Equation (C2), then

$$h_s(x_0)\approx\int_{-L}^{L}\frac{1}{\pi}\cdot\frac{H^3w(x)}{[(x-x_0)^2+H^2]^2}dx, \quad \text{(C3)}$$

where variables are as before. Putting this into non-dimensional form and substituting Equation (42) for $\xi w(x)$, we obtain

$$h_s(x_0)\approx\frac{H^3\bar{h}_U}{\xi\pi^2L^3}\int_{-1}^{1}\frac{\hat{w}(\hat{x})\,d\hat{x}}{[(\hat{x}-\hat{x}_0)^2+\hat{H}^2]^2}, \quad \text{(C4)}$$

where $\hat{H}\equiv H/L(t)$, $\hat{x}_0=x_0/L(t)$, $\hat{w}(\hat{x})$ is the scaled self-similar opening given in Equation (27), and other variables are as before. Thus, given a surface location x_0 (relative to the crack inlet at $x=0$ and in the plane of crack growth) and crack length $L(t)$, Equation (50) gives h_s in terms of our self-similar solution.

We can similarly account for the vertical displacement due to the horizontal opening of the vertical crack, and as shown below find that this contribution is negligible. Again as in *Walsh and Rice* [1979], the contribution due to the vertical crack's horizontal displacement u^* is

$$h^V_s=\int_{surf}\sigma^*_{xx}u^*dy, \quad \text{(C5)}$$

where σ^*_{xx} is given for a homogeneous halfspace by

$$\sigma^*_{xx}=\frac{2}{\pi}\cdot\frac{(x-x_0)^2(y-y_0)}{[(x-x_0)^2+(y-y_0)^2]^2}. \quad \text{(C6)}$$

Applying this to the vertical crack then

$$h^V_s(x_0)\approx\int_0^H\frac{2x_0^2yu^*(y)\,dy}{\pi(x_0^2+y^2)^2}. \quad \text{(C7)}$$

Noting that for the observations of *Das et al.* [2008], $x_0/H\approx 1.7$ then this contribution to h_s is bounded by

$$h^V_s(x_0)\le\int_0^1\frac{2\cdot 1.7^2\,\hat{y}\,d\hat{y}}{\pi(1.7^2+\hat{y}^2)^2}\cdot\max[u^*]=0.08\max[u^*]. \quad \text{(C8)}$$

Since $\max[u^*]$ is expected to be of similar (or smaller) magnitude to w^*, the contribution h^V_s is thus expected to be an order of magnitude less than that due to the basal crack opening, and we therefore neglect this contribution.

For horizontal surface displacements, we similarly expect an order of magnitude smaller contribution from vertical opening of the basal crack compared to horizontal opening of the (vertical) connecting crack, and hence ignore this former contribution. The horizontal displacement at a distance x_0 perpendicular to the center of the plane stress center crack (see Figure 8) can be obtained by integrating the results of *Tada et al.* [2000] as follows. *Tada et al.* [2000] provides the displacement at x_0 due to a pair of point forces of amplitude P_1 to be

$$u_{P1}(x_0)=\frac{4P_1}{\pi E}\left[\tanh^{-1}\sqrt{\frac{a^2-b^2}{a^2+x_0^2}}+\frac{1+\nu}{2}\cdot\frac{x_0^2}{b^2+x_0^2}\sqrt{\frac{a^2-b^2}{a^2+x_0^2}}\right], \quad \text{(C9)}$$

where b is the distance from the center of the crack of the pair of forces. Integrating this expression over the crack face ($0\le b\le a$) gives the corresponding expression, due to a constant pressure Δp_{avg} along the crack, of

$$u_s(x_0)=\frac{2\Delta p_{avg}a}{E}\left[\sqrt{1+(x_0/a)^2}-(x_0/a)+\frac{1+\nu}{2}(x_0/a)\left(1-\frac{x_0/a}{\sqrt{1+(x_0/a)^2}}\right)\right], \quad \text{(C10)}$$

which we take as an approximation to the horizontal surface displacement.

Appendix D: Estimates of Errors and Improvements on Approximations

Here, we first find that the approximations $L \ll H$ and $\Delta p_{loss} \ll \Delta p_{static}$ are of concern. Following estimates of how well these approximations are satisfied, we discuss possible approaches to addressing the two problems.

First, we can make an estimate of how large L becomes by equating the volume of water taken up by the basal crack plus vertical crack ($V_b + V_c$) with the initial volume of water in the surface lake (V_0). The initial lake volume was observed to be $V_0 = 4.4 \cdot 10^7$ m^3 [*Das et al.*, 2008], and we calculate the sum of the crack volumes to be

$$V_b(L) + V_c(a) = \pi \frac{\Delta p_{in}}{E} L^3 \left(\frac{16\xi C_1(1-\nu^2)}{3\pi} + \frac{a^2 H_w}{L^3} \right) \quad \text{(D1)}$$

Choosing $a = L$ as a plausible upper bound on V_c (as discussed in the next paragraph, which results in a lower bound on L) predicts that $L \gtrsim 5.25$ km is reached and thus suggests that the approximation $L \ll H$ should be revisited.

Second, we estimate the pressure loss from turbulent flow en route to the bed by applying the turbulent Manning-Strickler scaling of Equation (6) with each term estimated for flow through the vertical crack. As in our earlier plane stress calculation for this vertical crack, we assume a depth-averaged value of excess pressure $\Delta p_{in}/2$ opening the crack, giving a cross-sectionally averaged opening of $2u_{avg} \equiv \pi u_s(0)/2 \approx \pi \Delta p_{in} a/2E$. We expect that a lies in the range $0.1 \lesssim a/L < 1$ since significant opening will only occur over the region with minimal basal shear stress to counteract the excess pressure (i.e. $a < L$) but for $a \ll L$ the excess pressure should encourage a to grow (i.e. $a \gtrsim 0.1L$). Taking $L \approx 3$ km and $a/L \approx 0.8$ as plausibly representative, then $2u_{avg} \approx 0.48$ m. The average fluid velocity through this vertical crack U_{vert} can be estimated by equating the volumetric flow rate in the vertical crack $\pi a u_s(0) U_{vert} = 4 a u_{avg} U_{vert}$ to the volumetric flow rate into the basal crack $dV_b/dt \equiv dV_b/dL \cdot U_{tip}$ (where V_b is given by Equation (54)). Using the procedures of Section 3.1, we estimate dV_b/dt using $\bar{h}_U^{3D}$, which gives $U_{tip} \approx 1.4$ m/s and therefore $dV_b/dt \approx 8.5 \cdot 10^3$ m^3/s. Using these values, then $U_{vert} \approx 3.7$ m/s and the loss of pressure in excess of hydrostatic through the connecting conduit would be

$$\Delta p_{loss} = \frac{0.0357 \rho U_{vert}^2 k^{1/3} H}{(2u_{avg})^{4/3}} \approx 0.58 \text{ MPa}, \quad \text{(D2)}$$

which is a large fraction (67%) of the maximum excess pressure of 0.87 MPa, and is a higher fraction when L is smaller. Any sinuosity in the path from the surface to the base, or a smaller value of a/L, would also increase this pressure head loss. Thus, both the $L \ll H$ approximation and the approximation of no loss of excess pressure at the basal inlet are of concern.

For the uniform pressure loading Δp_{in} over a penny-shaped plate of radius L clamped on the edges, *Timoshenko and Woinowsky-Krieger* [1959] gives

$$h_U^P(\mathcal{R}) = \frac{3\Delta p_{in} L^4}{16 E' H^3} (1 - \hat{\mathcal{R}}^2)^2, \quad \text{(D3)}$$

where, as before, $\hat{\mathcal{R}} = \mathcal{R}/L$. The average opening is then

$$\bar{h}_U^P = \frac{1}{16} \cdot \frac{\Delta p_{in} L}{E'} \cdot \frac{L^3}{H^3}. \quad \text{(D4)}$$

Comparing Equation (D4) for $\bar{h}_U^P$, which applies when $L \gg H$, with Equation (53) for $\bar{h}_U^{3D}$, which applies when $L \ll H$, we suggest a summed version of $\bar{h}_U$ (the average opening under uniform pressure) defined by

$$\bar{h}_U^S \equiv \bar{h}_U^{3D} + \bar{h}_U^P = \frac{16\xi \Delta p_{in} L}{3\pi E'} \left[1 + \frac{3\pi}{256\xi} \cdot \frac{L^3}{H^3} \right]. \quad \text{(D5)}$$

To account for pressure loss in the connecting conduit, we let $\Delta p_{in} \equiv \chi \Delta p_{static}$, where $0 \le \chi \le 1$. We then solve for the unknowns χ and U_{vert} (average fluid velocity in the vertical crack) by equating the excess pressures at the juncture between the vertical crack and the basal crack inlet, and similarly equating the volumetric flow rates there. We use the same turbulent scaling as was used in Equation (D2), noting again that this depth-averaged, lumped-parameter treatment of flow in the vertical crack is a crude approximation to the true situation. With this caveat, the first equality is satisfied by

$$(1-\chi)\Delta p_{static} = \frac{0.0357 \rho U_{vert}^2 k^{1/3} H}{(\pi a \chi \Delta p_{static}/2E)^{4/3}}, \quad \text{(D6)}$$

where $\chi \Delta p_{static}$ has replaced Δp_{in}. The second (flow rate) equality is satisfied (as also discussed prior to Equation (D2)) by setting

$$\begin{aligned} 4 a u_{avg} U_{vert} &= \frac{\pi a^2 \Delta p_{static} U_{vert} \cdot \chi}{E} \\ &= \frac{dV_b}{dt} = \frac{dV_b}{dL} U_{tip}, \end{aligned} \quad \text{(D7)}$$

where U_{tip} is given by Equation (46) and dV_b/dL is calculated as

$$\frac{dV_b}{dL} = C_1 \pi \frac{d(L^2 \bar{h}_U)}{dL}. \quad \text{(D8)}$$

As discussed in the main text, all 3 models (with $\bar{h}_U$, $\bar{h}_U^{3D}$ or $\bar{h}_U^S$) combine a 2D approximation for surface displacements (Equation (50)) with a 3D approximation for volumes (Equation (54)), so all are hybrid models that should not be expected to precisely agree with any realistic situation. Proceeding nonetheless and using 'Model II' (with $\bar{h}_U^{3D}$) in Equation (46), for example, gives

$$U_{tip} = C_2 \sqrt{\frac{\Delta p_{static}}{\rho}} \left(\frac{16\xi \Delta p_{static}}{3\pi E'} \right)^{2/3} \left(\frac{L}{k} \right)^{1/6} \chi^{7/6} \quad \text{(D9)}$$

(where the exponent of 7/6 on χ comes from 1/2 + 2/3). Similarly, using 'Model II' in Equation (D8) gives

$$\begin{aligned} \frac{dV_b}{dL} &= \frac{16 C_1 \xi \Delta p_{static}}{3E'} \frac{d(\chi L^3)}{dL} \\ &= \frac{16 C_1 \xi \Delta p_{static}}{E'} \cdot \chi L^2 (1 + \frac{L}{3\chi} \frac{d\chi}{dL}), \end{aligned} \quad \text{(D10)}$$

where it will be shown that the $d\chi/dL$ term can be safely ignored compared with the other term (this is also true for 'Model I', but not for 'Model III'). Using these expressions in Equation (D7), and solving for U_{vert} gives

$$U_{vert} = 4.83 \sqrt{\frac{\Delta p_{static}}{\rho}} \left(\frac{\Delta p_{static}}{E} \right)^{2/3} \left(\frac{L}{k} \right)^{1/6} \left(\frac{L}{a} \right)^2 \cdot \chi^{7/6} \quad \text{(D11)}$$

Substituting U_{vert} into Equation (D6), and ignoring the $d\chi/dL$ term, allows us to solve algebraically for χ in terms of known quantities (and given L and a). Using values from Section 3, then

$$\chi = \frac{(a/L)^{16/3} \cdot (L/H)}{0.456 + (a/L)^{16/3} \cdot (L/H)}. \quad \text{(D12)}$$

Explicitly calculating $d\chi/dL$ with this solution, we find that $(L/3\chi) d\chi/dL \le 1/3$ regardless of L, and thus small compared to 1, which validates ignoring that contribution in

Equation (D10). If we had used 'Model I' (with $\bar{h}_U$) instead of 'Model II', Equation (D12) would have a numerical factor of 3.55 instead of 0.456, while not changing the rest of the expression. If we instead use 'Model III' (with $\bar{h}_U^S$) instead of 'Model II' to calculate χ, then we can no longer ignore the $d\chi/dL$ term and instead must numerically solve the differential equation to find $\chi(L)$. For plots of χ for these three cases for plausible choices of a/L, see Figure 4.10 of *Tsai* [2009]. For 'Model III' (including approximate plate bending), the strong dependence of U_{tip} on L implies the fast asymptote of $\chi \to 0$ as L grows. This asymptote of $\chi \to 0$ results in the rapid decrease in $\Delta p_{in} \to 0$ and thus rapid closing of the vertical crack which, in turn, is what stabilizes the growth rate of the basal crack. One should note that, in this model, the rapid closing of the vertical crack is complete since it involves a mathematical crack that can close completely under zero excess pressure Δp_{in}, whereas a realistic rough crack would not have complete closure to flow even with $\Delta p_{in} = 0$. The behavior of 'Model III' therefore may be unrealistic.

Finally, in the late stages of crack growth, when the surface lake is gone but there remains excess water pressure driving the basal crack open (with height of liquid water H_w now below the surface height of the glacier H), we assume that the crack system continues to grow while conserving the total water volume in the basal crack plus vertical crack. We now find it convenient to separate the contributions to pressure loss into a hydrostatic component due to $H_w < H$ such that $\Delta p_{hy} \equiv \chi_w \Delta p_{static}$ in hydrostatic equilibrium, and a fractional dynamic component on top of this such that $\Delta p_{in} \equiv \chi \Delta p_{hy} \equiv \chi \cdot \chi_w \Delta p_{static}$. H_w and χ_w can easily be related by expressing hydrostatic balance in terms of H_w, which yields

$$\frac{H_w}{H} = \frac{\rho_{ice}}{\rho} + \frac{\rho - \rho_{ice}}{\rho}\chi_w. \tag{D13}$$

As expected, when $\chi_w \to 1$, $H_w \to H$ and when $\chi_w \to 0$, $H_w \to 0.91H$. Since the geometric changes in H_w/H are small compared to the effects of χ_w on Δp_{in}, we continue to approximate $H_w \approx H$ when it enters equations geometrically. With this approximation, we then find that χ is still determined by Equation (D12). Maintaining $V_b + V_c = V_0$ in 'Model II' (i.e., using Equation (D1) implemented with $\bar{h}_U^{3D}$) then determines $\chi_0 \equiv \chi\chi_w$ to be

$$\chi_0 = \frac{EV_0}{\pi \Delta p_{static} L^3} \cdot \frac{L/H}{0.503L/H + (a/L)^2}. \tag{D14}$$

Thus, $\Delta p_{in}/\Delta p_{static} \equiv \chi_0$ is again determined algebraically as a function of L (and a/L) during the late stages of basal crack growth.

Notation

A	constant related to U_{tip} in Eq. (13).
A'	constant related to A.
A_k	self-similar series constants.
a	half length of connecting conduit.
b	distance along crack of force pair.
C_1	$h_{avg}/\bar{h}_U$.
C_2	scaling factor for velocity in Eq. (46).
c_0	coefficient in lubrication equation.
c_k	constants chosen to satisfy $K_{Ic} = 0$.
D	self-similar series constant.
D^*	size of largest entrained grains.
E	Young's modulus.
E'	effective modulus in plane strain.
E_{loss}	energy loss.
e_{loss}	energy loss per unit area.
F_{ij}	angular function associated with σ_{ij}.
$F(\hat{x})$	pressure term associated with first term of self-similar opening series.
f	Darcy-Weisbach friction factor.
f_0	value of f at reference scale.
G	shear modulus.
g	gravitational acceleration.
$g(z)$, $F(z)$	complex bimaterial functions.
H	height of ice sheet.
H_w	height of water.
h	basal crack opening.
h_0	crack opening estimate.
h_{avg}	average value of h.
h_s	surface uplift from crack opening.
h_s^V	surface uplift from V_c crack.
h_U^P	opening of plate for uniform p.
$\bar{h}_U$	average opening for uniform p.
$\bar{h}_U^{3D}$	average opening for 3D uniform p.
$\bar{h}_U^P$	average opening of plate for uniform p.
$\bar{h}_U^S$	average opening of 3D crack plus plate.
I_k	integral expressions.
K_I	stress intensity factor.
K_{Ic}	fracture toughness.
k	Nikuradse roughness height.
L	horizontal basal crack length.
$\hat{L}$	non-dimensional L.
L_0	characteristic scale for L.
L_c	surface crack length.
LHS	left-hand side of Eq. (26).
l	conductive length scale.
m	power-law index.
n	Manning roughness.
P	constant in Eq. (13).
p	fluid pressure in crack.
Δp	pressure in excess of hydrostatic.
$\hat{p}$	non-dimensional Δp.
Δp_0	characteristic scale for Δp.
Δp_{in}	excess pressure at crack inlet.
Δp_{hy}	hydrostatic component of Δp_{in}.
Δp_{static}	Δp from column of standing water.
Δp_{loss}	loss of pressure in excess of hydrostatic.
Q_{2D}	2D flow rate.
Q_b	flow rate contributed by V_b.
Q_c	flow rate contributed by V_c.
q	constant in complex potential.
R	distance along crack behind crack tip.
$\mathcal{R}$	distance from center of 3D crack.
$\hat{\mathcal{R}}$	non-dimensional $\mathcal{R}$.
R_h	hydraulic radius.
$\Re$	Reynolds number.
RHS	right-hand side of Eq. (26).
r	distance away from crack tip.
S	negative hydraulic head gradient.
s, $\hat{s}$	dummy variables.
T	timescale of drainage.
t	time.
$\hat{t}$	non-dimensional t.
U	fluid velocity averaged across h.
$\hat{U}$	non-dimensional U.

U_0 characteristic scale for U.
$U_{Manning}$ Manning velocity.
U_{vert} average velocity in vertical conduit.
U_{tip} crack-tip velocity.
u_{P1} horizontal displacement due to force pair.
u_s surface horizontal displacement.
$u + iv$ complex displacement.
V_0 initial lake volume.
V_{2D} 2D crack volume.
V_b basal crack volume.
V_c connecting crack volume.
W_L water level.
w model crack opening.
$\hat{w}$ non-dimensional w.
w_0 characteristic scale for w.
w_k terms of self-similar opening series.
w_U model opening for uniform p.
w_U^{3D} model opening of 3D crack for uniform p.
x horizontal position along crack.
x_0 horizontal sensor position.
y vertical position.
z complex variable.
α, β, γ self-similar constants.
$\dot{\gamma}$ shear rate.
δ elastic constant relating w and Δp.
ϵ bimaterial mismatch constant.
ϵ_m normalized squared error.
η water viscosity.
η_k $3 - 4\nu_k$.
ϕ self-similar constant for velocity scale.
φ dummy variable.
$\phi(z)$, $\psi(z)$ Muskhelishvili potentials.
κ thermal diffusivity.
ν, ν_k Poisson's ratio.
ρ water density.
ρ_{ice} ice density.
ρ_s grain density.
σ_0 hydrostatic ice overburden pressure.
σ_{ij} stress tensor components.
τ average shear stress on channel walls.
τ_{b0} initial basal shear stress.
τ_c^* dimensionless critical Shields stress.
τ_d diffusion timescale.
θ angle around crack tip.
ξ h/w.
χ $\Delta p_{in}/\Delta p_{hy}$.
χ_w $\Delta p_{hy}/\Delta p_{static}$.
χ_0 $\chi \cdot \chi_w$.

References

Das, S., I. Joughin, M. D. Behn, I. M. Howat, M. A. King, D. Lizarralde, and M. P. Bhatia (2008), Fracture propagation to the base of the Greenland Ice Sheet during supraglacial lake drainage, *Science*, *320*, doi:10.1126/science.1153360.

Desroches, J., E. Detournay, B. Lenoach, P. Papanastasiou, J. R. A. Pearson, M. Thiercelin, and A. Cheng (1994), The crack tip region in hydraulic fracturing, *Proc. R. Soc. Lond. A*, *447*, 39–48.

England, A. H. (1965), A crack between dissimilar media, *J. App. Mech.*, *32*, 400–402.

Erdogan, F. (1965), Stress distribution in bonded dissimilar materials with cracks, *J. App. Mech.*, *32*, 403–410.

Lay, T., and T. C. Wallace (1995), *Modern Global Seismology*, 521 pp., Academic Press, San Diego.

Muskhelishvili, N. I. (1953), *Some Basic Problems of the Mathematical Theory of Elasticity*, 3rd ed., 704 pp., Noordhoff Ltd, Groningen, Holland.

Rice, J. R., and G. C. Sih (1965), Plane problems of cracks in dissimilar media, *J. App. Mech.*, *32*, 418–423.

Tada, H., P. C. Paris, and G. R. Irwin (2000), *The Stress Analysis of Cracks Handbook*, 3rd ed., 677 pp., ASME Press, New York.

Timoshenko, S., and S. Woinowsky-Krieger (1959), *Theory of Plates and Shells*, 2nd ed., 580 pp., McGraw-Hill, New York.

Timoshenko, S. P., and J. N. Goodier (1987), *Theory of Elasticity*, 3rd ed., 567 pp., McGraw-Hill, New York.

Tsai, V. C. (2009), The use of simple physical models in seismology and glaciology, Ph.D. thesis, Harvard University.

Walsh, J. B., and J. R. Rice (1979), Local changes in gravity resulting from deformation, *J. Geophys. Res.*, *84*, 165–170.

V. C. Tsai, Department of Earth and Planetary Sciences, Harvard University, 20 Oxford St., Cambridge, MA 02138, USA. (vtsai@post.harvard.edu)

J. R. Rice, Department of Earth and Planetary Sciences and School of Engineering and Applied Sciences, Harvard University, 29 Oxford St., Cambridge, MA 02138, USA. (rice@seas.harvard.edu)