azimuthal plane \((x,y)\) normal to the axis \((z)\) of the cylinder and containing its midpoint \((0,0,0),\)

\[
\psi^\prime, A(\psi^\prime, \varphi) = f/(g+g'),
\]

where\(^6\)

\[
g = \frac{2\ell}{k}\left[\begin{array}{c}
si(\varphi) + \frac{1}{|A|^2} \left[ T R [M_{xy}] \right] M_{xy} + \frac{j}{|A|^2} \left[ T R [M_{yy}] \right] M_{yy} \right] + \frac{j}{|A|^2} \left[ T R [M_{zz}] \right] M_{zz} + \frac{1}{|A|^2} \left[ T R [M_{xy}] \right] M_{xy} - \frac{j}{|A|^2} \left[ T R [M_{yy}] \right] M_{yy} \right]
\]

\[
f = \frac{2\ell}{k} \left[ \frac{1}{|A|^2} \left[ T R [M_{xy}] \right] M_{xy} + \frac{j}{|A|^2} \left[ T R [M_{yy}] \right] M_{yy} + \frac{1}{|A|^2} \left[ T R [M_{zz}] \right] M_{zz} + \frac{j}{|A|^2} \left[ T R [M_{xy}] \right] M_{xy} - \frac{1}{|A|^2} \left[ T R [M_{yy}] \right] M_{yy} \right]
\]

It should be noted that this expression is independent of the azimuthal angle \(\psi = \cos^{-1}(\varphi/\ell),\) since the shadow remainant becomes isotropic for bodies of very small thickness.\(^7\) The back scattering cross section is proportional to \(|A|^4,\) and this result has been checked with experimental back scattering measurements\(^4\) on fine wires for lengths less than \(\lambda.\) The experimental results agree with the theoretical to within \(5\%\) for cases where losses cause as much as a \(75\%\) reduction in scattering.\(^4\) This indicates that the result given in Eq. (2) may be used in Eq. (1) to give the total cross section to about the accuracy quoted above.

A partial check on this may be made from the measurements quoted in reference 4. The absorption cross section \(\sigma_a\) of the cylinder may be computed by integration of the current squared times the skin resistance over the cylinder, where current is taken as the same function as was used in deriving Eq. (2), with the constants determined by the variational method. This yields

\[
\sigma_a/\lambda^2 = (4\pi R_{g} g/k) |A|^2,
\]

where

\[
R_L = \text{Re}(Z_L).
\]

The scattering cross section computed from Eqs. (1) and (2) is given by

\[
(\sigma_s/\lambda^2)_{\text{theo}} = (\sigma_s - \sigma_a)/\lambda^2.
\]

The scattering cross section may be obtained from the experimental measurements by integrating the differential cross section over the total solid angle. The \(\psi\) dependence is constant, and the \(\theta\) dependence of a half-wave dipole is known from antenna theory. Therefore,

\[
(\sigma_s/\lambda^2)_{\text{theo}} = \left(1.64 \times 10^{-4}\right) \frac{\lambda^2}{\lambda^2},
\]

where \(\sigma_b\) = scattering cross section in azimuthal plane, and 1.64 = gain of half-wave dipole in azimuthal plane.

Measurements of back scattering from a 0.001-in. diam bismuth wire illuminated by 10 cm radiation\(^4\) yield the following value at first resonance,

\[
(\sigma_s/\lambda^2)_{\text{meas}} = 0.121 \pm 0.005.
\]

The calculated value for this case is

\[
(\sigma_s/\lambda^2)_{\text{theo}} = 0.127,
\]

yielding a discrepancy of 5.1\%, similar to those obtained in the back scattering results.\(^4\) The scattering cross section of a wire with the same dimensions as above but of perfect conductivity is

\[
\sigma_s/\lambda^2 = 0.515,
\]

indicating the correction offered by the variational expression for the scattered amplitude.

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3. This scattering amplitude is dimensionless, differing from that of reference 2 by the factor \(k_0\).


Spontaneous Emission from an Inverted Spin System

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This paper summarizes the results of calculations made in an attempt to explain the origin of the observed modulation effects in two-level maser experiments.\(^1\)

In a spontaneously oscillating spin system any transverse magnetization, which couples rf power out of the spin system, is due to the presence of a transverse rf field, which is in turn induced by the transverse magnetization. Any consideration of the dynamics of the spin system or the build-up of the radiation field must consequently treat the two systems simultaneously.

Our analysis uses a model first formulated by Bloembergen and Pound,\(^4\) except that our problem is inherently nonlinear and we cannot make the simplifying assumption \(M_S=\text{const},\) which linearizes the equations of motion, and are forced to have recourse to an IBM 704.

The physical situation is that of an inverted, homogeneously broadened, magnetization of amplitude \(M_0\) placed inside a microwave cavity which can support a circularly polarized transverse magnetic field. If we take the spin precession frequency \(\gamma H_0\) to be equal to the cavity resonant frequency \((\omega C)^{(0)}\), assume that no external power is fed into the cavity, and that the spin lattice relaxation time \(T_1\) is infinite, we can write a set of three nonlinear simultaneous differential equations.

\[
\begin{align*}
\frac{2j\omega}{T_R} [M_{xy}] M_{xy} + \frac{2}{T_R} [M_{xy}] M_{xy} &+ \frac{1}{T_R} \frac{dM_1}{dt} + \omega H_1 = 0, \\
\frac{dM_1}{dt} &- j\gamma M_1 H_1 + M_1 \frac{M_1}{T_S} = 0, \\
\frac{dM_s}{dt} &+ \text{Im}[\gamma M_1^2 M_{xy}] = 0.
\end{align*}
\]

Equations (1)–(3) appear in a slightly different form in footnote 3. \(M_1\) and \(H_1\) are complex and are defined by

\[
\begin{align*}
M_1 + j M_1 &\equiv M_1^2 \omega^4, \\
H_1 &\equiv H_1 \omega^4,
\end{align*}
\]

and

\[
\begin{align*}
M_S + j M_r &\equiv M_S \omega^4, \\
H_r &\equiv H_r \omega^4.
\end{align*}
\]

where \(M_S, M_r, H_s,\) and \(H_r\) are the transverse components of the magnetization and magnetic field, respectively. \(\gamma\) is the magnetograviy ratio. \(T_S\) is the radiation damping time defined by

\[
T_R = 2\pi \gamma M_s^2 (Q)^{-1}.
\]

\(\eta\) is the sample filling factor, \(Q\) is the loaded "Q" of the cavity. \(T_s\) is the spin-spin relaxation time which for a homogenous Lorentzian line is given by

\[
T_s = 2/\gamma \Delta H = 2/\Delta \omega,
\]
where $\Delta H$ is the separation in gauss between the half power points. $M_i^*$ is the complex conjugate of $M_i$. Since $M_0, H_0$, and $M_1$ are functions of time. Equations (1)-(3) constitute a set of non-linear differential equations. The solution was carried out with the aid of an IBM 704. For numerical purposes it was necessary to consider the following set of equations which are the real and imaginary parts of Eqs. (1)-(3)

$$aM_r+bM_i - \frac{\gamma}{2}M_rH_t+C_M = 0,$$

$$aM_r-bM_i + \frac{\gamma}{2}M_rH_t + C_M = 0,$$

$$dM_r + |\gamma| M_rH_t + C_M = 0,$$

$$dM_i - |\gamma| M_rH_t + C_M = 0,$$

and

$$dM_t + |\gamma| M_rH_t - |\gamma| M_rH_t = 0,$$

where

$$M_1 = M_r + \gamma M_i,$$

$$H_t = H_r + \gamma H_i,$$

$$a = \frac{2\omega}{T_M |\sigma|},$$

$$b = \frac{2}{T_M |\sigma|},$$

and

$$c = \frac{1}{T_M}.$$ 

Since we are dealing with an inverted magnetization, our first boundary condition is $M_1(0) = -M_0$. This constitutes an unstable equilibrium and some mechanism is necessary to trigger the spontaneous emission. This could be either some residual transverse magnetization or some cavity current present at $t=0$. We used the residual current for a trigger and put $H_r(0) = 0.1 M_0$. The magnitude of $H_r(0)$ was found to have little effect on the subsequent behavior of the magnetization.

The values of $\omega, Q$, and $M_0$ were chosen to approximate the experimental conditions prevailing at the two-level maser experiment performed by Chester, Wagner, and Castle of Westinghouse. These are

$$M_0 = 5 \times 10^{-6} \text{ gauss (} 10^{18} \text{ spins at 4.20K)},$$

$$Q = 10000,$$

and

$$\omega = 2 \times 10^{10} (f = 10 \text{ KHz}).$$

The resulting behavior of $H_r^2$, which is equal to $H_r^2 + H_i^2$, is shown in the lower trace of Fig. 1. $H_r^2$ is proportional to the power coupled out of the cavity and is to be compared with the experimental observations in which the spontaneous emission power following inversion was monitored. The upper trace shows the oscillatory return of $M_r$ to its equilibrium, $M_r = -M_0$, value. The peaks of $M_r$ coincide with these of $H_r^2$ as required by conservation of energy considerations. It was found that as long as $T_r < T_s$ which is the start oscillation condition for a two-level maser oscillator, $T_s$ had a negligible effect on the shape of the curves and could be assumed to be infinite. The behavior of $H_r^2$ bears a close resemblance to the experimental results—especially those of the Westinghouse Group. A significant difference remains—the time interval between power peaks in Fig. 1 is $\sim 0.1 \mu\text{sec}$ while the observed results give $\sim 1 \mu\text{sec}$. This difference may be due to the actual inhomogeneous nature of the line, in contrast to the homogenous line assumed in our analysis. The envelope curve, traced through the successive oscillation peaks, decays with a characteristic time corresponding to the cavity $Q$, i.e.,

$$T_{envelope} = Q/\omega.$$ 

This feature is retained regardless of the magnitude of $M_0$. The interval between peaks depends on $M_0$. This dependence is not very strong and using a value of $M_0$ which is smaller by a factor of 15 than that used in Fig. 1 yielded a value of $\sim 0.3 \mu\text{sec}$ which is closer to the experimentally observed values.

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Backscattering of Carbon-14 Beta Radiation

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In connection with studies of radiostearic acid monolayers, we have been concerned about the backscattering of $^{14}$C $\beta$-particles by various substrates. It is commonly stated that the percentage saturation backscattering of $\beta$-particles is independent of $\beta$-energy for radiation of sufficiently high energy ($E_{\text{max}} > 0.6$ Mev). Zumwalt has shown that the same saturation backscattering results are obtained with Geiger-Müller counters for radiation of lower energy if the observed results are corrected for air- and window-absorption of the scattered radiation. Glendenin and Solomon have pointed out that the same sort of correction brings results for $^{22}\text{Na}$ $\beta$-particles (0.17 Mev) into line with those for more energetic radiation.

It is reasonable to suppose that the same results would be obtained with $^{14}$C (0.15 Mev), but we have found no reported experimental verification of this. Because of the considerable importance of this isotope in chemical and biological studies, it seems desirable to record the results which we have obtained. Yaffe and Justus have reported some results for $^{14}$C $\beta$-radiation with Geiger-Müller counters. Beincher has found that autoradiographic observations lead to similar backscattering results for $^{14}$C and more energetic radiation. In the present experiments we have compared the backscattering of $\beta$-radiation from $^{14}$C and $^{14}$N (0.76 Mev) with a mica end-window Geiger-Müller counter under conditions of similar geometry, and have measured the absorption (in aluminum) of the backscattered radiation.

The sources were evaporated deposits of $^{14}$C labelled stearic acid ($\sim 10$ mC/g) and TiO$_2$ ($\sim 50$ mC/g) on vinylite films of