RIGIDITY DEPENDENCE OF THE
INTERPLANETARY MEAN FREE PATH

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ABSTRACT
We use energy spectra of anomalous cosmic rays derived from data from experiments on the Voyager
and Pioneer spacecraft to infer the rigidity dependence of the perpendicular interplanetary mean free
path over the rigidity range ~0.3 to 4 GV. Data are from three intervals: 1993/157-209, 1994/157-313,
and 1996/1-52. We use two techniques to estimate that the rigidity dependence is approximately
proportional to \( R^2 \) at low rigidities with a flattening above ~1 GV, roughly consistent with recent
theoretical estimates (Bieber et al. 1995).

INTRODUCTION
Anomalous cosmic rays (ACRs) are thought to be accelerated pickup ions (Fisk et al. 1974). The
pickup ions that contribute to ACRs are thought to be interstellar neutral atoms that become ionized
after drifting into the heliosphere either by charge exchange with the solar wind or via photoionization
by solar photons. The final stage of acceleration presumably occurs at the solar wind termination
shock (Pesses et al. 1981). The flux of ACRs in the heliosphere is governed by the processes of dif-
fusion, convection, adiabatic deceleration, and large-scale drift in the solar wind. Convection can
be relatively accurately estimated because the solar wind speed is fairly well known. Likewise, the
process of adiabatic deceleration is determined by the expansion rate of the solar wind. However, the
parameters that govern diffusion and drift are less certain. In this paper we estimate the perpendicular
mean free path in the outer heliosphere as a function of rigidity. The estimate is based on ACR
and galactic cosmic ray (GCR) energy spectra acquired by instruments on the Voyager 1 and 2 (V1
and V2) spacecraft during three periods spanning mid-1993 to early 1996.

ESTIMATE OF \( \lambda_\perp \) FROM FORCE-FIELD MODEL
The force-field model of cosmic ray modulation (Gleeson & Axford 1968) is a spherically-symmetric
model which assumes that there is zero streaming of the particles. In this model, the intensity of
cosmic rays at V2 (\( j_2 \)) and V1 (\( j_1 \)) are related by:

\[
j_2 = j_1 \exp\left[ - \int \frac{CV}{\kappa_\perp} dr \right]
\]

(1)

where \( C \) is the Compton-Getting coefficient, \( V \) is the solar wind speed, and \( \kappa_\perp \) is the perpendicular
diffusion coefficient, which dominates the parallel diffusion coefficient in the outer heliosphere. For
non-relativistic particles, which we will primarily be concerned with in this study,

\[
C = \frac{2 - 2\gamma}{3}
\]

(2)

where \( \gamma \) is the power-law index determined from a power-law approximation in a local region of the
energy spectrum. We assume that \( \kappa_\perp \) is proportional to the radial distance, so that Eq. 1 becomes:

\[
\ln(j_1/j_2)/\ln(r_1/r_2) = < C > V/\kappa_\perp
\]

(3)

where \( < C > \) is the average of the V1 and V2 values of C. Since the mean free path, \( \lambda_\perp \), is related to
the diffusion coefficient and particle speed, \( \beta \), by:

\[
\lambda_\perp = \frac{3\kappa_\perp}{(\beta C)}
\]

(4)
we can derive an estimate of $\lambda_{\perp}$ from:

$$\lambda_{\perp} = 3 < r > < C > V / (e B r)$$  \hspace{1cm} (5)$$

where $< r >$ is the average radial position of V1 and V2 and

$$G_r = \ln(j_1/j_2)/\ln(r_1/r_2)$$  \hspace{1cm} (6)$$

For a V1-V2 pair of ACR or GCR energy spectra, Eqs. 5 and 6 can be used to estimate the perpendicular mean free path. Only intensities in the diffusive part of the spectra can be used in this approximation, i.e., only above the energy of the peak in intensity. Below the peak, the intensity is dominated by particles decelerated from higher energies and the force-field approximation is no longer valid.

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We have collected energy spectra at V1 and V2 of various ACR and GCR species for three different time periods and the resulting estimates of $\lambda_{\perp}$ are presented in Figure 1. The energy spectra of ACR He and O that were used for the period 1993/157-209 are shown in Stone et al. (1996) and the ACR spectra that were used for the period 1994/157-313 are shown in Cummings & Stone (1996). The values of $\lambda_{\perp}$ in Figure 1 are for the midpoint of the V1 and V2 radial positions. The V1 and V2 heliographic radial and latitudinal coordinates are shown in Table 1 for the three time periods.

Below $\sim 1$ GV the mean free path is generally increasing rapidly with rigidity but above 1 GV there is a pronounced flattening. This form of $\lambda_{\perp}$ is similar to the form derived from quasi-linear theory by Bieber et al. (1995) and the dashed lines in Figure 1 are fits of this form.

**Fig. 1:** Estimate of $\lambda_{\perp} = 3\kappa_{\perp}/(B_0)$ at the average midpoint radial position of V1 and V2 from the observed energy spectra of the species shown in the figure legend. The dashed line are fits to the function described by Eqs. 4 and 7.

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form:
\[ \lambda_\perp = 3\kappa/(\beta c) = \frac{3\kappa_0 r (R/R_c)^2}{\beta c[1 + (R/R_c)^2]} \] (7)

where \( r \) is radius, \( R \) is rigidity, and \( \kappa_0 \) and \( R_c \) are constants. \( \lambda_\perp \) is \( \propto R^2 \) at low rigidities with a flattening at high rigidities. Such a form for the mean free path was used successfully to fit V1 and V2 energy spectra in a spherically-symmetric model of modulation in Stone et al. (1996) and Cummings & Stone (1996).

ESTIMATE OF \( \lambda_\perp \) FROM ENERGIES OF PEAK INTENSITIES

Cummings et al. (1984) derived a scaling relationship for the peak energies in the energy spectra of two different species of cosmic rays. The relation follows from the one-dimensional result that the peak energies occur at an energy for each species for which the particles have the same interplanetary diffusion coefficient. The equation is:
\[ f_E(Z,A) \propto (A/Z)^{-2\alpha/(\alpha+1)} \] (8)

where \( A \) is the mass of the particle, \( Z \) is its charge, and \( \alpha \) is the power-law index in the rigidity dependence of the interplanetary mean free path, i.e., \( \lambda_\perp \propto R^\alpha \). For singly-charged ACR energy spectra acquired at Voyager 2 in 1987, Cummings & Stone (1990) and Cummings et al. (1994) showed a mass dependent energy scaling factor that was consistent with \( \lambda_\perp \propto R^{1.8} \), close to the above quasilinear form at low energies of \( R^2 \).

For the more recent data, the energy spectra presented in Stone et al. (1997) can be inspected to determine the energies of peak intensities of ACR and GCR H and He. For the spectra at V1 during the period 1996/184-365 in Stone et al. (1997) we estimate that the ACR H and He energies of peak intensity are \( \sim 40 \) MeV (0.28 GV) and \( \sim 6.3 \) MeV/nuc (0.43 GV), respectively. For GCRs, the intensity appears to peak at \( \sim 230 \) MeV (0.70 GV) and \( \sim 165 \) MeV/nuc (1.16 GV) for H and He, respectively. Using \( Z = 1 \) for ACR He and \( Z = 2 \) for GCR He and Eq. 8 we find that \( \alpha \sim 0.3 \) for GCR H to He scaling and \( \alpha \sim 2.0 \) for ACR H to He scaling. In Figure 2 we show the \( \lambda_\perp \) for 1996/1-52 from Figure 1 along with two solid lines representing the two estimates of the rigidity dependence of \( \lambda_\perp \) from the peak-energy scaling of ACR and GCR H and He. The agreement between the slopes of the dashed and solid lines is good, indicating that in order to fit the energy spectra of ACRs and GCRs in the outer heliosphere in a spherically-symmetric model, a perpendicular mean free path approximately proportional to \( R^2 \) at low rigidities with a flattening above 1 GV will be required.

DISCUSSION

A principal limitation in this approach is that it is based on a one-dimensional propagation model that ignores drifts and latitudinal gradients. Even so, the general similarity of the magnitude and rigidity dependence of \( \lambda_\perp \) for the 3 periods and the consistency with the slopes derived from the pathlength

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scaling argument suggests that the often used assumption that \( \lambda_\perp \) is a fixed power law in rigidity, i.e., \( \lambda_\perp \propto R^\alpha \), may not be appropriate in the outer heliosphere. Thus, in attempting to model the Voyager observations with 2-dimensional drift models, we recommend that \( \lambda_\perp \) be parameterized with a form similar to Eq. 7. Until such model calculations are available, it will be difficult to ascertain whether or not the apparent variation in \( \lambda_\perp \) among the three periods is temporal or the result of changing current sheet tilt and latitudinal gradients.

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REFERENCES