DOUBLE-PEAKED SUPERNOVAE AND THE SIGNATURE OF PROGENITORS WITH LOW-MASS EXTENDED ENVELOPES

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Submitted for publication in The Astrophysical Journal

ABSTRACT

Early observations of supernova light curves are powerful tools for shedding light on the pre-explosion structures of their progenitors and their mass-loss histories just prior to explosion. Some core-collapse supernovae that are detected during the first days after the explosion prominently show two peaks in the optical bands, where the first is presumably powered by the cooling of shocked surface material and the second peak is clearly powered by radioactive decay. Here we explore, first, whether this type of light curve can be generated by a progenitor with a “standard” density profile, such as a red supergiant or a Wolf-Rayet star. We find that such a progenitor: (i) cannot produce a double-peaked light curve in the \textit{R} and \textit{I} bands, and (ii) cannot exhibit a fast drop in the bolometric luminosity as is seen after the first peak. We then explore the signature of a progenitor with a massive, compact core surrounded by extended, low-mass material. This may be a hydrostatic low-mass envelope or material ejected just prior to the explosion. We show that it naturally produce both of these features. We use this result to provide simple formulae to estimate: (i) the mass of the extended material from the time of the first peak, (ii) the extended material radius from the luminosity of the first peak, and (iii) an upper limit on the core radius from the luminosity minimum between the two peaks.

\textit{Subject headings:} hydrodynamics — shock waves — supernovae: general

1. INTRODUCTION

The first hours to days of a supernova (SN) light curve holds valuable information on the structure of the progenitor and on its mass-loss history before the explosion. However, until recently, only a small number of events were caught sufficiently early to extract this information. This has changed with the advent of sensitive, large field of view, transient surveys, such as the Katzman Automatic Imaging Telescope (KAIT; Filippenko et al. 2001), the Palomar Transient Factory (PTF; Rau et al. 2009) and the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS; Kaiser et al. 2012). In the future, these efforts will continue to grow with SkyMapper (Keller et al. 2007), the Zwicky Transient Facility (ZTF; Law et al. 2009), the All-Sky Automated Survey for Supernovae (ASAS-SN; Shappee et al. 2014), and the Large Synoptic Survey Telescope (LSST Science Collaboration et al. 2009). Today a growing number of SNe are detected within one or two days of the explosion, opening a new window into the relatively unexplored early phase of these events.

Following the detection of some very young SNe, an unexpected discovery has been that a subset of these SNe (of various core-collapse types) show double-peaked light curves. In such events, in addition to the main optical peak that is powered by the decay of \textit{\textsuperscript{56}Ni}, there is also an earlier peak in the optical bands (including \textit{R} and/or \textit{I} bands) that fades on a time scale of days. The best known example of such a light curve is the Type Ib SN 1993J (Wheeler et al. 1993). More recent examples are the Type IIb SN 2011dh (a.k.a. PTF11eon; Arcavi et al. 2011) and 2013df (Van Dyk et al. 2014), the Type Ibn SN iPTF13beo (Gorbikov et al. 2013), and the Type Ic (broad lined) SN 2006aj (Campana et al. 2006). In the upper panel of Figure 1 we plot a typical double-peaked light curve that was calculated numerically by Bersten et al. (2012) in order to match the observed light curve of SN 2011dh. This shows how the first peak can rival or exceed the luminosity of the second peak as well as the characteristic timescales of each peak.

SNe 1993J, 2011dh and 2013df are among the rare cases where progenitors were identified in pre-explosion images. In all three of these SNe, they were found to be supergiants with radii $\gtrsim 10^{13}$ cm (Aldering et al. 1994; Maund et al. 2011; Van Dyk et al. 2011, 2014). Such a large radius was claimed to be in tension with the absence of bright, long-lived emission, as would have been expected from a cooling of shocked extended envelope (Arcavi et al. 2011). This discrepancy was explained by invoking a low mass for the envelope (Hofflich et al. 1993; Woosley et al. 1994; Bersten et al. 2012).

Motivated by these discoveries and previous theoretical work, we investigate the conditions required for double-peaked SN light curves and summarize what can be learned from such observations. We divide our discussion between “standard” core-collapse SN progenitors, where a large fraction of the mass reaches out to the stellar radius, and “non-standard” progenitors, which have a compact, massive core surrounded by extended, low-mass material. In §2 we show that the standard progenitors cannot produce a few day time scale peak in the \textit{R} and \textit{I} bands and that their bolometric luminosity cannot decay sufficiently quickly to reproduce the first peak of double-peaked events. Interestingly, standard progenitors with particularly extended envelopes (e.g., red supergiants) are predicted to produce a peak in all bands $\sim 10$ min after the shock breakout, followed by a slow decay over several hours (Nakar & Sari 2010). This feature has yet
R × \text{time of the first peak}

ture of a non-standard progenitor used for the calculation of the
of the first peak

provide information about the progenitor structure. The luminosity

of the optical light curve calculated numerically by Bersten et al. (2012).
The light curve shape is similar in all optical bands (here we present

optical light curve calculated numerically by Bersten et al. (2012).

to be detected, but its discovery would be an important
test for the understanding of massive stars before core
collapse.

In Table 3 we find that the density profiles of non-standard
progenitors naturally produce double-peaked light curves
with a sharp drop of the bolometric luminosity on the

time scales of hours to days after the explosion. We
then provide simple relations to be used in conjunction
with observations of these events to constrain the mass
(eq. 10) and radius (eq. 12) of the low-mass extended
material, along with the radius of the core (eq. 14).
These are confirmed with comparisons to previous de-
tailed modeling. We conclude with a summary of our
results in Section 3.

2. STANDARD PROGENITORS

In this section we explore the expected light curve from
standard core-collapse SN progenitors, in which most of
the mass is concentrated near the stellar radius, \(R_\ast\). To
understand what is meant by this, consider two typical
cases. The first is an extended progenitor, such as a red
supergiant, which has a massive hydrogen envelope in

hydrostatic equilibrium. Here, \(M_{\text{ext}} > M_{\text{core}}\) and \(R_{\text{ext}} = R_\ast \gg R_{\text{core}}\), where \(M_{\text{core}}\) and \(R_{\text{core}}\) are the core ejected
mass and radius, respectively, and \(M_{\text{ext}}\) and \(R_{\text{ext}}\) are the
mass and radius of the extended envelope, respectively
(a more specific definition of \(M_{\text{ext}}\) in the context of this
paper is given later). The second is a stripped progenitor,
such as a Wolf-Rayet star. Although such stars have little
or no envelope, most of the mass is again concentrated
near \(R_\ast\). A property shared by both of these cases is
that hydrostatic equilibrium dictates a density profile,
\(\rho(r)\), at a radius \(r \approx R_\ast\), that varies on a scale that is
comparable to the distance from the stellar edge such
that it is well approximated by a polytrope \(\rho \propto x^n\), where

\[x = (R_\ast - r)/R_\ast\] and \(n\) is typically in the range \(1 - 3\).

Near the stellar edge, \(x \ll 1\), the SN shock accelerates
with the decreasing density as \(v \propto \rho^{-\beta}\), where the
value of \(\beta\) depends weakly on \(n\) (Sakurai 1960; Grassberg
1981). For standard progenitors with \(n = 1 - 3\), \(\beta = 0.19\)
while for \(n = 15\), an extremely steep density profile,
\(\beta = 0.17\). Hereafter we use \(\beta = 0.19\). The shock heats
and accelerates the material and after it breaks out of the
stellar edge, the observed luminosity is determined by the
diffusion of photons through the hot expanding gas. The
light curve of this cooling phase for progenitors with a
\(\rho \propto x^n\) density profile has been calculated analytically
by many authors (e.g., Chevalier 1992; Piro et al. 2010;
Nakar & Sari 2010; Rabinak & Waxman 2011). Here we
focus on the results from Nakar & Sari (2010), which cal-
culated the observed temperature most accurately.

2.1. Planar Phase

At first, before the gas roughly doubles its radius, the
evolution of the surface layers is planar (as is discussed
in more detail by Piro et al. 2010; Nakar & Sari 2010).
Here we highlight the main results of the optical emis-
sion during this phase. Such emission is only expected
if the shock breakout radiation is in thermal equilib-
rium, namely if the progenitor is a supergiant with an
extended envelope. In a supergiant progenitor, the opti-
cal light curve peaks on a time scale of \(R_\ast/c\) after the
explosion, while the planar phase lasts for a time \(R_\ast/v\).
In compact progenitors, the optical emission during the
breakout and the planar phase is too faint to be observed
with current instruments.

At any given time the observed luminosity is gener-
ated at a mass depth \(m_{\text{obs}}\) (measured from the outside
inward), where the diffusion time equals the dynamical
time. During the entire planar phase photons diffuse out
from the breakout layer (i.e., \(m_{\text{obs}}\) is roughly constant
and equal to the mass from where the shock breaks out; see
Nakar & Sari 2010 for details). The resulting light
curve evolves as

\[L_{\text{bol}} \propto t^{-4/3},\]
\[T_{\text{obs}} \propto t^{-0.35}.
\]

Since \(T_{\text{obs}}\) is in the UV, the optical flux scales as
\(F_{\nu,\text{opt}} \propto t^{-0.35}\),

\[L_{\text{bol}} \propto t^{-4/3},\]
\[T_{\text{obs}} \propto t^{-0.35}.
\]

This is the mass from the top of the helium core inward, minus
the remnant mass (\(\approx 1.4 M_\odot\)) left over that will produce a neutron
star.
Thus, during the entire planar phase the optical flux is derived with respect to the $\alpha$-type, for the canonical values of $\alpha = 3/2$ (convective envelope) and $n = 3$ (radiative envelope), $\alpha = 0.17$ and $\alpha = 0.35$, respectively. Thus, for any standard progenitor the bolometric luminosity decrease during this phase can be at most moderate. If a more rapid luminosity drop is observed, it implies that either the density structure is highly non-standard or that the diffusion front has travelled through the entire envelope (i.e., $m_{\text{obs}} > M_{\text{ext}}$). This is similar to the fast drop seen from Type II-P SNe at the end of their plateau.

Another limit on double-peaked light curves can be derived with respect to the $R$ and $I$ band properties. This can be seen because during the spherical phase, before recombination becomes important, the temperature evolves roughly as

$$T_{\text{obs}} \propto t^{-0.6},$$

where the dependence on $n$ is weak. As a result, the observed flux in bands that are on the Rayleigh-Jeans tail of the spectrum rises as $t^{-1.5}$ (Piro & Nakar 2013). The flux starts falling only once the temperature falls below the observed band. However, once the observed temperature reaches about 6000 ~ 8000 K recombination commences. This has two effects. The most prominent one is that the observed temperature drop stops almost entirely. Thus, the roughly constant temperature is set to peak around the $R$ and $I$ bands. The second is that the bolometric luminosity falls more slowly (or even start rising slowly) when the recombination front reaches deep enough to affect $m_{\text{obs}}$. The result is that as long as $m_{\text{obs}} < M_{\text{ext}}$ the $R$ and $I$ band luminosity remains rather constant during recombination (this is the origin of the plateau in Type II-P SNe). This result is true both for hydrogen rich envelopes and for hydrogen stripped progenitors (Dessart et al. 2011).

To conclude, before recombination starts the $R$ and $I$ bands are rising. After recombination begins, these bands are rather constant, or at most the $R$ band is dropping very slowly. This implies that the cooling envelope phase of a standard progenitor with a massive envelope cannot produce a prominent peak in the $R$ or $I$ band as long as $m_{\text{obs}} < M_{\text{ext}}$.

3. NON-STANDARD PROGENITORS

Motivated by the inability of standard progenitors to reproduce the main features of double-peaked SN light curves, we now turn to considering non-standard progenitors. In particular, since the standard progenitors appear to fail when $m_{\text{obs}} < M_{\text{ext}}$ we look at lower amounts of material surrounding a compact core, i.e., $M_{\text{ext}} < M_{\text{core}}$ and $R_{\text{ext}} > R_{\text{core}}$. In such cases, $m_{\text{obs}} < M_{\text{ext}}$ will not be satisfied for long during the light curve evolution. An example of a non-standard progenitor is shown in the lower panel of Figure 1. We plot the mass measured from the stellar edge inward to highlight just how little mass is in the extended material. In this example, $M_{\text{ext}} \approx 6 \times 10^{-3} M_{\odot}$, even though it constitutes the outer 2/3 of the star in radius!

The exact density profile of the extended material is unimportant for our analysis. The only important properties are that $M_{\text{ext}}$ is concentrated around $R_{\text{ext}}$ and that the density at larger radii is low enough so interaction can be neglected. Thus, the extended material can be a shell ejected just prior to the explosion (e.g., Ofek et al. 2013) or a continuous wind, as long as it is terminated at $R_{\text{ext}}$. It can also be a low-mass extended envelope, either in or out off hydrostatic equilibrium. Note that in that case the mass $M_{\text{ext}}$ is not strictly the envelope mass. The reason is that $M_{\text{ext}}$ includes only mass that is concentrated around $R_{\text{ext}}$, while some envelope mass may be found at smaller radii (when we look at the profiles of specific models later in this section, it will be more clear why we must make this distinction). We also restrict the discussion here to cases where

$$M_{\text{ext}} \gtrsim 4\pi R_{\text{ext}}^2 c \frac{\kappa v}{\nu} = 5 \times 10^{-5} \kappa_{0.34} v_9^{-1} R_{13}^2 M_{\odot},$$

where $R_{13} = R_{\text{ext}}/10^{13}$ cm. This criterion ensures that the shock breaks out from the extended material and not from the core.

Equation 3 is implicit since $v$ is in itself a function of $m_{\text{obs}}$. However, later we will discuss methods to estimate $v$ independently for the mass of interest.

\[ \text{Footnote:} \]

5 In contrast, in the optical blue bands and UV, a mild decrease in the observed temperature results in a significant drop of the blue light. This is because these wavelengths are in the Wien part of the spectrum during recombination.

6 When the envelope mass is smaller than this criterion the shock...
When there is low-mass, extended material, the physical picture changes as follows. After crossing the core, the shock accelerates the low-density material to rather high velocities. Adiabatic losses due to expansion of the shocked extended material are relatively small (due to its initial large volume) so its cooling emission is bright, dominating the early-time light curve. However, this emission falls off very rapidly once \( m_{\text{obs}} > M_{\text{ext}} \), implying that if the extended material mass is low, then this phase ends within \( \sim 1 \) days (using eq. [3]). At that point the main source of the emission becomes the core. Here, adiabatic losses are severe as the radius before the expansion is much smaller, so that the main source of emission is the radioactive decay of \(^{56}\text{Ni}\). The observed radioactive luminosity increases as more mass of the core, and thus of \(^{56}\text{Ni}\), is exposed by the inward traveling diffusion front. The peak of this phase is observed roughly when \( n_{\text{obs}} \approx M_{\text{core}} \) (note that \( M_{\text{core}} \) includes only the ejected core mass and not any potential remnant mass that is left over from the SN).

Therefore, low-mass, extended material around a compact core naturally leads to a double-peak SN light curve in all wavelengths, including the \( R \) and \( I \) bands. This also results in a sharp drop in the bolometric luminosity, between the end of the cooling phase and the emergence of the \(^{56}\text{Ni}\) driven core luminosity. Calculating the main properties of the resulting light curve is simplified by the fact that the emission of the extended material and the core are independent of each other. *One can in effect treat the emission as that of two separate SNe.* The first SN is the cooling phase emission of a low energy explosion of an extended low-mass star. This emission is short lived and the time, luminosity and temperature at the peak are straightforward to calculate since, as we show below, recombination does not play a role. The second SN is a regular compact star explosion which has been calculated by many authors in the context of Ib and Ic SNe.

Below we discuss ways to estimate the extended material velocity and energy, and then we use these values to constrain the properties of the progenitor. Using arrow and color-coding, we highlight the connections between the double-peaked light curve and the progenitor structure in Figure 1.

### 3.1. Estimating the Velocity and Energy of the Extended Material

The characteristic velocity of the extended material \( v_{\text{ext}} \) can be estimated from observations if an early spectrum of the first peak emission is available and the photospheric velocity at this time can be measured (often the spectrum will be too hot to show spectral lines). The extended material velocity is smaller than the photospheric velocity at peak by a factor of order unity. For an envelope in hydrostatic equilibrium, this factor is in the range \( 1.3 \sim 1.5 \) \cite{nakar2010}. Alternatively, a photospheric velocity at a later time can be used, since for a layer with a given mass \( \tau \propto t^{-2} \). At the time of the first peak, \( \tau \approx c/v_{\text{ext}} \). Therefore, \( M_{\text{ext}} \) becomes the photosphere at \( t \approx t_p \sqrt{c/v_{\text{ext}}} \), where \( t_p \) is the time of the first peak. For typical parameters this is at \( \sim 5 \times t_p \), which is usually during the rising of the second peak.

If an observational constraint is not available, then \( v_{\text{ext}} \) can be estimated based on theory. Following the core collapse, a shock is driven through the remaining parts of the core. It accelerates once it encounters the sharp density drop at the edge of the helium core, bringing smaller amounts of mass to higher and higher velocities. This leads to a velocity profile \( v(m_e) \), where \( m_e \) is the amount of core mass accelerated to a velocity \( v \). Once the shock starts propagating into the shallower density profile of the extended material it decelerates again, leading to a reverse-forward shock structure. During deceleration the swept-up extended material mass is comparable to the core mass that crossed the reverse shock, implying that by the time that the entire extended material is shocked its velocity is \( v_{\text{ext}} \approx v(m_e = M_{\text{ext}}) \). We approximate \( v(m_e) \) by assuming that the core density profile is not significantly affected by the extended material, in which case the acceleration follows the self-similar solution of \cite{sakurai1960} with \( n = 3 \),

\[
v_{\text{ext}} \approx 1.5 \times 10^9 E_{51}^{0.5} \left( \frac{M_{\text{core}}}{3 M_\odot} \right)^{-0.35} \times \left( \frac{M_{\text{ext}}}{0.01 M_\odot} \right)^{-0.15} \text{ cm s}^{-1},
\]

where \( E \) is the total explosion energy and \( E_{51} = E/10^{51} \text{ erg} \). The energy carried by the extended material is then

\[
E_{\text{ext}} \approx 2 \times 10^{49} E_{51} \left( \frac{M_{\text{core}}}{3 M_\odot} \right)^{-0.7} \left( \frac{M_{\text{ext}}}{0.01 M_\odot} \right)^{0.7} \text{ erg}.
\]

Thus, the cooling phase is similar to a low mass, low energy SN of an extended progenitor, which produces a bright, short-lived signal.

### 3.2. Constraints on the Extended Material and the Core Properties

The peak optical flux is observed when \( m_{\text{obs}} \approx M_{\text{ext}} \). Thus the mass of the extended material can be measured, using equations (3) and (8), simply by identifying the time of the first optical peak, \( t_p \),

\[
M_{\text{ext}} \approx 5 \times 10^{-3} \kappa_{0.34}^{-1} \left( \frac{v_{\text{ext}}}{10^9 \text{ cm s}^{-1}} \right) \left( \frac{t_p}{1 \text{ day}} \right)^2 M_\odot
\]

\[
\approx 8 \times 10^{-3} E_{51}^{0.43} \kappa_{0.34}^{-0.87} \left( \frac{M_{\text{core}}}{3 M_\odot} \right)^{-0.3} \left( \frac{t_p}{1 \text{ day}} \right)^{1.75} M_\odot.
\]

The connection between \( t_p \) and \( M_{\text{ext}} \) is shown in green in Figure 1. As we discuss above the emission from the extended material is dominated by the mass at \( r \approx R_{\text{ext}} \). Thus, \( M_{\text{ext}} \) measures only the mass concentrated at \( r \approx R_{\text{ext}} \). If the envelope structure is such that a significant amount of mass is concentrated at \( r \ll R_{\text{ext}} \), then this mass does not contribute to the flux at \( t_p \) and is therefore not included in \( M_{\text{ext}} \). Note that \( t_p \) is also roughly the decay time scale of the observed flux after the peak. So even if the SN is detected only after the peak, then the decay time scale can provide a rough estimate of \( M_{\text{ext}} \).
The bolometric luminosity at the peak is set by the initial internal energy in the extended material and the adiabatic losses to expansion, namely

\[ L_{\text{bol}}(t_p) \approx \frac{E_{\text{ext}} R_{\text{ext}}}{v_{\text{ext}} t_p^2} \]  

(11)

Thus, the peak emission also provides an estimate of the extended material radius,

\[ R_{\text{ext}} \approx 2 \times 10^{13} \kappa_{0.34} L_{43} \left( \frac{v_{\text{ext}}}{10^9 \text{ cm s}^{-1}} \right)^{-2} \text{ cm} \]

\[ \approx 10^{13} \kappa_{0.34} E_{51}^{-0.87} L_{43} \left( \frac{M_{\text{core}}}{3 M_{\odot}} \right)^{0.61} \left( \frac{t_p}{1 \text{ day}} \right)^{0.51} \text{ cm} \]  

(12)

where \( L_{43} = L_{\text{bol}}(t_p)/10^{43} \text{ erg s}^{-1} \). The observed temperature at the peak can be approximated by the effective temperature, resulting in

\[ T_{\text{obs}}(t_p) \approx 3 \times 10^4 \kappa_{0.34}^{-1/4} \left( \frac{t_p}{1 \text{ day}} \right)^{-1/2} \left( \frac{R_{\text{ext}}}{10^{13} \text{ cm}} \right)^{1/4} \text{ K.} \]  

(13)

This temperature justifies ignoring recombination. It peaks in the UV and therefore cannot be measured easily by optical surveys, although it may be possible to probe using future UV surveys (e.g., Sagiv et al. 2013). Equations (12) and (13) enable constraints to be placed on \( R_{\text{ext}} \) with optical photometry at the peak alone. The connection between \( L(t_p) \) and \( R_{\text{ext}} \) is shown in blue in Figure 1. Note, however, that if \( R_{\text{ext}} \) is derived in this way, then for an observed frequency in the Rayleigh-Jeans tail \( R_{\text{ext}} \propto v^4 \). This implies that the order of unity uncertainty in the coefficients of equations (12) and (13) is translated to an uncertainty of an order of magnitude in the derived \( R_{\text{ext}} \).

Another property of the progenitor that can be constrained by the observations is \( R_{\text{core}} \). Since the emission from the core is similar to that of Type Ib/Ic SNe, its luminosity decreases following shock cooling emission to a roughly constant value before the \( ^{56}\text{Ni} \) driven emission becomes dominant (Dessart et al. 2011). This value, which we find analytically in Piro & Nakar (2013) (see their eq. [5]), sets a lower limit to the luminosity generated by the core. Thus, the minimal observed luminosity between the two peaks, \( L_{\text{min}} \), constrains the core radius,

\[ R_{\text{core}} \lesssim 2.5 \times 10^{11} \kappa_{0.2}^{0.9} E_{51}^{-1.1} \]

\[ \times \left( \frac{L_{\text{min}}}{10^{41} \text{ erg s}^{-1}} \right)^{1.3} \left( \frac{M_{\text{core}}}{3 M_{\odot}} \right)^{0.85} \text{ cm,} \]  

(14)

where we use a canonical value of \( \kappa_{0.2} = \kappa/0.2 \text{ cm}^2 \text{ g}^{-1} \), as appropriate for a hydrogen deficient ionized gas. The connection between \( L_{\text{min}} \) and \( R_{\text{core}} \) is shown in red in Figure 1. Note that the temperature during the rising phase of the second peak is typically in the optical and therefore \( L_{\text{min}} \) can be often estimated based on optical observations alone of the minimum between the two peaks.

3.3. Comparison to Numerical Work

In order to evaluate the accuracy of our analytic approximations we compare them to the results of detailed numerical simulations. Explosions of progenitors with low-mass, extended envelopes were carried out for two of the best studied double-peaked SNe, 2011dh (Bersten et al. 2012) and 1993J (Woosley et al. 1994). We compare to three models of Bersten et al. (2012), all of which have the same core structure and the same explosion energy (\( E_{51} = 1 \)), but the envelopes are extended to different radii of \( R_{\text{ext}} = 270, 200, \) and 150 \( R_{\odot} \). We also compare to model 13B of Woosley et al. (1994), which is found to produce a light curve that is similar to SN 1993J.

In all these simulations, a mass \( \approx 0.1 M_{\odot} \) that contains hydrogen is attached to a \( \approx 4 M_{\odot} \) He core. Most of this mass is concentrated right near the outer edge of the core radius, while a smaller amount of mass is spread over the extended parts of the envelope, around \( R_{\text{ext}} \). As discussed above, due to adiabatic losses, the first peak is dominated by the emission from the mass near \( R_{\text{ext}} \) and therefore, we take \( M_{\text{ext}} \) to be the mass between the radii of \( R_{\text{ext}}/3 \) and \( R_{\text{ext}} \) right before the explosion.

A comparison between the numerical results and our formulas is presented in Table 1. Our estimates of \( M_{\text{ext}} \) and \( R_{\text{ext}} \) agree very well, better than a factor of 2, in all cases. We expected such agreement for \( M_{\text{ext}} \) and the estimates of \( R_{\text{ext}} \) for the case studied by Woosley et al. (1994), where \( L_{\text{bol}} \) is given. The agreement, however, with the values of \( R_{\text{ext}} \) estimated for the three cases studied by Bersten et al. (2012), where only the absolute \( g' \)-band magnitude is given, are better than expected. It is probably not representative of the true uncertainty in \( R_{\text{ext}} \) in that case, which is accurate only to within an order of magnitude when only optical photometry is known (see discussion below equation (13)). Finally, the upper limits on \( R_{\text{core}} \) are all a factor of 3 – 5 larger than the actual core radius.

4. SUMMARY

We have explored the progenitor properties of double-peaked SNe, where the first peak is seen also in the \( R \) and/or \( I \) bands and the second peak is powered by radioactive decay. We consider the emission from two types of progenitors. Our main results are as follows.

Standard progenitors. The planar phase of an extended (e.g., red supergiant) progenitor produces an optical peak with a rise time of \( R_c/v \sim \text{minutes} \) and a decay time of \( R_c/v \sim \text{hours} \). This phenomenon has yet to be seen in observations, but would be an important test of SN theory. The first optical peak in all known double-peaked SNe occur on a longer time scale and are not explained by this planar emission.

During the spherical phase, for both compact and extended progenitors, we derive an upper limit to how quickly the bolometric luminosity can drop. This is found to be \( \alpha < 0.64 \), where \( L_{\text{bol}} \propto t^{-\alpha} \). For an envelope structure with a typical polytropic index, the limit is more stringent, \( \alpha < 0.35 \). Furthermore, the \( R \) and \( I \) band fluxes do not show a significant decay (never faster than \( L_{\text{bol}} \)) at any time because the temperature is either too high or recombination kicks in. These factors prevent standard progenitors from being able to explain the first peak in double-peaked SNe.

Non-standard progenitors. We show that progenitors with an extended, low-mass material on top of a compact, massive core naturally produce double-peaked SNe.
first peak is dominated by the cooling of the shock-heated extended material, and the second peak is the radioactive decay of $^{56}$Ni in the core. We show that the following properties can be constrained.

1. The time of the peak provides a strong constraint on the extended material mass $M_{\text{ext}}$ (eq. [10]).

2. The bolometric luminosity at $t_P$ measures the initial radius of the extended material $R_{\text{ext}}$ (eq. [12]). If only the specific luminosity at one or more optical bands is known, then $R_{\text{ext}}$ can still be constrained (using eq. [13] in addition), although less accurately.

3. The minimal observed luminosity, between the two peaks, sets an upper limit to the core radius $R_{\text{core}}$ (eq. [14]).

Note that the time of the minimum between the two peaks is dominated by the decay rate of the first peak, and thus by $M_{\text{ext}}$ and not by $R_{\text{ext}}$. This is consistent, for example, with the result of Van Dyk et al. (2014), which find that the radius of the progenitor of SN 1993J is comparable to, or larger than, that of SN 2013df, even though the emission following the first peak of the latter decays more slowly.

The observed signatures that we discuss here are insensitive to the exact density profile of the extended material. The point where the details of the structure affect the light curve is the rise to the first peak. When the extended material is in hydrostatic equilibrium, then the light curve before the first peak is expected to follow the planar and spherical phases of a standard progenitor that we discussed here. Thus, very early future observations of double-peaked SNe have the potential to detect a third peak on the time scale of $\sim 10$ min after the explosion.

We thank A. Gal-Yam, D. Maoz, E. Ofek, C. Ott, and D. Poznanski for helpful comments. EN was partially supported by an ERC starting grant (GRB-SN 278369) and by the I-CORE Program of the Planning and Budgeting Committee and The Israel Science Foundation (1829/12). ALP is supported through NSF grants AST-1205732, PHY-1068881, PHY-1151197, and the Sherman Fairchild Foundation.

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### TABLE 1

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<th>Ref.</th>
<th>$p_{\text{bol}}$</th>
<th>$L(p_{\text{bol}})$ ($\text{erg s}^{-1}$)</th>
<th>$L_{\text{min}}$ ($\text{erg s}^{-1}$)</th>
<th>$M_{\text{core}}$ ($\text{M}_\odot$)</th>
<th>$M_{\text{ext}}$ ($\text{M}_\odot$)</th>
<th>$R_{\text{ext}}$ ($10^{13}$ cm)</th>
<th>$R_{\text{core}}$ ($10^{13}$ cm)</th>
<th>$M_{\text{ext}}$ ($\text{M}_\odot$)</th>
<th>$R_{\text{ext}}$ ($10^{13}$ cm)</th>
<th>$R_{\text{core}}$ ($10^{13}$ cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B12</td>
<td>0.27</td>
<td>$M_0 = -15.5$</td>
<td>$2 \times 10^{41}$</td>
<td>2.5</td>
<td>$4 \times 10^{-4}$</td>
<td>1.1</td>
<td>1.7</td>
<td>7.1</td>
<td>2.2</td>
<td>&lt; 5</td>
</tr>
<tr>
<td>B12</td>
<td>0.5</td>
<td>$M_0 = -16.2$</td>
<td>$2 \times 10^{41}$</td>
<td>2.5</td>
<td>$2 \times 10^{-3}$</td>
<td>1.4</td>
<td>1.7</td>
<td>$2 \times 10^{-3}$</td>
<td>2.4</td>
<td>&lt; 5</td>
</tr>
<tr>
<td>B12</td>
<td>0.85</td>
<td>$M_0 = -16.8$</td>
<td>$2.5 \times 10^{41}$</td>
<td>2.5</td>
<td>$6 \times 10^{-3}$</td>
<td>1.9</td>
<td>1.7</td>
<td>$6 \times 10^{-3}$</td>
<td>3.5</td>
<td>&lt; 7</td>
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<tr>
<td>W94</td>
<td>3</td>
<td>$L_{\text{bol}} = 10^{43}$</td>
<td>$4 \times 10^{41}$</td>
<td>2.23</td>
<td>$4 \times 10^{-2}$</td>
<td>3.86</td>
<td>2</td>
<td>$6 \times 10^{-2}$</td>
<td>2</td>
<td>&lt; 9</td>
</tr>
</tbody>
</table>

A comparison of the analytic formula provided in this paper to numerical simulations presented in Bersten et al. (2012) ($R_{\text{ext}} = 270, 200,$ and 150 $R_\odot$) and Woosley et al. (1994) (model 13B). The numerical values include the relevant initial conditions and results of the simulations. The analytic values are calculated using equations (10), (12), and (14) with initial conditions taken from the numerical simulations. The agreement of the numerical and analytical results is better than a factor of 2 (see discussion in the text).

- The extended material velocity is used in equations (10) and (12) when provided ($v_{\text{ext}} = 10^3 \text{ cm s}^{-1}$) in model 13B of Woosley et al. (1994). Otherwise, $E_{\text{51}}$ and $M_{\text{core}}$ (from Bersten et al. 2012) are used in these equations.
- The time of the first optical peak.
- The bolometric luminosity (from Woosley et al. 1992) or $g'$-band absolute magnitude (from Bersten et al. 2012) at the first peak.
- The pre-explosion mass within the radius range of $R_{\text{ext}}/3$ to $R_{\text{ext}}$ (see text for discussion).