Measurement of the $B \to X_s \ell^+ \ell^-$ branching fraction and search for direct CP violation from a sum of exclusive final states

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We measure the total branching fraction of the flavor-changing neutral-current process \( B \rightarrow X_s \ell^+\ell^- \), along with partial branching fractions in bins of dilepton and hadronic system (\( X_s \)) mass, using a sample of 471 \( \times \) 10^6 \( \Upsilon(4S) \rightarrow B\bar{B} \) events recorded with the \( B\bar{B} \) detector. The admixture of charged and neutral \( B \) mesons produced at PEP-II are reconstructed by combining a dilepton pair with 10 different \( X_s \) final states. Extrapolating from a sum over these exclusive modes, we measure a lepton-flavor-averaged inclusive branching fraction \( \mathcal{B}(B \rightarrow X_s \ell^+\ell^-) = (6.73^{+0.70}_{-0.64}[\text{stat}]^{+0.34}_{-0.25}[\text{exp syst}] \pm 0.50[\text{model syst}]) \times 10^{-6} \) for \( m_{\ell^+\ell^-} > 0.1 \text{GeV}/c^2 \). Restricting our analysis exclusively to final states from which a decaying \( B \) meson’s flavor can be inferred, we additionally report measurements of the direct \( CP \) asymmetry \( A_{CP} \) in bins of dilepton mass; over the full dilepton mass range, we find \( A_{CP} = 0.04 \pm 0.11 \pm 0.01 \) for a lepton-flavor-averaged sample.

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The \( b \rightarrow s\ell^+\ell^- \) transition, where \( b \) is a bottom quark, \( s \) is a strange quark, and \( \ell^+\ell^- \) is an \( e^+e^- \) or \( \mu^+\mu^- \) pair, is forbidden at lowest order in the standard model (SM) but is allowed at one loop via electroweak penguin and \( W \)-box diagrams. The amplitude for this decay is expressed in terms of perturbatively calculable effective Wilson coefficients, \( C_{\text{eff}}, C_{9\text{eff}} \), and \( C_{10\text{eff}} \), which represent the electromagnetic penguin diagram, and the vector part and the axial-vector part of the linear combination of the \( Z \) penguin and \( W^+W^- \) box diagrams, respectively [1]. Non-SM contributions can enter these loops at the same order as the SM processes, modifying the Wilson coefficients from their SM expectations and allowing experimental sensitivity to possible non-SM physics [2–11].

We study the inclusive decay \( B \rightarrow X_s \ell^+\ell^- \), where \( X_s \) is a hadronic system containing exactly one kaon, using a sum over exclusive final states, which provides a basis for extrapolation to the fully inclusive rate. We measure the total branching fraction (BF), as well as partial BFs in five disjoint dilepton mass-squared \( q^2 \equiv m_{\ell^+\ell^-}^2 \) bins and four hadronic mass \( m_{X_s} \) bins, which are defined in Table I. We additionally search for direct \( CP \) violation in the same \( q^2 \) bins. The relative precision of our results is approximately a factor of two better than the combined precision of all similar previously published measurements [12].

The \( X_s \) system in the lowest mass \( m_{X_s} \) bin \( m_{X_s,1} \) contains a single kaon with no other hadrons present; the \( m_{X_s,2} \) bin is populated only above the \( K\pi \) threshold. Results are also reported in an additional \( q^2 \) region \( q^2 = 1 < q^2 < 6 \text{GeV}^2/c^4 \), i.e., the perturbative window away from the photon pole at low \( q^2 \) and the \( \epsilon \tau \) resonances at higher \( q^2 \), where theory uncertainties are well controlled [13–24]. The most recent SM predictions in this region are \( B_{\text{low}}(B \rightarrow X_s \mu^+\mu^-) = (1.59 \pm 0.11) \times 10^{-6} \) and \( B_{\text{low}}(B \rightarrow X_s e^+e^-) = (1.64 \pm 0.11) \times 10^{-6} \) [22].

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Theory uncertainties in the $q^2$ range above the $\psi(2S)$ are also well-characterized but relatively much larger than above, with SM predictions for $q^2 > 14.4 \text{GeV}^2/c^4$ of $B^\text{high}(B \rightarrow X_s \mu^+\mu^-) = (0.24 \pm 0.07) \times 10^{-6}$ and $B^\text{high}(B \rightarrow X_s e^+e^-) = (0.21 \pm 0.07) \times 10^{-6}$ [22]. The SM expectation in the $q^2 > 4m_B^2$ range is $B(B \rightarrow X_s \ell^+\ell^-) = (4.6 \pm 0.8) \times 10^{-6}$ [20]. Direct CP violation, defined as $A_{CP} = (B(B_b^\rightarrow B_b) - \overline{B}(B_b^\rightarrow B_b))/\overline{B}(B_b^\rightarrow B_b + \overline{B}(B_b^\rightarrow B_b))$, where $b(\overline{b})$ denotes a $B(\overline{B})$ parent, is expected to be suppressed well below the $1\%$ level in both exclusive and inclusive $b \rightarrow s\ell^+\ell^-$ transitions [25–28]; however, in beyond-SM models with four quark generations, significant enhancements are possible, particularly in the high-$q^2$ region [10, 11].

The BaBar [29] and Belle [30] Collaborations have previously published $B \rightarrow X_s \ell^+\ell^-$ BFs based on a sum over exclusive final states using only $\sim 25\%$ of each experiment’s final dataset. More recently, both collaborations (along with LHCb and CDF) have published BFs, and time-integrated rate and angular asymmetries, for the exclusive decays $B \rightarrow K^{(*)}\ell^+\ell^-$ [31–37]. The present analysis uses the $424.2 \pm 1.8 \text{fb}^{-1} e^+e^- \rightarrow \Upsilon(4S)$ data sample [38], corresponding to $\sim 471$ million $BB\overline{B}$ pairs, collected with the BaBar detector [39, 40] at the PEP-II collider at the SLAC National Accelerator Laboratory.

The decays $B \rightarrow X_s \ell^+\ell^-$ are reconstructed in 10 separate $X_s$ hadronic final states ($K^+\pi^0\pi^0$, $K^{+}\pi^{-}\pi^{0}$, $K^{0}\pi^{0}\pi^{0}$, $K^{0}\pi^{0}\pi^{+}$, $K^{0}\pi^{0}\pi^{-}$, and $K^{0}\pi^{0}\pi^{0}$) [41], combining these with an $e^+e^-$ or $\mu^+\mu^-$ pair for a total of 20 final states. The selection of charged and neutral particle candidates, as well as the reconstruction of $\pi^0 \rightarrow \gamma\gamma$ and $K^{0}\rightarrow \pi^+\pi^-$, is described in Refs. [31, 36]. Based on studies including up to 18 $X_s$ modes with a maximum of four pions and $m_{X_s}$ as large as $2.2 \text{ GeV}/c^2$, we limit the number of $X_s$ final states to the 10 listed above and require $m_{X_s} < 1.8 \text{ GeV}/c^2$ since the expected signal-to-background ratio rapidly decreases with increasing $X_s$ pion multiplicity and mass.

We assume that the fraction of modes containing a $K^{0}_S$ is equal to that containing a $K^{+}\pi^{-}$ and account for these decays, as well as $K^{0}_S \rightarrow \pi^0\pi^0$ and $\pi^0$ Dalitz decays, in our reconstruction efficiencies. With these efficiencies taken into account, the reconstructed states represent $\sim 70\%$ of the total inclusive rate.

We account for missing hadronic final states, as well as for states with $m_{X_s} > 1.8 \text{ GeV}/c^2$, based on the formalism of Refs. [8, 13, 22, 42–44], with hadronization of the $X_s$ system provided by the JETSET [45] event generator. Given that we observe no statistically significant non-resonant $B \rightarrow K\pi\ell^+\ell^-$ decays in our data [31], signal decays with a two-body $X_s$ system and $m_{X_s} < 1.1 \text{ GeV}/c^2$ are assumed to proceed through the $K^{*}(892)$ resonance. The simulation of such events, as well as those with a single kaon and no pions, is similar to that for inclusive events but incorporates the form factor models of Refs. [46, 47].

The kinematic variables $m_{ES} = \sqrt{E_{CM}^2/4 - p_B^2}$ and $\Delta E = E_B^* - E_{CM}/2$, where $p_B^*$ and $E_B^*$ are the $B$ momentum and energy in the $\Upsilon(4S)$ center-of-mass (CM) frame with $E_{CM}$ the total CM energy, are used to distinguish signal from background events. We require $m_{ES} > 5.225 \text{ GeV}/c^2$ and $-0.10 < \Delta E < 0.05 \text{ GeV}$ ($-0.05 < \Delta E < 0.05 \text{ GeV}$) for dielectron (dimuon) final states. Signal-like $B$ backgrounds with $J/\psi(\psi(2S))$ daughters are removed by vetoing events with $6.8 < q^2 < 10.1 \text{ GeV}^2/c^4$ ($12.9 < q^2 < 14.2$). We reconstruct $X_s h^+\mu^-$ final states, where $h$ is a track with no particle identification (PID) requirement applied, to characterize backgrounds from hadrons misidentified as muons. Such backgrounds occur only in dimuon final states because of the significantly higher probability to misidentify $K^+$ or $\pi^+$ as a muon rather than an electron. Similarly, backgrounds from $B \rightarrow D(\rightarrow K^{(*)}\pi)\pi$ decays occur only in dimuon modes and, assigning the pion mass hypothesis to both muon candidates, we reject candidates with $K^{(*)}\pi$ mass values in the range $1.84 < m_{K^{(*)}\pi} < 2.04 \text{ GeV}/c^2$.

We suppress $e^+e^- \rightarrow q\overline{q}$ events (where $q$ is a $u, d, s$ or $c$ quark) and $BB\overline{B}$ combinatoric backgrounds using boosted decision trees (BDTs) [48, 49] identical in construction to those used in our $B \rightarrow K^{(*)}\ell^+\ell^-$ analysis [31]. These BDTs are respectively trained with simulated $uds\overline{c}$ or $BB\overline{B}$ backgrounds and correctly reconstructed signal events. Ensembles of simulated event samples are used to simultaneously optimize the $\Delta E$ windows and selection on the $uds\overline{c}$ BDTs for each individual $q^2$ and $m_{X_s}$ bin. After all selection criteria are applied, the average multiplicity of $B$ candidates per event is approximately $2.6 \pm 2.2$ for $e^+e^-$ ($\mu^+\mu^-$) final states. We allow only one candidate per event, selecting the candidate with the smallest $|\Delta E|$. Signal efficiencies after event selection range from about 1 to 30% depending on mode and the $q^2$ or $m_{X_s}$ bin.

In each $q^2$ and $m_{X_s}$ bin, we extract the signal yield with a two-dimensional maximum likelihood (ML) fit using $m_{ES}$ and a likelihood ratio $L_R$ based on the $BB\overline{B}$ BDT, $L_R \equiv P_S/(P_S + P_B)$, where $P_S$ and $P_B$ are, respectively, probabilities for genuine-signal and $BB\overline{B}$ backgrounds. For correctly reconstructed signal events, $L_R$ sharply peaks near one, while $BB\overline{B}$ backgrounds peak at zero. Events with $L_R > 0.42$ are selected. This selection rejects greater than 95% of the $BB\overline{B}$ background events remaining after all other event selections have been applied, with only a trivial reduction in signal efficiency.

Five (six) event classes contribute to the dielectron (dimuon) ML fit: (1) correctly reconstructed signal; (2) events that contain a partially or incorrectly reconstructed $B \rightarrow X_s \ell^+\ell^-$ decay (signal cross-feed); (3) $uds\overline{c}$ and (4) $BB\overline{B}$ combinatorial backgrounds; (5) charmonium backgrounds; and, for dimuon modes, (6) events with hadrons misidentified as muons.

There is no correlation between $m_{ES}$ and $L_R$ for correctly reconstructed signal events. Therefore, the probability distribution function (PDF) for these events is chosen as a product of two one-dimensional (1D) PDFs,
with $m_{ES}$ parameterized with a Crystal Ball (CB) function [50–52] and $L_R$ described by a non-parametric histogram PDF. The CB shape parameters are fixed using simulated signal events, as is the $L_R$ PDF. These PDFs describe well the $m_{ES}$ and $L_R$ distributions derived from the high-statistics control samples of vetoed signal-like charmonium events. The signal cross-feed is modeled as a two-dimensional (2D) $m_{ES}$ versus $L_R$ histogram PDF using simulated signal samples, with normalization $N_{xsf}$ scaled as a fixed fraction of the fit signal yield $N_{sig}$.

The udsc combinatoric background PDF is derived from simulated events using a 2D non-parametric kernel density estimator with adaptive bandwidth [49, 53, 54], which is validated using data collected with $e^+e^-$ center-of-mass energy 40 MeV below the $Y(4S)$ resonance. The udsc normalization $N_{udsc}$ is obtained by scaling the 43.9 $\pm$ 0.2 fb$^{-1}$ of off-resonance data [38] by the ratio of on- to off-resonance integrated luminosity.

The shape of the 2D PDF for the $B\bar{B}$ combinatoric background is modeled similarly to the udsc background. Its normalization in the $5.225 < m_{ES} < 5.270$ GeV/$c^2$ sideband, where no correctly reconstructed signal events are expected, is obtained by subtracting the $N_{xsf}$, $N_{udsc}$, $N_{SB}^{c_{chm}}$, and $N_{SB}^{h_{had}}$ (for dimuon events) contributions from the total number of sideband events, giving the $B\bar{B}$ yield in the sideband region $N_{SB}^{B\bar{B}}$. We use simulated events to obtain the ratio of the number of $B\bar{B}$ combinatoric events in the $m_{ES} > 5.27$ GeV/$c^2$ signal region to the number in the sideband region to scale $N_{SB}^{B\bar{B}}$ into the expected contribution $N_{B\bar{B}}$ in the full fit region.

Charmonium backgrounds escaping the vetoed $q^2$ regions are similarly described by a 2D kernel estimator, with normalization $N_{chm}$ derived from a fit to the data in the vetoed regions that is extrapolated into the non-vetoed regions. The normalization $N_{had}$ and shape of the 2D PDF for misidentified dimuon events are characterized by a weighted 2D histogram taken directly from data using event-by-event weights obtained from PID control samples [31, 55].

We extract the $N_{sig}$ central value and associated upper and lower limits using the negative log-likelihood (NLL) for $N_{sig}$. We calculate partial BFs taking into account the efficiency for each final state in each $q^2$ and $m_{X_s}$ bin, as well as the multiplicative factors that provide extrapolation to the fully inclusive BFs. The results are shown in Table I, where the fully inclusive total rate and the $m_{X_s}$ binned results include estimated signal contributions in the vetoed charmonium $q^2$ regions. Fit projections for all $q^2$ and $m_{X_s}$ bins are available as supplemental EPAPS material [56], along with a table giving the raw numerical results from our fits. Figure 1 shows our $q^2$ binned results overlaid on the nominal SM expectations derived from our $B \rightarrow X_s \ell^+\ell^-$ signal model. A similar plot for $m_{X_s}$ is included as supplemental material.

We consider systematic uncertainties associated with purely experimental systematic uncertainties and the model-dependent extrapolation to the fully inclusive rate. The experimental systematics can either be additive, affecting the extraction of the signal yield from the data, or multiplicative, affecting the calculation of a BF from an observed signal yield. Sources of multiplicative systematic uncertainty include $B\bar{B}$ counting as well as tracking, PID and reconstruction efficiencies. The only significant additive systematic uncertainties are associated with the PDF parameterizations and normalizations. The total experimental systematic uncertainty is the sum-in-quadrature of the above terms, with the exception that uncertainties related to charged particle tracking efficiencies are assumed to be fully correlated among all charged particles. The evaluation of all experimental systematics is fully described in Ref. [31]. Tables quantifying each individual contribution to the experimental and model-dependent extrapolation systematic uncertainties are available as supplemental EPAPS material [56].

The uncertainty in the extrapolation to the inclusive rate is characterized through variations that attempt to quantify our lack of knowledge of the true dilepton mass-squared distribution and hadronization of the $X_s$ system beyond the specific final states and $m_{X_s}$ range that we observe. We average the most recent $B \rightarrow K^{(*)}\ell^+\ell^-$ BFs [57], excluding BaBar results, and use the latest BaBar result [58] for the ratio of charged-to-neutral $Y(4S) \rightarrow b\bar{b}$ decays, $\Gamma(B^+B^-)/\Gamma(B^0\bar{B}^0) = 1.006 \pm 0.036 \pm 0.031$. Each of these terms is varied by its one-standard-deviation uncertainty. We examine an alternate $m_{X_s}$ transition point of 1.0 GeV/$c^2$ between the $B \rightarrow K^{(*)}\ell^+\ell^-$ and $B \rightarrow X_s \ell^+\ell^-$ models. To account for hadronization uncertainties in $m_{X_s} > 1.1$ GeV/$c^2$ events, we generate 20 simulated datasets with varied JETSET tunings, two different values for the B-meson Fermi motion, and two different b-quark mass values. We take the full spread of the extrapolation factors derived from these variations to estimate this systematic uncertainty. Additionally, for $m_{X_s} > 1.1$ GeV/$c^2$, the fraction of modes with more than one $\pi^0$ is varied around the generator value of 0.20 by $\pm 50\%$; the fraction of modes with either no $\pi^0$ and more than two charged pions, or one $\pi^0$ and more than one charged pion, is varied by $\pm 50\%$ around the $q^2$-dependent generator value; and the fraction of modes with more than one neutral or charged kaon is varied around the generator value of 0.034 by $\pm 50\%$. Contributions from final states with photons that do not come from $\pi^0$ decays but rather from $\eta$, $\eta'$, $\omega$, etc., are expected to be insignificant, and we do not vary the fractions of these decays. Each of the above variations is added in quadrature to obtain the final model-dependent systematic. Table I lists both the experimental and model-dependent systematics.

We calculate the total inclusive rate by summing the $q^2$ results taking into account correlations in the systematic uncertainties and estimating signal contribu-
TABLE I: $B \to X_s e^+ e^-$, $B \to X_s \mu^+ \mu^-$ and $B \to X_s \ell^+ \ell^-$ partial BFs (in units of $10^{-6}$) and $A_{CP}$ by $q^2 (\text{GeV}^2/c^4)$ and $m_{X_c} (\text{GeV}/c^2)$ bin. The number in parentheses after each result is the multiplier which is applied to the measured semi-inclusive rate to account for unreconstructed $m_{X_c} > 1.8 \text{GeV}/c^2$ final states. Estimated contributions from the vetoed charmonium $q^2$ regions are included in the total and $m_{X_c}$ binned results, but not in the total $A_{CP}$. The first uncertainties are statistical, the second experimental systematics and the third model-dependent systematics associated with the multiplicative factor. There are no model-dependent $A_{CP}$ systematics and $A_{CP}$ is not measured as a function of $m_{X_c}$; the multiplicative factors are not used in calculating the total $A_{CP}$.

<table>
<thead>
<tr>
<th>Bin</th>
<th>Range</th>
<th>$B \to X_s e^+ e^-$</th>
<th>$B \to X_s \mu^+ \mu^-$</th>
<th>$B \to X_s \ell^+ \ell^-$</th>
<th>$A_{CP, B \to X_s e^+ e^-}$</th>
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</thead>
<tbody>
<tr>
<td>$q^2_{00}$</td>
<td>$1.0 &lt; q^2 &lt; 6.0$</td>
<td>$1.93^{+0.72}_{-0.45} \pm 0.18$ (1.71)</td>
<td>$1.60^{+0.41}_{-0.39} \pm 0.07$ (1.78)</td>
<td>$1.60^{+0.41}_{-0.39} \pm 0.13$ (1.86)</td>
<td>$-0.06 \pm 0.22 \pm 0.01$</td>
</tr>
<tr>
<td>$q^2_{10}$</td>
<td>$0.1 &lt; q^2 &lt; 2.0$</td>
<td>$3.05^{+0.52}_{-0.49} \pm 0.35$ (1.96)</td>
<td>$1.85^{+0.50}_{-0.48} \pm 0.20$ (2.02)</td>
<td>$2.70^{+0.42}_{-0.41} \pm 0.35$ (1.98)</td>
<td>$-0.13 \pm 0.18 \pm 0.01$</td>
</tr>
<tr>
<td>$q^2_{20}$</td>
<td>$2.0 &lt; q^2 &lt; 4.3$</td>
<td>$0.69^{+0.11}_{-0.28} \pm 0.07$ (1.73)</td>
<td>$-0.15^{+0.01}_{-0.43} \pm 0.14$ (1.80)</td>
<td>$0.46^{+0.26}_{-0.23} \pm 0.07$ (1.70)</td>
<td>$0.42 \pm 0.50 \pm 0.01$</td>
</tr>
<tr>
<td>$q^2_{30}$</td>
<td>$4.3 &lt; q^2 &lt; 6.8$</td>
<td>$0.69^{+0.13}_{-0.29} \pm 0.05$ (1.53)</td>
<td>$0.34^{+0.19}_{-0.06} \pm 0.03$ (1.59)</td>
<td>$0.60^{+0.27}_{-0.25} \pm 0.05$ (1.55)</td>
<td>$-0.45 \pm 0.44 \pm 0.01$</td>
</tr>
<tr>
<td>$q^2_{40}$</td>
<td>$10.1 &lt; q^2 &lt; 12.9$</td>
<td>$1.14^{+0.22}_{-0.40} \pm 0.04$ (1.16)</td>
<td>$0.87^{+0.11}_{-0.47} \pm 0.08$ (1.18)</td>
<td>$1.02^{+0.10}_{-0.06} \pm 0.04$ (1.10)</td>
<td>$-0.57 \pm 0.16 \pm 0.01$</td>
</tr>
<tr>
<td>$q^2_{50}$</td>
<td>$14.2 &lt; q^2$</td>
<td>$0.56^{+0.03}_{-0.19} \pm 0.00$ (1.02)</td>
<td>$0.60^{+0.05}_{-0.29} \pm 0.04$ (1.02)</td>
<td>$0.57^{+0.16}_{-0.15} \pm 0.02$ (1.02)</td>
<td>$0.19 \pm 0.18 \pm 0.01$</td>
</tr>
<tr>
<td>$q^2_{60}$</td>
<td>$q^2_{30} \cup q^2_{50}$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Total: $0.1 < q^2$ $7.69^{+0.82}_{-0.77} \pm 0.50$ $4.41^{+1.31}_{-1.17} \pm 0.42$ $0.27$ $6.73^{+0.70}_{-0.64} \pm 0.25 \pm 0.50$ $0.04 \pm 0.11 \pm 0.01$

---

FIG. 1: Differential BF as a function of $q^2$ for electron (blue circles), muon (black squares) and lepton-flavor-averaged final states (red triangles). The errors correspond to the total uncertainties. The histogram shows the SM expectation, which has uncertainties of approximately 10-30% in different $q^2$ regions. The shaded boxes denote the vetoed charmonium regions. The horizontal spread of points in each bin is meant only to aid visibility.
pected to be trivially small [69, 70], with the same fitting methodology used for the signal $q^2$ bins; we find $A_{CP} = 0.0046 \pm 0.0057$ [stat]. Observing no significant bias, we assign the statistical uncertainty here as the systematic uncertainty for the $A_{CP}$ results. To extract $A_{CP}$ for the full dilepton mass range, we sum the $A_{CP}$ $q^2$ bins; excluding the charmonium veto windows, we find $A_{CP} = 0.04 \pm 0.11$ [stat] $\pm 0.01$ [syst]. We observe no significant asymmetry in any $q^2$ region or for the full dilepton mass range.

In summary, we have measured the total and partial BFs, as well as $A_{CP}$, for the inclusive radiative electroweak process $B \to X_s \ell^+ \ell^-$. Our results are in general agreement with SM expectations with the exception of our partial BF results in the high-$q^2$ region, which show a $\sim 2\sigma$ excess compared to both the SM expectation and the most favored value of the beyond-SM contribution $C_{6}^{BSM}$ advanced to explain recent observations by LHCb [35].

We are grateful to Enrico Lunghi, Tobias Hurth and Tobias Huber for useful discussions, as well as providing dilepton mass-squared theory distributions derived using the most up-to-date corrections. We are additionally grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support $BaBar$. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MES (Russia), MINECO (Spain), STFC (United Kingdom). Individuals have received support from the Marie Curie EIF (European Union) and the A. P. Sloan Foundation (USA).

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The use of charge conjugate reactions is implied unless otherwise indicated.


T. Skwarnicki, DESY-F31-86-02.


A URL OR OTHER METHOD FOR ACCESSING THE SUPPLEMENTAL MATERIAL WILL BE PROVIDED BY APS EDITORIAL STAFF. THE LINK/INFO WILL BE HERE.


Introduction

This Supplemental Material includes:

- Figure 2, plotting the $A_{CP}$ results as a function of $q^2$;
- Figure 3, plotting the differential branching fraction as a function of $m_{X_s}$;
- Table II, giving in each individual $q^2$ and $m_{X_s}$ bin the fitted raw number of signal events $N_{\text{sig}}$, as well as the fitted number of random combinatorial $B\bar{B}$ background events $N_{B\bar{B}}$ present in the signal enhanced region with $m_{ES} > 5.27$ GeV/c$^2$;
- Tables III-VIII, detailing individual contributions to the “additive” and “multiplicative” branching fraction systematics (as defined in the article main text), and the model-dependent extrapolation systematics; and
- Figures 4-23, which show the projections of our branching fraction fits onto their respective datasets.
$A_{CP}$ results.

FIG. 2: Results for $A_{CP}$ as a function of $q^2$. The black points show the $q^2_{15}$ results; the red triangle denotes $q^2_0$. The $q^2_{45}$ $A_{CP}$ result does not include events in the $\psi(2S)$ veto window.
Differential Branching Fraction in $m_{X_s}$ bins.

FIG. 3: Differential BF as a function of $m_{X_s}$ for electron (blue circles), muon (black squares) and lepton-flavor-averaged final states (red triangles). The errors correspond to the total uncertainties. The histogram shows the SM expectation, which has uncertainties of approximately 10-30% as a function of $q^2$. Estimated contributions from the vetoed charmonium $q^2$ regions are included. The horizontal spread of points in each bin is meant only to aid visibility.
Fitted Signal and Background Yields

Table II gives the fitted number of signal events $N_{\text{sig}}$ in each individual $q^2$ and $m_{X_s}$ bin, along with the fitted number of random combinatorial $B\bar{B}$ background events $N_{B\bar{B}}$ present in the signal enhanced region with $m_{ES} > 5.27\text{ GeV}/c^2$. The quoted uncertainties are statistical only.

TABLE II: Fitted number of signal events $N_{\text{sig}}$ and random combinatorial $B\bar{B}$ background events $N_{B\bar{B}}$ present in the signal enhanced region with $m_{ES} > 5.27\text{ GeV}/c^2$ by $q^2(\text{GeV}/c^2)$ and $m_{X_s}(\text{GeV}/c^2)$ bin.

<table>
<thead>
<tr>
<th>Bin</th>
<th>Range</th>
<th>$B \rightarrow X_s e^+ e^-$</th>
<th>$B \rightarrow X_s \mu^+ \mu^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$N_{\text{sig}}$</td>
<td>$N_{B\bar{B}}$</td>
</tr>
<tr>
<td>$q_0^2$</td>
<td>$1.0 &lt; q^2 &lt; 6.0$</td>
<td>58.5$^{+14.4}_{-13.5}$</td>
<td>348.8 ± 22.2</td>
</tr>
<tr>
<td>$q_1^2$</td>
<td>$0.1 &lt; q^2 &lt; 2.0$</td>
<td>60.4$^{+11.9}_{-11.1}$</td>
<td>95.3 ± 12.4</td>
</tr>
<tr>
<td>$q_2^2$</td>
<td>$2.0 &lt; q^2 &lt; 4.3$</td>
<td>20.8$^{+9.4}_{-8.5}$</td>
<td>168.2 ± 15.3</td>
</tr>
<tr>
<td>$q_3^2$</td>
<td>$4.3 &lt; q^2 &lt; 6.8$</td>
<td>25.4$^{+10.3}_{-9.5}$</td>
<td>181.3 ± 16.2</td>
</tr>
<tr>
<td>$q_4^2$</td>
<td>$10.1 &lt; q^2 &lt; 12.9$</td>
<td>59.1$^{+14.8}_{-14.0}$</td>
<td>201.0 ± 20.3</td>
</tr>
<tr>
<td>$q_5^2$</td>
<td>$14.2 &lt; q^2$</td>
<td>41.0$^{+8.3}_{-7.7}$</td>
<td>40.2 ± 10.0</td>
</tr>
<tr>
<td>$m_{X_s,1}$</td>
<td>$0.4 &lt; m_{X_s} &lt; 0.6$</td>
<td>63.0$^{+9.9}_{-9.2}$</td>
<td>3.0 ± 2.9</td>
</tr>
<tr>
<td>$m_{X_s,2}$</td>
<td>$0.6 &lt; m_{X_s} &lt; 1.0$</td>
<td>68.1$^{+11.5}_{-10.9}$</td>
<td>38.0 ± 8.9</td>
</tr>
<tr>
<td>$m_{X_s,3}$</td>
<td>$1.0 &lt; m_{X_s} &lt; 1.4$</td>
<td>38.1$^{+11.3}_{-11.9}$</td>
<td>168.1 ± 17.9</td>
</tr>
<tr>
<td>$m_{X_s,4}$</td>
<td>$1.4 &lt; m_{X_s} &lt; 1.8$</td>
<td>28.5$^{+12.3}_{-12.9}$</td>
<td>483.9 ± 28.6</td>
</tr>
</tbody>
</table>
Tables III-VIII detail the individual contributions to the branching fraction systematics. Uncertainties quoted without a preceding “+” or “−” are ± symmetric.

#### TABLE III: $B \to X_s e^+ e^-$ branching fraction “multiplicative” systematic uncertainties.

<table>
<thead>
<tr>
<th>Variation</th>
<th>$q_0^2$</th>
<th>$q_1^2$</th>
<th>$q_2^2$</th>
<th>$q_3^2$</th>
<th>$q_4^2$</th>
<th>$q_5^2$</th>
<th>$m_{X_{s,1}}$</th>
<th>$m_{X_{s,2}}$</th>
<th>$m_{X_{s,3}}$</th>
<th>$m_{X_{s,4}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_B\Pi$</td>
<td>0.012</td>
<td>0.018</td>
<td>0.004</td>
<td>0.004</td>
<td>0.007</td>
<td>0.003</td>
<td>0.004</td>
<td>0.007</td>
<td>0.010</td>
<td>0.011</td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.031</td>
<td>0.049</td>
<td>0.011</td>
<td>0.011</td>
<td>0.018</td>
<td>0.009</td>
<td>0.011</td>
<td>0.019</td>
<td>0.026</td>
<td>0.030</td>
</tr>
<tr>
<td>Particle Identification efficiency</td>
<td>0.033</td>
<td>0.052</td>
<td>0.012</td>
<td>0.012</td>
<td>0.019</td>
<td>0.010</td>
<td>0.012</td>
<td>0.020</td>
<td>0.027</td>
<td>0.032</td>
</tr>
<tr>
<td>$K^0\phi$ efficiency</td>
<td>0.004</td>
<td>0.006</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>$\pi^0$ efficiency</td>
<td>0.012</td>
<td>0.024</td>
<td>0.004</td>
<td>0.006</td>
<td>0.004</td>
<td>0.002</td>
<td>0.000</td>
<td>0.018</td>
<td>0.014</td>
<td>0.019</td>
</tr>
<tr>
<td>BDT efficiency correction</td>
<td>0.004</td>
<td>0.006</td>
<td>0.001</td>
<td>0.001</td>
<td>0.009</td>
<td>0.004</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>Monte Carlo statistics</td>
<td>0.002</td>
<td>0.009</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.006</td>
</tr>
<tr>
<td>Total</td>
<td>0.048</td>
<td>0.079</td>
<td>0.017</td>
<td>0.017</td>
<td>0.030</td>
<td>0.014</td>
<td>0.017</td>
<td>0.030</td>
<td>0.041</td>
<td>0.050</td>
</tr>
</tbody>
</table>

#### TABLE IV: $B \to X_s \mu^+ \mu^-$ branching fraction “multiplicative” systematic uncertainties.

<table>
<thead>
<tr>
<th>Variation</th>
<th>$q_0^2$</th>
<th>$q_1^2$</th>
<th>$q_2^2$</th>
<th>$q_3^2$</th>
<th>$q_4^2$</th>
<th>$q_5^2$</th>
<th>$m_{X_{s,1}}$</th>
<th>$m_{X_{s,2}}$</th>
<th>$m_{X_{s,3}}$</th>
<th>$m_{X_{s,4}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_B\Pi$</td>
<td>0.004</td>
<td>0.011</td>
<td>0.001</td>
<td>0.002</td>
<td>0.005</td>
<td>0.004</td>
<td>0.004</td>
<td>0.005</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.009</td>
<td>0.027</td>
<td>0.002</td>
<td>0.005</td>
<td>0.011</td>
<td>0.007</td>
<td>0.008</td>
<td>0.011</td>
<td>0.010</td>
<td>0.003</td>
</tr>
<tr>
<td>Particle Identification efficiency</td>
<td>0.015</td>
<td>0.042</td>
<td>0.003</td>
<td>0.008</td>
<td>0.020</td>
<td>0.014</td>
<td>0.017</td>
<td>0.017</td>
<td>0.015</td>
<td>0.004</td>
</tr>
<tr>
<td>$K^0\phi$ efficiency</td>
<td>0.001</td>
<td>0.004</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>$\pi^0$ efficiency</td>
<td>0.004</td>
<td>0.013</td>
<td>0.001</td>
<td>0.002</td>
<td>0.004</td>
<td>0.002</td>
<td>0.000</td>
<td>0.005</td>
<td>0.005</td>
<td>0.002</td>
</tr>
<tr>
<td>BDT efficiency correction</td>
<td>0.002</td>
<td>0.005</td>
<td>0.000</td>
<td>0.001</td>
<td>0.010</td>
<td>0.007</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Monte Carlo statistics</td>
<td>0.001</td>
<td>0.005</td>
<td>0.000</td>
<td>0.001</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Total</td>
<td>0.019</td>
<td>0.054</td>
<td>0.004</td>
<td>0.010</td>
<td>0.026</td>
<td>0.018</td>
<td>0.020</td>
<td>0.022</td>
<td>0.019</td>
<td>0.006</td>
</tr>
</tbody>
</table>

#### TABLE V: $B \to X_s e^+ e^-$ branching fraction “additive” systematic uncertainties.

<table>
<thead>
<tr>
<th>Variation</th>
<th>$q_0^2$</th>
<th>$q_1^2$</th>
<th>$q_2^2$</th>
<th>$q_3^2$</th>
<th>$q_4^2$</th>
<th>$q_5^2$</th>
<th>$m_{X_{s,1}}$</th>
<th>$m_{X_{s,2}}$</th>
<th>$m_{X_{s,3}}$</th>
<th>$m_{X_{s,4}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal $m_{\phi}$ pdf shape</td>
<td>+0.039</td>
<td>+0.092</td>
<td>+0.007</td>
<td>+0.014</td>
<td>+0.034</td>
<td>+0.011</td>
<td>+0.021</td>
<td>+0.036</td>
<td>+0.032</td>
<td>+0.057</td>
</tr>
<tr>
<td>Signal $L_R$ pdf shape</td>
<td>+0.077</td>
<td>+0.159</td>
<td>+0.051</td>
<td>+0.006</td>
<td>+0.142</td>
<td>+0.002</td>
<td>+0.002</td>
<td>+0.050</td>
<td>+0.127</td>
<td>+0.474</td>
</tr>
<tr>
<td>Crossfeed pdf shape</td>
<td>+0.032</td>
<td>+0.074</td>
<td>+0.026</td>
<td>+0.013</td>
<td>+0.067</td>
<td>+0.000</td>
<td>+0.002</td>
<td>+0.013</td>
<td>+0.055</td>
<td>+0.201</td>
</tr>
<tr>
<td>Crossfeed normalization</td>
<td>+0.034</td>
<td>+0.039</td>
<td>+0.014</td>
<td>+0.011</td>
<td>+0.021</td>
<td>+0.008</td>
<td>+0.002</td>
<td>+0.014</td>
<td>+0.027</td>
<td>+0.011</td>
</tr>
<tr>
<td>$B\bar{B}$ pdf shape</td>
<td>+0.159</td>
<td>+0.139</td>
<td>+0.080</td>
<td>+0.086</td>
<td>+0.010</td>
<td>+0.014</td>
<td>+0.006</td>
<td>+0.020</td>
<td>+0.126</td>
<td>+0.367</td>
</tr>
<tr>
<td>$udsc$ pdf shape</td>
<td>+0.123</td>
<td>+0.096</td>
<td>+0.052</td>
<td>+0.070</td>
<td>+0.038</td>
<td>+0.009</td>
<td>+0.005</td>
<td>+0.019</td>
<td>+0.099</td>
<td>+0.351</td>
</tr>
<tr>
<td>$udsc$ normalization</td>
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<td>+0.106</td>
<td>+0.026</td>
<td>+0.032</td>
<td>+0.035</td>
<td>+0.014</td>
<td>+0.005</td>
<td>+0.021</td>
<td>+0.087</td>
<td>+0.181</td>
</tr>
<tr>
<td>Charmonium pdf shape</td>
<td>+0.019</td>
<td>+0.059</td>
<td>+0.009</td>
<td>+0.011</td>
<td>+0.019</td>
<td>+0.006</td>
<td>+0.003</td>
<td>+0.010</td>
<td>+0.030</td>
<td>+0.055</td>
</tr>
<tr>
<td>Charmonium normalization</td>
<td>+0.007</td>
<td>+0.039</td>
<td>+0.005</td>
<td>+0.006</td>
<td>+0.009</td>
<td>+0.003</td>
<td>+0.003</td>
<td>+0.013</td>
<td>+0.031</td>
<td>+0.057</td>
</tr>
<tr>
<td>Total</td>
<td>+0.055</td>
<td>+0.020</td>
<td>+0.019</td>
<td>+0.032</td>
<td>+0.115</td>
<td>+0.005</td>
<td>+0.006</td>
<td>+0.034</td>
<td>+0.126</td>
<td>+0.231</td>
</tr>
</tbody>
</table>

13
### TABLE VI: $B \to X_s \mu^+ \mu^-$ branching fraction “additive” systematic uncertainties.

<table>
<thead>
<tr>
<th>Variation</th>
<th>$q^2_1$</th>
<th>$q^2_2$</th>
<th>$q^2_3$</th>
<th>$q^2_4$</th>
<th>$q^2_5$</th>
<th>$m_{X_s,1}$</th>
<th>$m_{X_s,2}$</th>
<th>$m_{X_s,3}$</th>
<th>$m_{X_s,4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal $m_{ps}$ pdf shape</td>
<td>$+0.012$</td>
<td>$+0.008$</td>
<td>$+0.008$</td>
<td>$+0.012$</td>
<td>$+0.022$</td>
<td>$+0.019$</td>
<td>$+0.019$</td>
<td>$+0.019$</td>
<td>$+0.006$</td>
</tr>
<tr>
<td>Signal $L_R$ pdf shape</td>
<td>$-0.007$</td>
<td>$-0.018$</td>
<td>$-0.004$</td>
<td>$-0.006$</td>
<td>$-0.008$</td>
<td>$-0.012$</td>
<td>$-0.015$</td>
<td>$-0.019$</td>
<td>$-0.004$</td>
</tr>
<tr>
<td>Crossfeed pdf shape</td>
<td>$+0.018$</td>
<td>$+0.037$</td>
<td>$+0.029$</td>
<td>$+0.010$</td>
<td>$+0.039$</td>
<td>$+0.014$</td>
<td>$+0.003$</td>
<td>$+0.031$</td>
<td>$+0.077$</td>
</tr>
<tr>
<td>Crossfeed normalization</td>
<td>$-0.020$</td>
<td>$-0.123$</td>
<td>$-0.011$</td>
<td>$-0.006$</td>
<td>$-0.035$</td>
<td>$-0.015$</td>
<td>$-0.003$</td>
<td>$-0.006$</td>
<td>$-0.026$</td>
</tr>
<tr>
<td>$B^0$ decay pdf shape</td>
<td>$-0.020$</td>
<td>$-0.008$</td>
<td>$-0.002$</td>
<td>$-0.013$</td>
<td>$-0.017$</td>
<td>$-0.006$</td>
<td>$-0.001$</td>
<td>$-0.007$</td>
<td>$-0.039$</td>
</tr>
<tr>
<td>udsc pdf shape</td>
<td>$+0.050$</td>
<td>$+0.083$</td>
<td>$+0.013$</td>
<td>$+0.005$</td>
<td>$+0.024$</td>
<td>$+0.013$</td>
<td>$+0.001$</td>
<td>$+0.033$</td>
<td>$+0.064$</td>
</tr>
<tr>
<td>udsc normalization</td>
<td>$-0.050$</td>
<td>$-0.005$</td>
<td>$-0.007$</td>
<td>$-0.078$</td>
<td>$-0.021$</td>
<td>$-0.012$</td>
<td>$-0.000$</td>
<td>$-0.029$</td>
<td>$-0.070$</td>
</tr>
<tr>
<td>Charmionium pdf shape</td>
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<td>$-0.030$</td>
<td>$-0.032$</td>
<td>$+0.567$</td>
<td>$+0.019$</td>
<td>$+0.003$</td>
<td>$+0.002$</td>
<td>$+0.024$</td>
<td>$+0.026$</td>
</tr>
<tr>
<td>Charmionium normalization</td>
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<td>$+0.083$</td>
<td>$+0.085$</td>
<td>$+0.102$</td>
<td>$+0.039$</td>
<td>$+0.004$</td>
<td>$+0.007$</td>
<td>$+0.032$</td>
<td>$+0.098$</td>
</tr>
<tr>
<td>Hadronic misidentification pdf shape</td>
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<td>$+0.099$</td>
<td>$+0.061$</td>
<td>$+0.060$</td>
<td>$+0.051$</td>
<td>$+0.030$</td>
<td>$+0.029$</td>
<td>$+0.034$</td>
<td>$+0.099$</td>
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<tr>
<td>Hadronic misidentification normalization</td>
<td>$-0.087$</td>
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<td>$-0.054$</td>
<td>$-0.044$</td>
<td>$-0.026$</td>
<td>$-0.028$</td>
<td>$-0.035$</td>
<td>$-0.169$</td>
</tr>
<tr>
<td>Total</td>
<td>$+0.098$</td>
<td>$+0.099$</td>
<td>$+0.061$</td>
<td>$+0.060$</td>
<td>$+0.051$</td>
<td>$+0.030$</td>
<td>$+0.029$</td>
<td>$+0.034$</td>
<td>$+0.099$</td>
</tr>
</tbody>
</table>

### TABLE VII: $B \to X_s e^+ e^-$ branching fraction model-dependent extrapolation systematic uncertainties.

<table>
<thead>
<tr>
<th>Variation</th>
<th>$q^2_1$</th>
<th>$q^2_2$</th>
<th>$q^2_3$</th>
<th>$q^2_4$</th>
<th>$q^2_5$</th>
<th>$m_{X_s,2}$</th>
<th>$m_{X_s,4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jetset tunings</td>
<td>$+0.060$</td>
<td>$+0.011$</td>
<td>$+0.011$</td>
<td>$+0.011$</td>
<td>$+0.001$</td>
<td>$+0.001$</td>
<td>$+0.001$</td>
</tr>
<tr>
<td>$\pm 50%$ $N_{e\theta} &gt; 1$</td>
<td>$0.249$</td>
<td>$0.047$</td>
<td>$0.038$</td>
<td>$0.025$</td>
<td>$0.002$</td>
<td>$0.130$</td>
<td>$0.030$</td>
</tr>
<tr>
<td>$\pm 50%$ $K$ multiplicity</td>
<td>$0.046$</td>
<td>$0.008$</td>
<td>$0.006$</td>
<td>$0.002$</td>
<td>$0.000$</td>
<td>$0.022$</td>
<td>$0.000$</td>
</tr>
<tr>
<td>$\pm 50%$ $\pi^+$ multiplicity</td>
<td>$0.196$</td>
<td>$0.036$</td>
<td>$0.028$</td>
<td>$0.012$</td>
<td>$0.000$</td>
<td>$0.100$</td>
<td>$0.024$</td>
</tr>
<tr>
<td>$\pm 1\sigma$ $B \to K^{(*)} \ell^+ \ell^-$ BF's</td>
<td>$+0.115$</td>
<td>$+0.024$</td>
<td>$+0.021$</td>
<td>$+0.018$</td>
<td>$+0.002$</td>
<td>$+0.007$</td>
<td>$+0.004$</td>
</tr>
<tr>
<td>Total</td>
<td>$+0.448$</td>
<td>$+0.053$</td>
<td>$+0.038$</td>
<td>$+0.003$</td>
<td>$+0.124$</td>
<td>$+0.012$</td>
<td>$+0.128$</td>
</tr>
</tbody>
</table>

### TABLE VIII: $B \to X_s e^+ e^-$ branching fraction model-dependent extrapolation systematic uncertainties.

<table>
<thead>
<tr>
<th>Variation</th>
<th>$q^2_1$</th>
<th>$q^2_2$</th>
<th>$q^2_3$</th>
<th>$q^2_4$</th>
<th>$q^2_5$</th>
<th>$m_{X_s,3}$</th>
<th>$m_{X_s,4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jetset tunings</td>
<td>$+0.035$</td>
<td>$+0.002$</td>
<td>$+0.005$</td>
<td>$+0.009$</td>
<td>$+0.001$</td>
<td>$+0.025$</td>
<td>$+0.015$</td>
</tr>
<tr>
<td>$\pm 50%$ $N_{e\theta} &gt; 1$</td>
<td>$-0.041$</td>
<td>$-0.003$</td>
<td>$-0.006$</td>
<td>$-0.012$</td>
<td>$-0.002$</td>
<td>$-0.020$</td>
<td>$-0.014$</td>
</tr>
<tr>
<td>$\pm 50%$ $K$ multiplicity</td>
<td>$0.154$</td>
<td>$0.011$</td>
<td>$0.020$</td>
<td>$0.021$</td>
<td>$0.002$</td>
<td>$0.047$</td>
<td>$0.012$</td>
</tr>
<tr>
<td>$\pm 50%$ $\pi^+$ multiplicity</td>
<td>$0.029$</td>
<td>$0.002$</td>
<td>$0.003$</td>
<td>$0.002$</td>
<td>$0.000$</td>
<td>$0.008$</td>
<td>$0.000$</td>
</tr>
<tr>
<td>$\pm 1\sigma$ $B \to K^{(*)} \ell^+ \ell^-$ BF's</td>
<td>$+0.027$</td>
<td>$+0.002$</td>
<td>$+0.004$</td>
<td>$+0.007$</td>
<td>$+0.001$</td>
<td>$+0.015$</td>
<td>$+0.001$</td>
</tr>
<tr>
<td>Total</td>
<td>$+0.130$</td>
<td>$+0.032$</td>
<td>$+0.020$</td>
<td>$+0.007$</td>
<td>$+0.003$</td>
<td>$+0.124$</td>
<td>$+0.012$</td>
</tr>
</tbody>
</table>

### Fit Projections

The pages following show the $B \to X_s e^+ e^-$ and $B \to X_s \mu^+ \mu^-$ branching fraction fit projections for each $q^2$ and $m_{X_s}$ bin.
FIG. 4: Fit to $B \to X_s e^+e^-$ in the $q_0^2$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
**FIG. 5:** Fit to $B \to X_s e^+ e^-$ in the $q_1^2$ bin. Top row is the $m_{ES}$ fit projection, top right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 6: Fit to $B \to X_s e^+ e^-$ in the $q_2^2$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
**B → X_s e^+ e^- Fit Projections: \( q_3^2 \)**

**FIG. 7:** Fit to \( B \rightarrow X_s \, e^+ e^- \) in the \( q_3^2 \) bin. Top row left is the \( m_{ES} \) fit projection, top row right is the \( L_R \) fit projection; middle row left is a signal-enhanced \( m_{ES} \) fit projection for events with \( L_R > 0.8 \), middle row right is a signal-enhanced \( L_R \) fit projection for events in the \( m_{ES} > 5.27 \text{GeV/c}^2 \) signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 8: Fit to $B \rightarrow X_s e^+ e^-$ in the $q^2_4$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 9: Fit to $B \rightarrow X_s e^+ e^-$ in the $q^2_5$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27 \text{ GeV/c}^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 10: Fit to $B \rightarrow X_s e^+ e^-$ in the $m_{X_s,1}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c² signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
B$\to X_s e^+ e^-$ Fit Projections: $m_{Xs,2}$

**FIG. 11:** Fit to $B \to X_s e^+ e^-$ in the $m_{Xs,2}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 12: Fit to $B \to X_s e^+ e^-$ in the $m_{X_s,3}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 13: Fit to $B \to X_s e^+ e^-$ in the $m_{X_s,4}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 14: Fit to $B \to X_s \mu^+\mu^-$ in the $q_0^2$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 15: Fit to $B \rightarrow X_{s} \mu^{+} \mu^{-}$ in the $q_{1}^{2}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_{R}$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_{R} > 0.8$, middle row right is a signal-enhanced $L_{R}$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^{2}$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
$B \rightarrow X_s \mu^+ \mu^-$ Fit Projections: $q^2$

![Graphs showing fit projections for $B \rightarrow X_s \mu^+ \mu^-$ in the $q^2$ bin.](image)

**FIG. 16:** Fit to $B \rightarrow X_s \mu^+ \mu^-$ in the $q^2$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 17: Fit to $B \to X_s \mu^+ \mu^-$ in the $q_3^2$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 18: Fit to $B \to X_s \mu^+ \mu^-$ in the $q^2$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 19: Fit to $B \rightarrow X_s \mu^+\mu^-$ in the $q_5^2$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/$c^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 20: Fit to $B \rightarrow X_s \mu^+ \mu^-$ in the $m_{X_s,1}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
FIG. 21: Fit to $B \rightarrow X_s \mu^+ \mu^-$ in the $m_{X_s,2}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
B$\rightarrow X_s \mu^+ \mu^-$ Fit Projections: $m_{Xs,3}$

**Full Fit Range**

- **Signal Enhanced Range: $L_R > 0.8$**
- **Signal Enhanced Range: $m_{ES} > 5.27$ GeV/c$^2$**

**Fit Likelihood Projection**

- **Total PDF**
- **Signal**
- **Signal Crossfeed**
- **BB Bkgd.**
- **udsc Bkgd.**
- **Charmonium Bkgd.**
- **Hadronic Misid Bkgd.**

**FIG. 22:** Fit to $B \rightarrow X_s \mu^+ \mu^-$ in the $m_{Xs,3}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.
**FIG. 23:** Fit to $B \rightarrow X_s \mu^+ \mu^-$ in the $m_{X_s,4}$ bin. Top row left is the $m_{ES}$ fit projection, top row right is the $L_R$ fit projection; middle row left is a signal-enhanced $m_{ES}$ fit projection for events with $L_R > 0.8$, middle row right is a signal-enhanced $L_R$ fit projection for events in the $m_{ES} > 5.27$ GeV/c$^2$ signal region. The lower left hand plot is the profile likelihood curve for the 2D data fit.