Progress Report for July - November 1949

MODEL STUDIES OF
MOBILE BREA MWATERS

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The cover shows the Hydraulic Structures Laboratory in Azusa, California. The building is a modified Army Advance Base-Hangar.
This report covers further work on the refraction of waves by opposing currents, and is essentially an elaboration of earlier results. Two types of currents have been considered, a horizontal flow of substantially constant velocity from surface to bottom, and a horizontally stratified flow characterized by high velocities near the surface and relatively low velocities in the opposite direction across the rest of the vertical section.

For the first type of current, a revised theoretical treatment is presented, together with experimental results which are in good agreement with the theory. For the second type of current, experimental results have been obtained for a range of current intensities much higher than would be practical in a prototype installation.

The results of these experiments do not change the earlier conclusion that the refraction of waves by opposing currents, and hence the pneumatic breakwater, is not a practical means for materially reducing wave disturbances. These experiments do extend the experimental investigation to cover the complete range of wave types, from deep-water to shallow-water and of current values to well over any conceivable prototype range.
A revised theory applicable to either deep- or transition, or shallow-water waves has been developed which substantially modifies the conclusions reached from the preliminary theory for current refraction of shallow-water waves presented in the Progress Report for May-June 1949, and reduced to the Scripps Institution theory for the case of deep-water waves.

This theory predicts the shortening of wave length and the current velocity required to stop given waves, but is not concerned with change in wave height, hence the theory is developed from kinematic considerations and is independent of energy relations. It is interesting to note, however, that the current velocity required to stop all waves predicted by this theory is in exact agreement with the value obtained by energy considerations.

The theory is developed as follows:

Let the wave length, period and velocity in the undisturbed region be \( L_0 \), \( T_0 \) and \( C_0 \), the same quantities in the current region be \( L \), \( T \) and \( C \), and the current velocity be \( V \).

Then, by definition:

\[
T_0 = \frac{L_0}{C_0}
\]

and

\[
T = \frac{L}{C+V}
\]

After a steady-state has been established, the wave period must be the same in the undisturbed and disturbed regions, or:

\[
\frac{L_0}{C_0} = \frac{L}{C+V}
\] (1)
In the general case, \( C \) will be given by the complete form of the Airy wave equation:

\[
C = \sqrt{\frac{gL}{2\pi}} \tanh \frac{2\pi d}{L}
\]

where \( d \) is the water depth.

This may be re-written:

\[
C = \sqrt{gd} \sqrt{\frac{\tanh \frac{2\pi d}{L}}{2\pi d/L}}
\]

and:

\[
\frac{L}{L_0} = \sqrt{\frac{gd}{C_0}} \sqrt{\frac{\tanh \frac{2\pi d}{L}}{2\pi d/L}} + \frac{V}{C_0}
\]

Note that when the waves are initially of the deep-water type, the effect of depth may be neglected, and

\[
C_0 = \sqrt{\frac{gL_0}{2\pi}}
\]

\[
C = \sqrt{\frac{gL}{2\pi}}
\]

hence

\[
\frac{L}{L_0} = \left( \frac{C}{C_0} \right)^2
\]

and, from eq. (2);

\[
\left( \frac{C}{C_0} \right)^2 = \frac{C}{C_0} + \frac{V}{C_0}
\]

as in the Scripps Institution derivation.

Returning to eq. (4), we have a transcendental equation which can be solved by trial, since it is assumed that the initial wave parameters \( L_0 \) and \( C_0 \) are known, as are the current velocity and water depth. For convenience of computation, let \( \tanh \frac{2\pi d}{L} = K \), then:

\[
\frac{L}{L_0} = \sqrt{\frac{gd}{C_0}} \sqrt{K} + \frac{V}{C_0}
\]
For values of $\frac{V}{c_o}$ less than the critical value for which no waves propagate into the stream, this quadratic equation has two solutions for the variable $\frac{L}{L_o}$; at the critical value the equation has but one solution, and for greater values, no real solutions can be obtained. This behavior is illustrated in Fig. 1, in which the results for a hypothetical case are plotted. The fact that the slope of the curve $\left( \frac{\Delta \frac{V}{c}}{\Delta \frac{L}{L_o}} \right)$ is zero at the critical current value suggests the possibility of an analytical solution for the critical current value. This may be investigated as follows:

Rearranging eq. (1), and differentiating, we have,

$$\frac{d}{dL} \frac{V}{c_o} = 1 - \frac{d}{dL} \frac{c}{c_o} = 0$$

(6)

or:

$$\frac{L}{c_o} \frac{dC}{dL} = 1$$

$$\frac{dC}{dL} = \frac{c_o}{L} = \frac{1}{T_o}$$

(7)

Performing the differentiation indicated:

$$\frac{1}{2} \left( \frac{gL}{2\pi} \tanh \frac{2\pi d}{L} \right)^{-\frac{1}{2}} \left[ \frac{gL}{2\pi} \tanh \frac{2\pi d}{L} + \frac{gL}{2\pi} \text{sech}^2 \frac{2\pi d}{L} \left( -\frac{2\pi d}{L^2} \right) \right] = \frac{1}{T_o}$$

(8a)

$$\frac{1}{2} \left[ \frac{1}{L} C^2 - \frac{2\pi d}{L^2} \frac{2\pi d}{\text{sech}^2 \frac{2\pi d}{L}} \right] = \frac{1}{T_o}$$

(8b)

$$\frac{C}{2L} \left[ 1 - \frac{L\pi d/L}{\sinh \frac{L\pi d}{L}} \right] = \frac{1}{T_o}$$

(8c)

$$\frac{C}{2L} \left[ 1 - \frac{L\pi d/L}{\sinh \frac{L\pi d}{L}} \right] = \frac{L}{T_o}$$

(8d)

$$\frac{C}{2L} \left[ 1 - \frac{L\pi d/L}{\sinh \frac{L\pi d}{L}} \right] = \frac{L}{T_o}$$

(8e)
\[ C - \frac{C}{2} \left[ 1 + \frac{L \mu d}{L} \frac{1}{\sinh \frac{L \mu d}{L}} \right] = \frac{L}{T_0} \]  

(8f)

Now, the second term on the left hand side of eq. (8f) is the group velocity, \( C_g \), of the wave in the current region, or

\[ \frac{L}{T_0} = C - C_g \]  

(8g)

Noting that eq. (2) may be written

\[ V + C = \frac{L}{T_0} \]  

(9)

we have: \( V + C = C - C_g \)

\[ V = - C_g \]

(10)

or, the opposing current velocity required to stop a given wave train, corresponding to the point on the \( \frac{L}{T_0} \), \( \frac{V}{C} \), curve where \( \frac{d C}{d \frac{L}{T_0}} = 0 \), is equal to the group velocity of the wave train in this current.

It is of course impossible to use this result to calculate critical current velocities, but it is of great interest to note that this conclusion can also be reached from energy considerations. The rate of energy advance in a wave train is equal to the group velocity, hence as long as the current velocity is less than this value, there is some net forward transfer of energy and the wave will propagate into the stream, but when the opposing current velocity equals the rate of energy transfer with respect to the water, then there can be no further advance of energy with respect to a fixed reference system and the waves no longer propagate upstream.

Returning to eq. (5), it should be remarked that this equation can always be solved by trial to calculate either the change in
wave length for given current conditions, or by constructing a plot such as Fig. 1, to find the critical current to completely stop the given initial wave train. The lower of each set of simultaneous values of \( \frac{L}{L_0} \) for a given \( \frac{V}{V_0} \) will never be obtained in practice, since the wave length reaches and stays at the higher value in the physical process of shortening, hence the lower branch of the curve has no physical significance.

B. Experimental Results

1. Technique

The experimental work was conducted in a special channel adapted from the main model basin. The model basin at the Azusa laboratory is equipped with a peripheral trench and a 2½-inch propeller pump with intake near the bottom of the trench, and this equipment formed the nucleus of the special channel required. The section of trench upstream of the pump was covered at the model basin floor level with steel plates and plywood sheets, except for a section 6 feet long located 30 feet from the pump. This open section was partially closed by 65 wooden slats 1" x 1" x 3/4", placed transverse to the channel. The resulting series of narrow slats constituted sufficient restriction to insure a uniform pressure gradient across the full length of the opening. Temporary metal walls were installed to produce a channel 4 feet wide, closed by a wave machine at the pump end and open to the model basin at the other end. By this means, the pump circulated water from the model basin, through the channel above the cover plates to the slotted section, thence through the trench beneath the cover plates to the pump and back to the model.
basin. This resulted in a channel of approximately constant depth in which the velocity was nearly zero for a distance of 22 feet from the wave machine, and then of any value up to 3.5 ft/sec, depending on pump speed. Thus, waves were generated in still water and propagated into the current, enabling simultaneous measurements of initial and refracted conditions. Fig. 2 illustrates this channel.

Due to the back water curve associated with the flow in the channel, the water depth could not be made the same in the still-water and current regions, thus the waves experienced refraction both by the current and by the changing water depth. The effect of varying depth can be accounted for in the analysis, however, permitting valid comparisons of predicted and observed current refraction effects.

The measurement of wave lengths was accomplished by photographing wave profiles against a gridded wall section. A 4" x 5" still camera was used in conjunction with a 5-foot long electronic flash tube and reflector for these photographs. Fig. 3 shows some examples of the data obtained.

2. Results

Experiments were conducted to determine the validity of the theory for a range of wave types from the deep-water end of the transition region \( \frac{L}{d} = 2.25 \) through the transition region \( \frac{L}{d} = 10 \) and \( \frac{L}{d} = 17.5 \) to shallow-water waves \( \frac{L}{d} = 25 \). For each wave type, measurements were made of the refracted wave length corresponding to several values of opposing current velocity less than the critical, and the critical velocity for no wave propagation was determined.
Table 1

Summary of Refracted Wavelength Measurements

<table>
<thead>
<tr>
<th>$\frac{L_0}{d}$</th>
<th>$\frac{V}{V_0}$</th>
<th>$\frac{d'_*}{d}$</th>
<th>$\frac{L}{L_0}$ Experiment</th>
<th>$\frac{L}{L_0}$ Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td>-.093</td>
<td>1.0</td>
<td>.94</td>
<td>.98</td>
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<tr>
<td></td>
<td>-.113</td>
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<td>.85</td>
<td>.84</td>
</tr>
<tr>
<td></td>
<td>-.170</td>
<td>1.0</td>
<td>.65</td>
<td>.64</td>
</tr>
<tr>
<td></td>
<td>-.226</td>
<td>1.0</td>
<td>.60</td>
<td>.54</td>
</tr>
<tr>
<td>10.5</td>
<td>-.188</td>
<td>1.0</td>
<td>.83</td>
<td>.78</td>
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<tr>
<td></td>
<td>-.374</td>
<td>.98</td>
<td>.49</td>
<td>.50</td>
</tr>
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<td></td>
<td>-.438</td>
<td>.92</td>
<td>.27</td>
<td>.35</td>
</tr>
<tr>
<td>17.5</td>
<td>-.177</td>
<td>1.0</td>
<td>.77</td>
<td>.79</td>
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<td>-.275</td>
<td>.98</td>
<td>.67</td>
<td>.68</td>
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<td>-.372</td>
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<tr>
<td></td>
<td>-.663</td>
<td>.84</td>
<td>.32</td>
<td>.26</td>
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</tbody>
</table>

Table 2

Summary of Critical Current Velocity Determination Experiments

<table>
<thead>
<tr>
<th>$\frac{L_0}{d}$</th>
<th>$\frac{d'_*}{d}$</th>
<th>$\frac{V}{C_0}$ critical Experimental</th>
<th>$\frac{V}{C_0}$ critical Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>.98</td>
<td>-.307</td>
<td>-.29</td>
</tr>
<tr>
<td>11.3</td>
<td>.93</td>
<td>-.474</td>
<td>-.47</td>
</tr>
<tr>
<td>19.7</td>
<td>.86</td>
<td>-.555</td>
<td>-.55</td>
</tr>
</tbody>
</table>

$d'_* \over d$ is the ratio of the water depth in the stream to that in still water. This change accounts for some refraction and is accounted for in the calculations.
The results of these experiments are shown in Tables 1 and 2 where they are compared with calculated values obtained by numerical solution of eq. (5).

The deviation of experimental and theoretical results is due partly to experimental error in the measurement of the water depth in the current region and in the measurement of the length of long, flat crested waves, and partly to the fact that the theory is based on the linear Airy wave equation which is strictly valid only for wave heights very small compared with the water depth. However, the good agreement of experiment and theory indicates that this simple theory is sufficiently accurate for the prediction of prototype behavior except in extreme cases of very large original wave heights relative to water depth.

No measurements were made of the change in wave height, since it was apparent that the relatively long wavelength transition and shallow-water waves which were of greatest interest in this study did not obey the simple conservation of energy law assumed in the original analysis for the refraction of deep-water waves. Instead, these waves behave as suggested in the April-May Progress Report, partially breaking at initial contact with the current and continuing to propagate at all current velocities up to the critical with moderate amplitude. This behavior is well illustrated by Fig. 3d, the breaking wave being visible in the left portion of the picture while a wave of about the original wave height but of shorter length propagates into the stream.

Qualitative observations of the presence of standing waves in the still-water region suggest that some of the wave energy is reflected at the boundary of the current and the still-water regions,
but this partition of the energy loss between reflection and turbulence was not investigated quantitatively.
III. REFRACTION OF WAVES BY STRATIFIED CURRENT

A. Theoretical Considerations

The effect of a surface current on waves has been of continuing interest on this contract since it has been shown that such a current is the only phenomenon associated with the pneumatic breakwater that can account for an appreciable performance of that device.

The theoretical basis for the effect of a surface current on deep-water waves has been given by G.I. Taylor, as summarized in the Progress Report for June, 1949, and the problem has been solved in analytic form for the case of shallow-water waves by the Laboratory's consultants, but difficulties of computation have prevented the presentation of this theory in a usable form.

The outstanding result of this theoretical work and previous experimental work has been the observation that a surface current is tremendously more effective in stopping deep-water than shallow-water waves. It is possible to analyse this result without recourse to complicated mathematical theory by a consideration of the physical processes of wave motion in relation to a surface current. The most fundamental aspect of wave motion is not the undulating water surface with its succession of advancing crests and troughs, but is rather the motion of the particles of water from surface to bottom, of the region in which the wave motion exists. Thus, deep-water and shallow-water waves may present identical surface appearances, but their true differences are at once apparent upon examination of their respective particle motions. Ignoring minor deviations, the water particles in a deep-water wave may be considered to describe
circular orbits whose amplitude decreases exponentially from the surface downwards. The amplitude at the surface is equal to the wave height, and the rate of decrease is such that there is no sensible motion at a depth of one-half the wave length. On the other hand, for a shallow-water wave whose surface appearance, length and period are identical with the deep-water wave just considered, the water particles move in elliptical orbits, the horizontal major diameters being of substantially constant length from surface to bottom while the vertical minor diameters decrease linearly from a value equal to the wave height at the surface to zero at the bottom. The magnitude of the horizontal diameter of the water particle orbits depends on the ratio of wave length to water depth, a typical value for ocean waves that have advanced into shallow coastal water where \( \frac{L}{d} = 12 \) being twice the wave height.

If we now consider the effect of a surface current on the flow regimes characteristic of these wave types, it is easy to understand the wide difference in behavior observed for the two conditions. When the deep-water wave encounters a surface current, the entire forward momentum of the wave is influenced by the surface current since all of the moving water particles that contribute to the momentum of the wave are concentrated near the surface, and as a result, the momentum of the current is efficiently utilized to oppose the wave momentum. On the other hand, in the shallow-water wave the momentum is due to the movement of all the water particles from surface to bottom, and moreover each particle is moving with an average horizontal velocity approximately twice that reached by the surface particle of the deep-water wave. A surface current, therefore, can influence the upper portions of the wave flow to a
much lesser extent than it could a deep-water wave, and does not influence the equally important deeper portion of the wave flow at all, with the result that the momentum of the shallow-water wave is little changed and the wave can propagate into the current with small loss of energy.

B. Experimental Studies

An additional reason for the continuing interest in surface currents as a wave damping system is that such a current is produced by the relatively simple procedure of creating a vertical current in the fluid region. This is in fact, the effect of the pneumatic breakwater, the rising bubbles acting as an air lift pump to produce a vertical water current above the air discharge pipe. This observation naturally led to the suggestion, both here and in England, that the same result could be achieved more efficiently by the direct mechanical production of a vertical water jet or current from the ocean bottom.

The object of the present series of experiments was, therefore, to model the ultimate that could be expected of any practical development along these lines.

1. Experimental Technique

The experiments were conducted in the 4-foot wide wave channel at the Azusa Laboratory. This channel is provided with a false bottom 5 inches above the concrete laboratory floor, which permitted the installation of sheet metal suction and discharge ducts with openings flush with the channel bottom. These ducts delivered
water to and from a \( \frac{1}{4} \)-inch propeller pump mounted outside the channel; since the pump and channel together constitute a closed system, the water level in the channel remained constant during a run. The channel was subdivided by a temporary wall 32 feet long into parallel 1-foot and 3-foot wide sections, the discharge opening being in the 1-foot and the symmetrically located suction openings in the 3-foot section.

The 12-inch wide discharge duct was provided with movable covers plates to form a discharge slot or orifice adjustable from zero to \( \frac{1}{4} \) inches, and the 20-inch wide suction duct openings were covered with expanded metal mesh to reduce the effect on wave motion of the sudden change in depth which they entailed. In operation, the vertical jet from the discharge orifice produced symmetrical outward-flowing surface currents in the 1-foot channel section, the velocity and depth being augmented near the point of issuance of the jet by the vertical eddy induced by the high-speed jet. The surface currents decay in velocity and depth at increasing distances from the jet, their momentum being spread uniformly across the entire channel section by mixing, and the circulation is completed by a very low velocity movement of water from the 1-foot channel section to the 3-foot section and finally into the suction ducts. Fig. 4 shows the general arrangement of the experimental facilities.

The characteristics of the surface flow and discharge jet were investigated with a Prandtl-type pitot tube and precision manometer reading to .001-foot of water. Velocity profiles for two discharge conditions are shown in Fig. 5.
The effect of the surface current on wave trains was measured by means of conductivity elements as described in previous reports of this laboratory. Elements were positioned on either side of the discharge orifice in the 1-foot channel section to measure the waves influenced by the current, and in the relatively undisturbed 3-foot channel for reference measurement of the undisturbed waves.

2. Results

The first part of this investigation was the determination of the effect of vertical jet parameters (jet width and velocity) on the surface current produced. It was determined that a limiting value of jet velocity existed, above which the jet broke into violent transverse oscillation and generated large amplitude wave trains. This phenomenon was not investigated further due to lack of time, and also to the fact that this critical velocity was of the order of the velocity of a shallow-water wave in the water depth used, hence of greater than practical magnitude for any prototype installation.

For jet velocities below this critical value, it was observed that the velocity and depth of the surface current are substantially independent of total discharge (cu ft/sec), but are a function of jet velocity. This relation is shown in Fig. 5, where the surface currents are nearly identical and the jet velocities are nearly the same, although the ratio of total discharge for the two cases is nearly 2 to 1. This observation indicates that the larger portion of the surface current is due to the vertical eddy induced by the jet, which augments the pump flow for a distance outwards from
the center, then sinks and returns beneath the surface to again mix with the jet. Time was not available to determine the optimum discharge conditions for most efficient surface current production, hence activities along these lines ceased with the determination of the probable maximum discharge conditions that could be expected in any prototype installation.

In the second part of these investigations, measurements were made of the effect of maximum practical surface currents on a series of wave trains which included deep-, transition, and shallow-water waves. Typical results of these measurements are shown in Fig. 6, where the ratio of transmitted to incident wave amplitude for various wave types interacting with the two currents shown in Fig. 5 are plotted. It is readily apparent that the results are similar to previous measurements with models of the pneumatic breakwater and of lower energy flow devices, appreciable reduction of wave amplitude being accomplished only for deep-water waves.
IV. CONCLUSIONS

1. A simple kinematic theory has been developed which permits the calculation of the change of wave length when a gravity wave of any type encounters a uniform current, and which predicts the current velocity required to halt completely the propagation of waves of specified characteristics. This theory has been amply verified by experimental measurements.

2. It does not appear possible to calculate current velocities to cause wave-breaking for transition or shallow-water waves, since it has been observed that the assumption of constant rate of energy transfer up to the breaking point is not valid for these waves. Rather, a definite energy loss by partial breaking and reflection is observed for current velocities less than, but approaching the critical.

3. Laboratory experiments with a surface flow barrier indicate that with a driving jet of great size and velocity (20 per cent of the water depth in width, and 60 per cent of the shallow-water wave speed in velocity), no significant wave attenuation is realized for transition or shallow-water waves. With a prototype depth of 50 feet, this jet would be 10 feet wide and have a velocity of 24 ft/sec, with a flow of 240 ft³/sec per foot of breakwater and an energy content of 24.5 HP per foot of breakwater.
4. A qualitative analysis of the mechanics of the various types of wave motion indicates that whereas deep-water waves may be greatly influenced by surface phenomena, any device or process intended to influence transition or shallow-water waves must act across the entire fluid section, from surface to bottom.
Solution of the Refraction Equation

\[
\frac{L}{L_0} = \frac{\sqrt{c_0^2 + 4c_a^2}}{c_a} K + \frac{\sqrt{c_0^2}}{c_a}
\]

For: \( L_0 = 500 \text{ ft} \)
\( d = 50 \text{ ft} \)

Change in wave length, \( \frac{L}{L_0} \)

Ratio of current to original wave velocity, \( \frac{V}{c_a} \)

Fig. 1 - Effect of Opposing Current on Typical Wave Train

Fig. 2 - Arrangement of Facilities for Current Refraction Studies
Fig 3 - Refraction of Waves by Opposing Current

(a) $\frac{V}{C_0} = -0.093$

(b) $\frac{V}{C_0} = -0.226$

(c) $\frac{V}{C_0} = -0.349$

(d) $\frac{V}{C_0} = -0.663$

$T_0 = 0.7$ sec

$L_0 \frac{d}{d} = 2.25$

$T_0 = 4.5$ sec

$L_0 \frac{d}{d} = 25.0$
Fig. 4 – Arrangement of Facilities for Surface Current Studies

Distance from orifice in per cent of water depth (not to scale)

Width of orifice, 2 inches
\[ V_{\text{Jet}} = 3.25 \, \text{ft/sec} \]

Width of orifice, 1 inch
\[ V_{\text{Jet}} = 3.62 \, \text{ft/sec} \]

Fig. 5 – Horizontal Velocity Profiles due to Vertical Water Jet
Fig. 6 - Effect of Surface Currents shown in Fig. 5 on Waves of Various Periods.