Bulletin of the Seismological Society of America. Vol. 54, No. 4, pp. 1035–1058. August 1964

CONTINUUM DISTRIBUTION OF DISLOCATIONS ON FAULTS WITH FINITE FRICTION

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ABSTRACT

An analysis is made of continuous distributions of infinitesimal dislocations on faults with finite friction. The analysis was undertaken in an attempt to explain the fact that dislocations produced by earthquakes commonly lie at depths that are shallower than the average depth of earthquake foci in continents. (The depths of dislocations are determined from displacements around faults.) It is found that this discrepancy can be explained if, at some depth, there exists a region where the frictional stress on faults is anomalously low.

Introduction

In this paper we will make use of the concept of continuously distributed infinitesimal dislocations to study problems of slippage on faults with finite friction. This concept was originated by Leibfried (1951, 1954). He pointed out that the problem of determining the equilibrium positions of discrete dislocations all lying on the same slip plane in a crystalline lattice is considerably simplified if the dislocations are considered to be "smeared out" into a continuum of infinitesimal dislocations.

The idea of a continuum of infinitesimal dislocations is somewhat artificial when applied to dislocations within crystals because of the discrete nature of the atomic lattice. On the other hand it is natural to describe non-uniform displacements across faults within the earth's crust in terms of continuous infinitesimal dislocations. A number of papers have been written about dislocations on faults within the earth (see Scheidegger, 1963; Chinnery, 1961, 1963, and the references cited by them). It is tacitly assumed in these papers that dislocations on faults are discrete rather than continuously distributed. It seems worthwhile, therefore, to look at the problems of slippage on faults with friction from the viewpoint of infinitesimal dislocations.

The fact that the measured displacements in the vicinity of faults which have broken through the surface are approximately the same as the elastic displacement around dislocations with a total Burgers vector* of from 2 to 6 meters (Byerly and DeNoyer, 1958; Chinnery, 1961) is a firm piece of evidence that the dislocation approach to faulting is reasonable. However there is a problem connected with this approach. The displacements in the vicinity of faults which seem to indicate the existence of dislocations place the depth of these dislocations at about 4 to 8 kilometers. (The observed displacements would be produced if a dislocation were to enter the fault at the earth's surface and move down 4 to 8 kilometers. The same displacements would result if a pre-existing dislocation at this depth moved up to the surface. The former situation gives rise to an earthquake focus at the surface; the

* The Burgers vector of a discrete dislocation is a vector which is parallel to the direction of the displacement produced when the dislocation moves across its slip plane. The length of the Burgers vector is equal to the magnitude of this displacement. The Burgers vector is described in detail in any text on dislocations (Cottrell, 1953; Read, 1953; Weertman and Weertman, 1964).

latter to a focus at a 4 to 8 kilometer depth.) This result is difficult to reconcile with the observation that the average depth of the foci of earthquakes within continental blocks is of the order of 15 kilometers (at least in California). This paper was written primarily in an attempt to explain these contradictory observations.

Orowan and others (Orowan, 1960; Scheidegger, 1963) have pointed out a serious objection to the theory that earthquakes are produced by slippage on faults with friction. Even at shallow depths the hydrostatic pressure is so great that the ordinary law of static friction predicts frictional stresses exceeding the theoretical strength of crystalline material. (It is assumed that the coefficient of friction is of the order of 1.) However calculations based on the energy radiated by an earthquake indicate that the stresses which produce earthquakes are low: of the order of 25 bars (Benioff, 1955). This difficulty is not faced up to in this paper. We assume that faults at moderate depths within continental blocks (very deep faults are not considered) are lubricated in some manner and thereby have anomalously low frictional stresses.

Leibfried used the theory of continuous dislocations to analyse the behavior both of long straight parallel dislocations (1951) and of dislocation loops (1954). Only the former problem will be considered in this paper. Leibfried's theory for straight, parallel dislocations actually is a particular case of a boundary value problem for a half space in two dimensional elasticity theory. In particular, it is an example of the mixed boundary value problem in which displacements are specified over some regions of the boundary and the stresses are specified over the remainder. The general solution of this problem is known (Muskhelishvili, 1953a, 1953b; Mikhlin, 1957). This general solution has been applied to several problems in the continuum theory of infinitesimal dislocations (Head and Louat, 1955; Leonov and Shvaiko, 1961; Bilby, Cottrell and Swinden, 1963).

The continuous distribution of dislocations is specified by a distribution function $B(x)$, where $B(x)\,dx$ represents the total length of the Burgers vectors of the infinitesimal dislocations lying between x and $x + dx$ on the slip plane. The coordinate x measures distance along the slip plane in a direction perpendicular to the dislocation lines. A negative value for $B(x)$ implies that the dislocations have negative Burgers vectors.

Once the distribution function $B(x)$ is known, the shear stress $\tau(x)$ on the slip plane which arises from the dislocations can be calculated. The shear stress is

$$\tau(x) = \frac{\mu}{2\pi\alpha} \int_{-\infty}^{\infty} \frac{B(y)\,dy}{x - y} \tag{2}$$

In this equation μ is the shear modulus and α is a constant. The constant α has the value 1 if the dislocations are screw dislocations; its value is $(1 - \nu)$, where ν is Poisson's ratio, if the dislocations are edge dislocations; it has an intermediate value if the dislocations are of mixed character (partly edge and partly screw). The Cauchy principal value of the integral is used.

If the shear stress $\tau(x)$ is specified over the slip plane, the dislocation distribution $B(x)$ is given by:

$$B(x) = -\frac{2\alpha}{\mu\pi} \int_{-\infty}^{\infty} \frac{\tau(y)\, dy}{x - y} \tag{3}$$

The functions $B(x)$ and $\tau(x)$ are Hilbert transforms of each other.

In the problems that we will consider, the dislocation density $B(x)$ will be equal to zero everywhere on the slip plane except in a region such as $a \leqq x \leqq b$. However, the exact value of $B(x)$ will not be known within this region. The stress $\tau(x)$ will be known within $a \leqq x \leqq b$ but its value will not be known elsewhere on the slip plane. The solution for this mixed boundary value problem is (Muskhelishvili, 1953a, 1953b; Mikhlin, 1957):

$$B(x) = \frac{C}{\sqrt{(b - x)(x - a)}} + \frac{2\alpha}{\pi\mu\sqrt{(b - x)(x - a)}} \cdot \int_a^b \frac{\tau(y)\sqrt{(b - y)(y - a)}\, dy}{y - x} \tag{4}$$

where C is a constant. It will be necessary to set C equal to zero in all the problems we will consider in order to avoid infinite stresses on the slip plane. Equation (4) also can be written in the form

$$\begin{aligned} B(x) &= \frac{C}{\sqrt{(b - x)(x - a)}} + \frac{2\alpha}{\pi\mu}\sqrt{(b - x)(x - a)} \cdot \int_a^b \frac{\tau(y)\, dy}{(y - x)\sqrt{(b - y)(y - a)}} = \frac{C}{\sqrt{(b - x)(x - a)}} \\ &\quad + \frac{2\alpha}{\pi\mu}\sqrt{\frac{(x - a)}{(b - x)}} \int_a^b \sqrt{\frac{b - y}{y - a}} \frac{\tau(y)\, dy}{(y - x)} \end{aligned} \tag{5}$$

In order for eqns. (4) and (5) to give a solution it is necessary that the following equation be satisfied:

$$\int_a^b \frac{\tau(y)\, dy}{\sqrt{(b - y)(y - a)}} = 0 \tag{6}$$

If the dislocation density $B(x)$ is nonvanishing on a series of n strips, say $a_i \leqq x \leqq b_i$ where $i = 1, 2, \cdots$, the solution becomes

$$B(x) = \frac{\sum_1^{n-1} C_i x^i}{\sqrt{\prod_1^n (b_i - x)(a_i - x)}} + \frac{2\alpha}{\pi\mu} \prod_1^n \sqrt{\frac{(x - a_i)}{(b_i - x)}} \cdot \int \prod_1^n \sqrt{\frac{b_i - y}{y - a_i}} \frac{\tau(y)\, dy}{(y - x)} \tag{7}$$

The integration is carried out only on the strips $a_i \leqq x \leqq b_i$. The terms C_i are constants. If a solution is to exist the following equation must hold

$$\int \prod_1^n \sqrt{\frac{b_i - y}{y - a_i}}\, y^m \, dy = 0 \tag{8}$$

where again the integration is carried out only over the strips $a_i \leqq x \leqq b_i$. Equation (8) must be satisfied for everyone of the values $m = 0, 1, 2, \cdots (n - 1)$.

Movement of Localized Dislocations

In the study of crystals it is common to think of dislocations as discrete entities that move across their slip planes under an applied stress. With the electron micro-

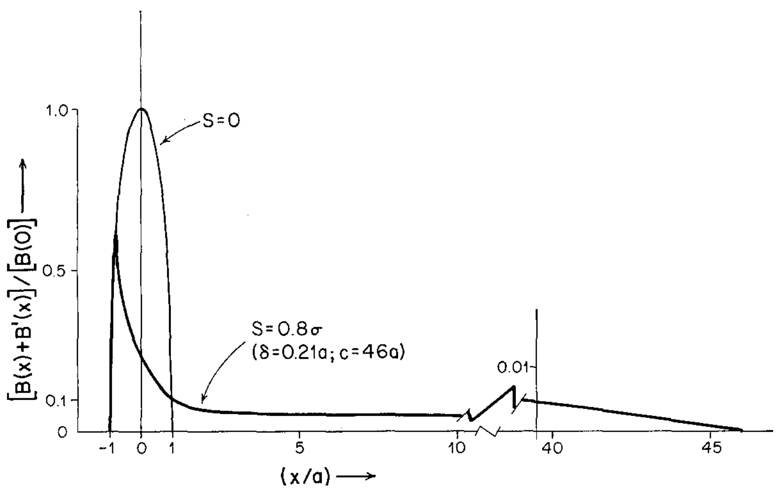

Fig. 1. Plot of the normalized dislocation density of a localized group of dislocations when the applied stress S equals zero and when S equals 0.8 times the frictional stress of the fault.

scope it is possible to watch individual dislocations in motion. Naively we might think that a localized distribution of infinitesimal dislocations can move in a similar fashion. We wish to show in this section that a localized "wave packet" of dislocations cannot move *slowly* as an entity across a fault with friction. (Fast moving dislocations, however, can move as a packet.) Rather, a localized group of slow moving dislocations becomes diffuse as the applied stress which moves it is increased.

For the sake of mathematical convenience let the distribution function of the localized dislocations be given by $B(x) = A(a^2 - x^2)^{1/2}$ between $-a \leqq x \leqq a$, where A is a constant. Let $B(x)$ be zero elsewhere. The stress $\tau(x)$ on the fault produced by this distribution can be found from equation (2)

$$\begin{aligned} \tau(x) &= \frac{A\mu}{2\alpha}\left[x + (x^2 - a^2)^{1/2}\right] && -\infty < x \leqq -a \\ &\ \frac{A\mu}{2\alpha}\, x && -a \leqq x \leqq a \\ &\ \frac{A\mu}{2\alpha}\left[x - (x^2 - a^2)^{1/2}\right] && a \leq x < \infty \end{aligned} \tag{9}$$

The distribution function $B(x)$ is shown in figure 1 and $\tau(x)$ in figure 2a. These curves are plotted for the case in which $A = \sigma(2\alpha/\mu a)$, where σ is the frictional stress on the fault which must be overcome to cause slippage. The frictional stress is considered to be independent of position. The maximum and minimum values of the stress $\tau(x)$ occur at $x = \pm a$.

Let an applied stress S be increased slowly from 0 to a value less than the frictional stress σ. What is the effect of this stress S on the distribution function shown in figure 1? The answer is found most easily by regarding the stress of figure 2a as an additional contribution to the frictional stress. If this viewpoint is adopted it can be seen that the additional stress required to obtain slippage at any point x

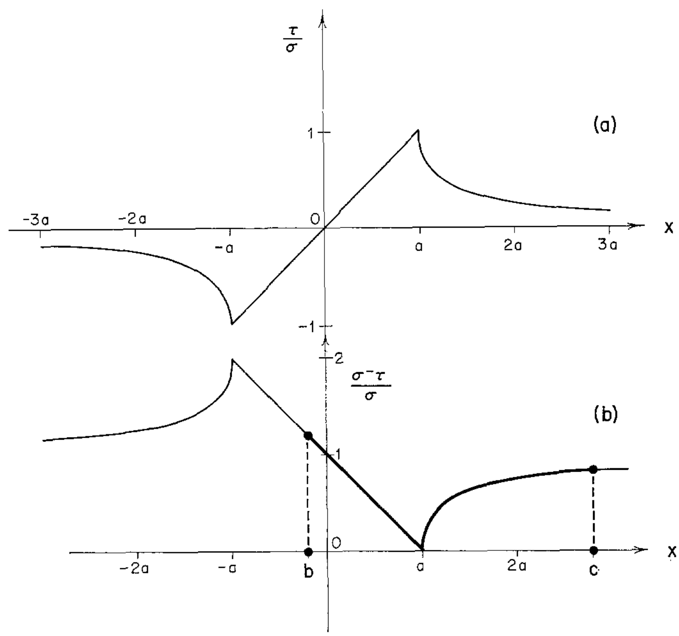

FIG. 2. (a) Plot of the stress arising from the localized dislocation group in Figure 1 ($S = 0$); (b) The effective frictional stress on a fault arising from the stress in (a).

is that shown in figure 2b. Here it is assumed that slip occurs in that direction which favors the movement of the dislocation distribution of figure 1 towards the right. The problem now is reduced to finding the dislocation distribution on a fault with a frictional stress given by figure 2b and upon which there are no dislocations when the applied stress is equal to zero. When this new dislocation distribution $B'(x)$, is added to the old, $B(x)$, the problem is solved.

As the applied stress S is increased from zero, slippage begins where the resistance to sliding is least. Slippage starts at $x = a$. The slipped zone will spread both to the left and the right of this point. Let the slipped zone extend between the points b and c, where $b \leqq x \leqq c$. Because only pairs of dislocations whose Burgers vectors are equal in magnitude but opposite in sign can be created in the interior of a block of material, $B'(x)$ must satisfy the equation:

$$\int_b^c B'(x)\, dx = 0 \tag{10}$$

The distribution function $B'(x)$ may be found without difficulty if the slipped zone is long, that is, $c \gg a$. In this situation it is to be expected that the value of b is only slightly larger than $-a$. The following analysis confirms this prediction. We shall set b equal to $-a + \delta$, where δ is a positive quantity and $\delta \ll a$.

The stress $\tau'(x)$ due to the dislocation distribution $B'(x)$ must exactly balance the stress $S + \tau(x) - \sigma$ in the slipped region. The dislocation distribution $B'(x)$ therefore can be found by substituting $\tau'(x) = \sigma - S - \tau(x)$ into equations (4) or (5). If high order terms in δ are neglected, it is found that the distribution function is equal to

$$B'(x) = \frac{2\sigma\alpha}{\mu}\left[\frac{\delta}{x+a}\sqrt{\frac{(c-x)(x-b)}{2a(c+a)}} - \frac{g}{a}\sqrt{(a-x)(x-b)}\right] \tag{11}$$

In this equation $g = 1$, for those values of x where $b \leqq x < a$, and $g = 0$ where $a < x \leqq c$. The dislocation density $B'(x)$ is equal to zero outside of the slipped zone.

When equation (11) is substituted into (10) the following relationship is obtained

$$\delta = a\sqrt{\frac{2a}{c+a}} \tag{12}$$

The values of b, or δ, and c must be such that equations (6) also is satisfied. That equation gives rise to the requirement that

$$\delta = a\left(\frac{\sigma - S}{\sigma}\right) \bigg/ \left(1 - \sqrt{\frac{a}{2(c+a)}}\right) \tag{13}$$

Equations (12) and (13) between them determine δ, or b, and c, and thus the dislocation distribution $B'(x)$ is completely specified. Equations (12) and (13) show that as S approaches σ, c becomes very large and δ approaches zero, in agreement with our assumption.

The distribution $B(x) + B'(x)$ gives the dislocation distribution of the original dislocations at any stress S up to σ. Figure 1 shows the distribution that occurs at the stress $S = 0.8\sigma$. It can be seen that the dislocations have spread themselves out in the direction favored by the stress.

The mean position $\bar{x}$ of the dislocations is defined by

$$\bar{x} = \int_{-a}^{c} x[B(x) + B'(c)]\, dx \bigg/ \int_{-a}^{a} B(x)\, dx \tag{14}$$

When S has a value close to σ this expression becomes:

$$\bar{x} = \frac{a}{4}\left(\frac{\sigma}{\sigma - S}\right)^2 \left[1 + \sqrt{1 - 2(\sigma - S)/\sigma}\right]^2 \tag{15}$$

The results of this section show that it is not possible for a localized group of infinitesimal dislocations to move slowly as an entity across a fault plane. The applied stress spreads out a localized group of dislocations. Clearly, it is not to be expected that an earthquake is produced by a sudden movement of a *pre-existing* localized group of dislocations.

Dislocation Movement from a Dislocation Source

Figure 2b of the previous section illustrates what could be called a dislocation source. The effective frictional stress around the point a is lower than elsewhere and here slip started. Any region of lower frictional stress on a fault plane will act

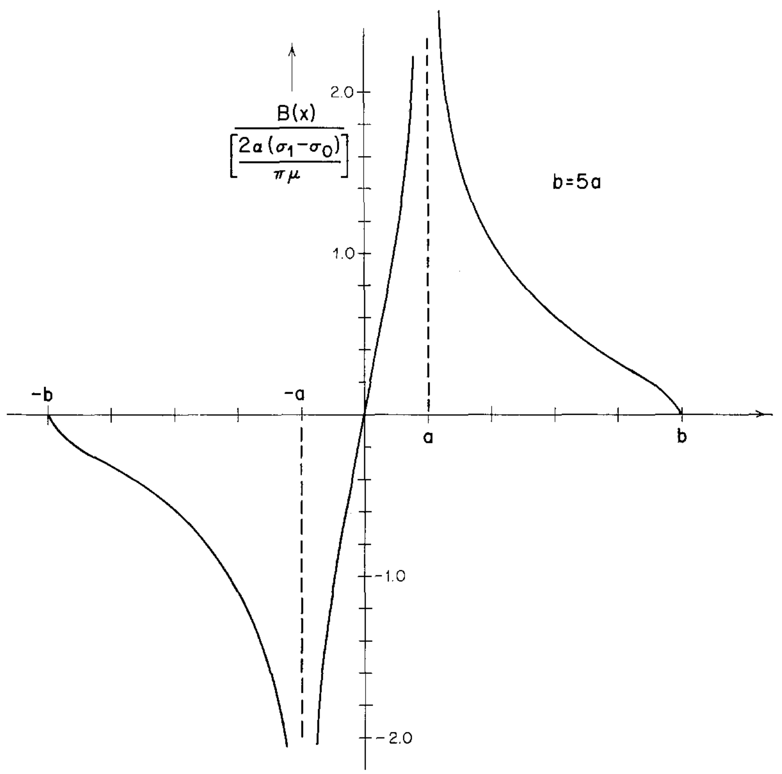

Fig. 3. The dislocation distribution around a dislocation source of the Bilby-Cottrell-Swinden type.

as a dislocation source. The lower frictional stress need not arise from pre-existing dislocations on the fault. A fault may cut across rock which has different physical properties, in particular, a different coefficient of friction. The amount of lubrication on a fault may vary with depth.

A simple example of a dislocation source is given by the case in which the frictional stress equals σ_0 in the region $-a < x < a$ and σ_1 where $|x| > a$. It is assumed that $\sigma_0 < \sigma_1$. The solution for this problem first was obtained by Bilby *et al.* (1963), and was applied by them to the phenomenon of plastic yielding from a notch cut in a metal specimen. The solution is simple to obtain. Suppose the applied stress S is larger than σ_0 and slip has occurred in the region $-b \leqq x \leqq b$, where $b > a$. The dislocations that are created must produce a stress $\tau(x)$ within the slipped region which is equal to $\sigma_0 - S$ from $-a < x < a$, and $\sigma_1 - S$ where $a < |x| \leqq b$. When substituted into equations (4), (5) and (6) this stress gives

$$B(x) = \frac{2\alpha(\sigma_1 - \sigma_0)}{\pi\mu}\left[\log\left|\frac{a+x}{a-x}\right| + \log\left|\frac{b^2 - ax + \sqrt{(b^2-a^2)(b^2-x^2)}}{b^2 + ax + \sqrt{(b^2-a^2)(b^2-x^2)}}\right|\right] \tag{16}$$

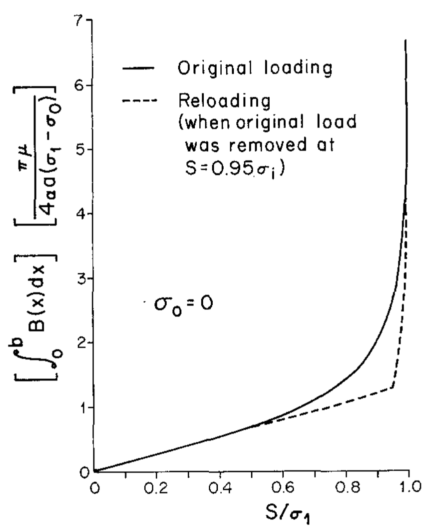

FIG. 4. Plot of the displacement at the center of the source of Figure 3 versus applies stress. Shown here are the two cases of a source not previously loaded and a source reloaded after previously having been stressed to $S = 0.95\,\sigma_1$.

where

$$\sin^{-1}(a/b) = \frac{\pi}{2}\left(\frac{\sigma_1 - S}{\sigma_1 - \sigma_0}\right) \tag{17}$$

The distribution $B(x)$ is plotted in figure 3. The infinite values in the dislocation distribution at $x = \pm a$ come about from the discontinuity in the frictional stress at these points. If the frictional stress varied smoothly the infinities would be eliminated. The total length of the Burgers vectors of the dislocations on either side of the origin is finite for finite b (that is, when $S < \sigma_1$). The total length is

$$\int_0^b B(x)\,dx = \frac{4\alpha a(\sigma_1 - \sigma_0)}{\pi\mu}\log\left(\frac{b}{a} + \sqrt{\frac{b^2}{a^2} - 1}\right) \tag{18}$$

The total displacement of the fault at $x = 0$ is equal to $\int_0^b B(x)\,dx$ and thus is identical to this last equation. Figure 4 shows a plot of the amount of slip at the origin versus the applied stress. The amount of slip goes to infinity as S approaches σ_1.

The Source Unloaded and Reloaded

Let us extend the analysis of Bilby *et al.* to include the case in which the applied stress is removed and then reapplied. The results obtained on the behavior of the source in this situation will have important application to the study of dislocations produced by earthquakes.

We can accomplish the process of unloading by adding an additional applied stress $-S'(S' > 0)$ to the original applied stress, and letting S' increase from zero to the value S. The stress due to the dislocations already on the fault can be added to the frictional stress in the manner described in the section on localized dislocations. A new dislocation distribution function $B'(x)$ thus is determined which, when added to the old distribution function $B(x)$, gives the complete dislocation distribution. (The function $B(x)$ corresponds to the situation at the start of unloading when the applied stress-S' equals zero). During unloading slip takes place in the reverse direction. Therefore the stress $\tau'(x)$ of the distribution function $B'(x)$ must equal $-2\sigma_0 + S'$ in the region $-a < x < a$, and $-2\sigma_1 + S'$ in the region $a < |x| \leqq b'$. Here b' is the half width of the zone in which reverse slip occurs. The factor 2 in these expressions arises because during reverse slip not only must the frictional stress be overcome but also the stress arising from the dislocations of the original distribution and the stress S. (Note that if S is smaller than $2\sigma_0$ it is not possible to have reverse slip. In this situation $B'(x)$ will equal zero in the completely unloaded state. The original dislocation distribution will not be changed.)

If $\tau'(x)$ is again substituted into equations (4), (5) and (6), the following equation is obtained:

$$B'(x) = -\frac{4\alpha(\sigma_1 - \sigma_0)}{\pi\mu}\left[\log\left|\frac{a+x}{a-x}\right| + \log\left|\frac{b'^2 - ax + \sqrt{(b'^2 - a^2)(b'^2 - x^2)}}{b'^2 + ax + \sqrt{(b'^2 - a^2)(b'^2 - x^2)}}\right|\right] \tag{19a}$$

where

$$\sin^{-1}(a/b') = \frac{\pi}{4}\left(\frac{2\sigma_1 - S'}{\sigma_1 - \sigma_0}\right)$$

The length of the Burgers vectors on either side of the origin is

$$\int_0^{b'} B'(x)\,dx = -\frac{8\alpha a(\sigma_1 - \sigma_0)}{\pi\mu}\log\left(\frac{b'}{a} + \sqrt{\frac{b'^2}{a^2} - 1}\right) \tag{19b}$$

When $S' = S$ the fault is unloaded and b' has its maximum value, which is less than b.

If the fault is reloaded by applying still another stress S^* we are led, using the same arguments, to a third distribution function:

$$B^*(x) = \frac{4\alpha(\sigma_1 - \sigma_0)}{\pi\mu}\left[\log\left|\frac{a+x}{a-x}\right| + \log\left|\frac{b^{*2} - ax + \sqrt{(b^{*2}-a^2)(b^{*2}-x^2)}}{b^{*2} + ax + \sqrt{(b^{*2}-a^2)(b^{*2}-x^2)}}\right|\right] \tag{20}$$

where

$$\sin^{-1}(a/b^*) = \frac{\pi}{4}\left(\frac{2\sigma_1 - S^*}{\sigma_1 - \sigma_0}\right)$$

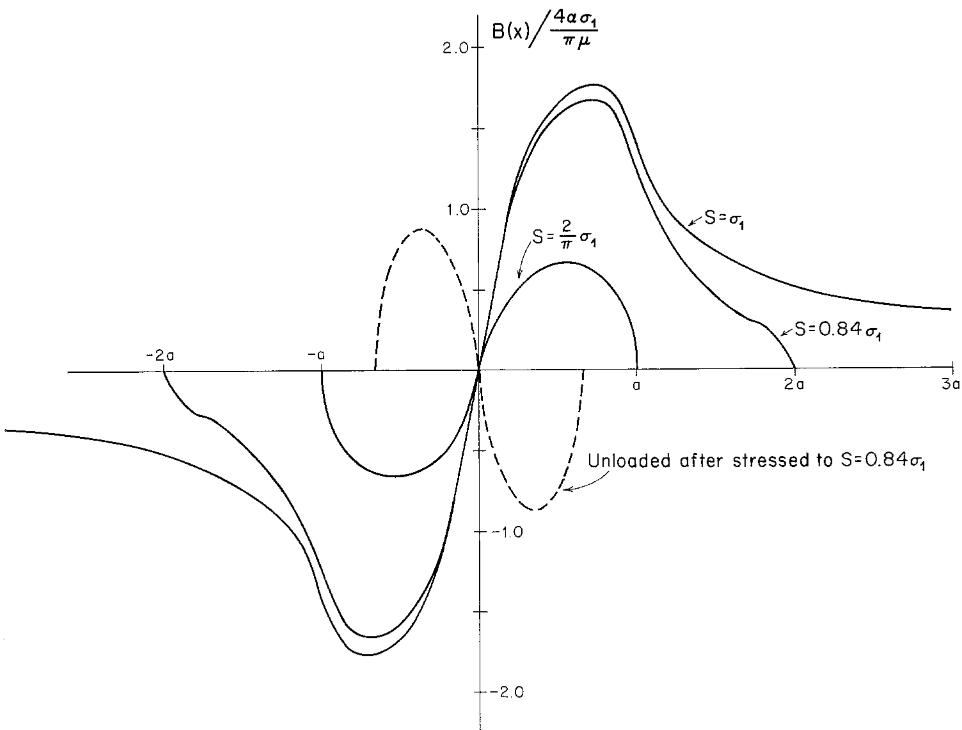

FIG. 5. Plot of the dislocation distribution around a dislocation source with linearly increasing friction. The dashed curve shows the additional distribution that arises when the source is unloaded.

This equation is valid only for stresses $S^* \leqq S$. When $S^* = S$, $B^*(x) + B'(x) = 0$ and the total dislocation distribution again is given by $B(x)$. If the stress is increased further the dislocation distribution is given by equation (16).

The displacement that occurs at the origin ($x = 0$) during reloading from the unloaded state is plotted in figure 4 for the case in which the fault previously was loaded to the level $S = 0.95\sigma_1$ and $\sigma_0 = 0$. It should be noted that the amount of slip increases rapidly when the reloading stress exceeds the stress level of the first loading. A curve with a much sharper break has been obtained. The closer the first loading approached σ_1, the sharper is the break.

Dislocation Source with Linearly Increasing Friction

Suppose the friction stress on a fault plane increases linearly from the origin to the points $x = \pm a$. Suppose further that the frictional stress has a constant value σ_1 for all values of $|x| \geqq a$. Thus the frictional stress in the region $-a \leqq x \leqq a$ is equal to $\sigma_1 |x|/a$. The dislocation distribution which results when a stress S is applied to the fault can be found from equations (4), (5) and (6). If b, the half width of the slipped zone, is less than a, the distribution function is

$$B(x) = \frac{4\alpha\sigma_1 x}{\pi\mu a} \log \left| \frac{b + \sqrt{b^2 - x^2}}{x} \right| \tag{21a}$$

where

$$b = \frac{a\pi S}{2\sigma_1}$$

The slipped zone has the width a at the stress $S = 2\sigma_1/\pi$.

When b exceeds a the dislocation distribution is given by:

$$\begin{aligned} B(x) = \frac{2\alpha\sigma_1 x}{\pi\mu a} \Bigg\{ & \log |a^2 - x^2| + 2 \log \left| \frac{b^2 + b\sqrt{b^2 - x^2}}{x} \right| \\ & - \log |(b^2 + \sqrt{(b^2 - a^2)(b^2 - x^2)})^2 - a^2x^2| \Bigg\} \\ & + \frac{2\alpha\sigma_1}{\pi\mu} \left\{ \log \left| \frac{a + x}{a - x} \right| + \log \left| \frac{b^2 - ax + \sqrt{(b^2 - a^2)(b^2 - x^2)}}{b^2 + ax + \sqrt{(b^2 - a^2)(b^2 - x^2)}} \right| \right\} \end{aligned} \tag{21b}$$

where

$$\frac{2}{a}(b - \sqrt{b^2 - a^2}) + \pi\left(1 - \frac{S}{\sigma_1}\right) = 2 \sin^{-1}\left(\frac{a}{b}\right)$$

The dislocation density given by these equations is plotted in figure 5 for the cases $b = a$ and $S = 2\sigma_1/\pi$; $b = 2a$ and $S = 0.84\sigma_1$; and $b = \infty$ and $S = \sigma_1$.

If the fault is unloaded our previous arguments may be used to find the additional dislocation density $B'(x)$

$$B'(x) = \frac{-8\alpha\sigma_1 x}{\pi\mu a} \log \left| \frac{b' + \sqrt{b'^2 - x^2}}{x} \right| \tag{22}$$

where b', the width of the reversed slip zone, is given by:

$$b' = \frac{a\pi S'}{4\sigma_1}$$

The total length of the Burgers vectors in the region from 0 to b' is

$$\int_0^{b'} B'(x)\,dx = -\frac{8\alpha\sigma_1 b'^2}{\pi\mu a} = -\frac{\pi\alpha S'^2 a}{2\mu\sigma_1} \tag{23}$$

where S' has the same meaning as before.

Figure 5 also shows a plot of $B'(x)$ for a completely unloaded fault which previously had been stressed to $S = 0.84\sigma_1$.

If the fault is reloaded a new distribution function can be obtained which equals $-B'(x)$ when the stress reaches the maximum value of the first loading. The dislocation density at still higher stress levels again is determined by equations (21).

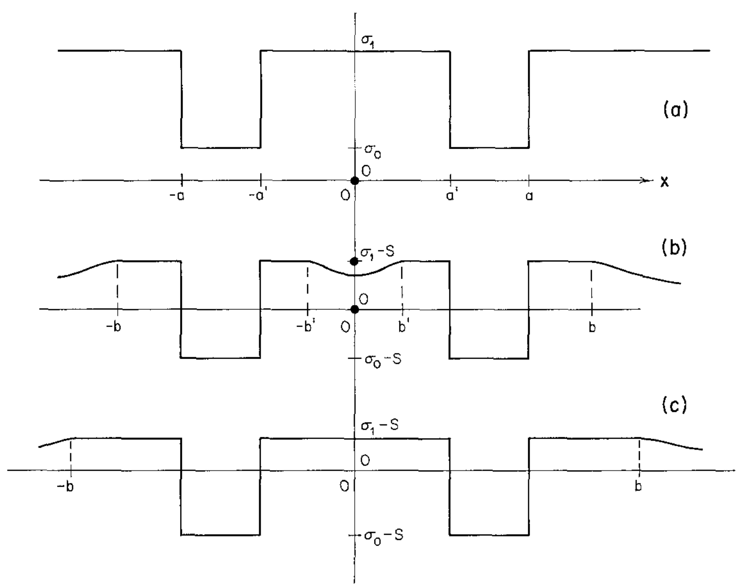

FIG. 6. Double source: the upper figure illustrates the dependence of the frictional stress upon distance. The middle figure shows schematically the stress arising from the dislocation distribution when slip has not extended from one source to the other. The lower figure shows schematically the stress arising from the dislocation distribution when slip does extend from one source to the other.

Double Source

Suppose the frictional stress on a fault is that shown in Figure 6a. There are two regions on the fault plane where the frictional stress has a lower value σ_0. These regions extend from $a' \leqq |x| \leqq a$. When the applied stress S is slightly larger than σ_0 slip occurs in a region $b' \leqq |x| \leqq b$, where $b' < a'$ and $b > a$. The newly created dislocations give rise to a stress $\tau(x)$ shown schematically in Figure 6b. (The value of $\tau(x)$ is known only between $b' \leqq |x| \leqq b$.)

When S exceeds σ_0 only slightly, the dislocation distribution around each source must be approximately the same as the distribution function given by equation (16) for a single source. The exact solution for the situation shown in Figure 6b can be found by using the more complicated equations (7) and (8). The exact solution contains a number of elliptic integrals and is extremely complicated.

As the applied stress is increased, eventually slip occurs everywhere in the region between the two sources. That is, b' becomes equal to zero. The stress produced by the dislocations then is that shown schematically in figure 6c. Slip has occurred everywhere between $-b \leqq x \leqq b$. The dislocation distribution for this situation is found easily from equations (4), (5) and (6). The dislocation density is

$$\begin{aligned} B(x) = \frac{2\alpha}{\pi\mu}(\sigma_1 - \sigma_0)\Bigg\{\log\left|\frac{a+x}{a-x}\right| - \log\left|\frac{a'+x}{a'-x}\right| \\ + \log\left|\frac{b^2 - ax + \sqrt{(b^2-a^2)(b^2-x^2)}}{b^2 + ax + \sqrt{(b^2-a^2)(b^2-x^2)}}\right| \\ - \log\left|\frac{b^2 - a'x + \sqrt{(b^2-a'^2)(b^2-x^2)}}{b^2 + a'x + \sqrt{(b^2-a'^2)(b^2-x^2)}}\right|\Bigg\} \end{aligned} \tag{24}$$

where

$$(\sigma_1 - \sigma_0)\left[\sin^{-1}\left(\frac{a}{b}\right) - \sin^{-1}\left(\frac{a'}{b}\right)\right] = \frac{\pi}{2}(\sigma_1 - S)$$

Equation (24) is not necessarily a satisfactory solution of the problem. In order for the density function to make physical sense it must satisfy the obvious requirement that $\int_0^b B(x)\,dx \geqq 0$. Equation (24) does not satisfy this requirement when b is only slightly greater than a. The only permissible values of b in equation (24) are those for which this integral is greater than or equal to zero.

If the applied stress is increased from zero, b' of figure 6b approaches the origin. Up to and including the moment the value of b' first becomes zero, the sum of the Burgers vectors of all the dislocations lying to the right of the origin must equal zero. Therefore at the stress which corresponds to $b' = 0$ the dislocation density given by equation (24) must satisfy the condition $\int_0^b B(x)\,dx = 0$. This relationship enables us to find the applied stress and the smallest value of b for which equation (24) is a physically meaningful solution. Setting $\int_0^b B(x)\,dx = 0$ gives

$$a \log\left(\frac{b}{a} + \sqrt{\frac{b^2}{a^2} - 1}\right) = a' \log\left(\frac{b}{a'} + \sqrt{\frac{b^2}{a'^2} - 1}\right) \tag{25}$$

The heavy curve of Figure 7 is a plot of the displacement, $\int_0^b B(x)\,dx$, at the origin for a double source with $a' = a/2$. (It is assumed here that the frictional stress σ_0 is zero.) For comparison purposes there also is plotted in this figure the displacements which occur at the center of single sources having half widths of $a/4$, $a/2$, and a. It can be seen that as the stress approaches σ_1 the displacement at the origin of a double source approaches the displacement of a single source having a half width equal to $a/2$. It should be noted that the extent of the region of lower frictional stress of a single source with half width of $a/2$ is identical to the extent of the low friction region of figure 6a. Therefore at large stresses a double source acts like a single source with an equivalent low friction region.

The stress at which $b' = 0$ for a double source with $a' = a/2$ is $S = 0.71\sigma_1$ when $\sigma_0 = 0$. It is interesting to compare this stress with the stress required to make a single source of half width $a/4$ produce slip out to the distance $(\frac{3}{4})a$ from its center (the half width of each of the sources in figure 6a is $a/4$). This distance is equal to the distance from the origin to the center of either of the sources in figure 6a. The stress required to produce slip over this area from the single source is $0.78\sigma_1$ when $\sigma_0 = 0$. This stress is close to the value $0.71\sigma_1$ for the double source. We concluded therefore that it is permissible to consider the dislocation distribution around each source in the situation pictured in Figure 6b, as approximately the same as that found when each source is an isolated single source. This approximation has useful application when earthquake faults with two or more source regions are considered.

Periodic Sources

Suppose the frictional stress on a fault can be expressed as $\sigma_1 + \sigma_p$, where σ_p is a periodic function of distance.

It is obvious that at the stress $S = \sigma_1 + \bar{\sigma}_p$, where $\bar{\sigma}_p$ is the average value of σ_p on the fault, slip will have occurred over the whole fault. The dislocation density on the fault is given by

$$B(x) = -\frac{2\alpha}{\mu\pi}\int_{-\infty}^{\infty}\frac{\sigma_p(y)\,dy}{x-y} \tag{26a}$$

If σ_p has a period λ such that $\sigma_p(y+\lambda) = \sigma_p(y)$, this equation can be rewritten as

$$B(x) = -\frac{2\alpha}{\mu\lambda}\int_{y}^{y+\lambda}\sigma_p(y)\cot\left[\frac{\pi}{\lambda}(x-y)\right]dy \tag{26b}$$

Consider two specific examples of σ_p. When $\sigma_p = (\sigma_1 - \sigma_0)\cos(2\pi x/\lambda)$:

$$B(x) = [2\alpha(\alpha_1 - \sigma_0)/\mu\pi]\sin(2\pi x/\lambda) \tag{27a}$$

for $S = \sigma_1 \equiv \sigma_1 + \bar{\sigma}_p$. Since a periodic function can be expressed as a Fourier series the values of $B(x)$ can be found for any function of σ_p which is periodic.

If $\sigma_p = \sigma_0 - \sigma_1$ for $-a < x < a$ and $-a + n\lambda < x < s + n\lambda$, where $n = \pm 1$, ± 2, ± 3, etc., and if $\sigma_p = 0$ elsewhere, then at the stress $S = \sigma_1 - (2a/\lambda)(\sigma_1 - \sigma_0) \equiv \sigma_1 + \bar{\sigma}_p$:

$$B(x) = \frac{2\alpha(\sigma_1 - \sigma_0)}{\mu\pi}\log\left|\frac{\sin\left[\frac{\pi}{\lambda}(x+a)\right]}{\sin\left[\frac{\pi}{\lambda}(x-a)\right]}\right| \tag{27b}$$

The maximum displacement (at the centers of the regions of lowest frictional stress) is $\alpha(\sigma_1 - \sigma_0)\lambda/\mu$ for the distribution (27a) and $2\alpha(\sigma_1 - \sigma_0)\lambda/\pi\mu$ for the distribution (27b). As would be expected the maximum displacement is proportional to the wavelength.

When the applied stress $S = \sigma_1 + \bar{\sigma}_p$ an infinite amount of slip can occur everywhere on the fault. Only a *finite* amount of slip takes places up to the moment when S reaches the level $\sigma_1 + \bar{\sigma}_p$. At this critical stress the fault plane will behave exactly like a fault plane whose frictional stress is a constant everywhere. The finite amount of slip on a fault with a periodic frictional stress has the effect of "priming" the fault into the condition in which it can slip catastrophically. This result suggests that creep motion along a fault (Steinbrugge and Zacher, 1960; Tocher, 1960; Whitten and Claire, 1960) and perhaps the movements produced by small earthquakes help prime the fault into a condition where a large earthquake can occur on it.

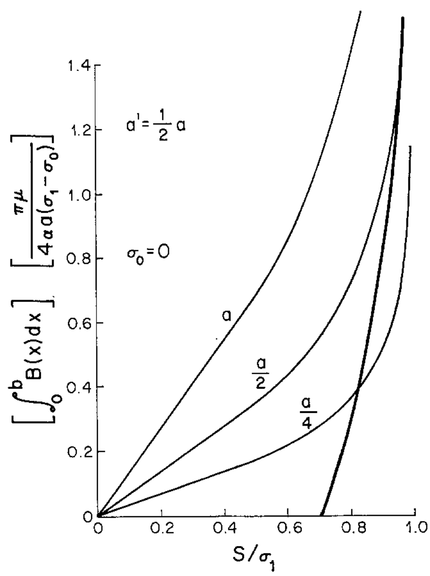

FIG. 7. Plot of displacement (heavy line) at $x = 0$ at the center of the double source of Fig. 6 as a function of applied stress. Also shown is the displacement at the center of single sources of various half widths.

EFFECT OF A FREE SURFACE

Suppose a fault terminates at a plane free surface which is perpendicular to the plane of the fault. The results of the previous sections still can be applied in this situation.

We learn from dislocation theory that when a screw dislocation is near a free surface, the additional stresses which arise because of the presence of the surface are identical to those which would be produced by an "image" screw dislocation having an opposite sign. Consider figure 8a. Here is shown a screw dislocation lying at a distance d from a free surface. The dislocation is parallel to this surface. The coordinate x is measured perpendicular to the free surface and y runs parallel to the surface and perpendicular to the dislocation. The stress acting at any point (x, y) for which $x > 0$ is identical to that which would be felt if all the free space $x < 0$ were filled up and a dislocation of opposite sign but equal strength were placed at the point $(-d, 0)$. This image dislocation is shown in the figure.

The stress fields of the real and the image dislocations combine to make the plane $x = 0$ free of traction. Therefore this surface could be cut without disturbing the stress field.

The fault can be considered as extending out into free space with the same frictional stress at $-x$ as the real fault has at x. Where the real fault has a dislocation density $B(x)$ of screw dislocations, the image fault has a density of image dislocations $-B(x)$ at $-x$. The real distribution of screw dislocations and the image distribution need only satisfy equations (2) and (3) integrated between $-\infty$ and ∞ in order to be a satisfactory solution to the problem of screw dislocations on a fault that terminates at a free surface. For example, in figures 3, 5, and 6, if a free surface were placed at $x = 0$, the solution found previously for those problems would still be valid.

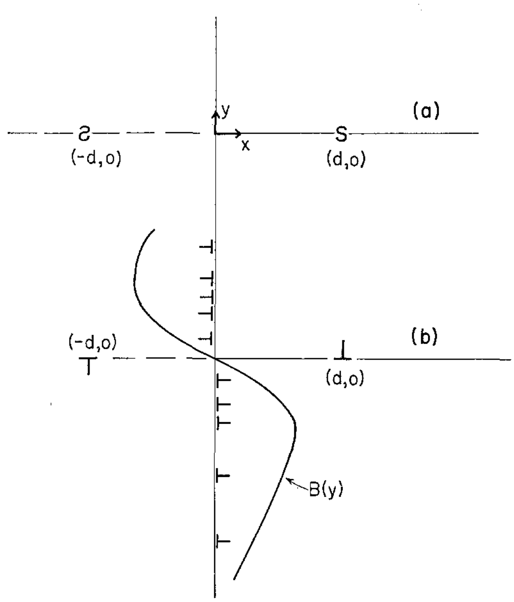

FIG. 8 (a) Image screw dislocation. (b) Image edge dislocations.

The problem of an edge dislocation near a free surface is more complicated than the corresponding case of a screw dislocation. If the edge dislocation is at $(d, 0)$, as in figure 8b, an image dislocation of opposite sign at $(-d, 0)$ will not completely satisfy the problem. A distribution of infinitesimal edge dislocations must be placed on the $x = 0$ plane. This distribution is shown in figure 8b. The density is given by the relationship $B(y) = (4byd^2)/(y^2 + d^2)^2$ where b is the length of the Burgers vector of the original dislocation. This distribution is required to make the plane $x = 0$ traction free.

The presence of an infinitesimal dislocation distribution on the plane $x = 0$ makes it impossible to apply rigorously the analysis of the previous sections to the problem of edge dislocations distributed on a fault terminating at a free surface. However if the continuous distribution on the free surface is ignored and only the image edge dislocations considered, our previous analysis gives a reasonably approximate solution to this problem.

Application to Slippage on Vertical Faults Near the Earth's Surface

We now will attempt to apply the results of the previous sections to the problem of slippage on vertical faults within continental blocks. These faults are imagined to continue as image faults above the earth's surface, which is a free surface.

Amonton's law of friction probably holds for those areas of a fault which are very close to the earth's surface. The frictional stress σ near the earth's surface may be given by:

$$\sigma = n\rho g x \tag{28}$$

where n is the coefficient of friction, g is the gravitational acceleration, and ρ is the average density of rock. The distance x is measured from the earth's surface. Equation (28) assumes that the only compressive stress acting across the vault at the depth x is the hydrostatic pressure $\rho g x$. If appreciable non-hydrostatic tensile or compressive stresses exist they would have to be included in this equation.

We assume that equation (28) breaks down at some depth a below the surface because the fault is lubricated or because of some unknown mechanism. The fact that foci of earthquakes within continental blocks occur at an average depth of around 15 kilometers suggests that the frictional stress may have a minimum value at this depth. We assume therefore that the frictional stress first increases with depth below the surface, reaches a maximum, decreases to a minimum value near 15 kilometers, and then increases. If this be the case the frictional stress may have a functional dependence approximating the function shown in Figure 9a. The image fault frictional stress also is shown in this figure. It is a mirror image of the frictional stress of the real fault.

The magnitude of the frictional stress can be estimated from the energy released in earthquakes. Benioff (1955), for example, calculated that the average elastic stress existing in the rock before a particular earthquake on the White Wolf Fault was 26 bars. We assume therefore that the maximum stress σ_1 of figure 9a must be of this magnitude.

The frictional stress at a constant depth undoubtedly varies as a function of the horizontal distance. If we restrict ourselves to faults whose horizontal dimensions are great, the horizontal variation of frictional stress is relatively unimportant. [Slippage occurred on the San Andreas Fault for a distance of the order of 400 kilometers during the San Francisco Earthquake. Movement appeared to have occurred over 1100 kilometers during the great Chilean Earthquake of 1960 (Press *et al.* 1961).] From the results of a previous section we know that after a finite amount of slippage a fault with a periodic frictional stress is primed and acts like a fault with constant friction. The same behavior should be found in a fault in which the variation in the frictional stress occurs over distances small compared with the length of the fault. We assume therefore that at a constant depth below the surface the horizontal variation in the frictional stress is effectively removed as a result of this priming action after an appreciable amount of slippage has occurred.

The fault of figure 9a contains three regions which may act as sources. They are located around the points $x = 0$ and $x = \pm f$, where f is the average depth of foci of earthquakes in continents. As the stress within the crust rises to a value approaching $\bar{\sigma}$, where $\bar{\sigma}$ is the average frictional stress on the fault, the infinitesimal dislocation distribution becomes that shown schematically in figure 9b. During this increase in the applied stress, slow creep slippage occurs across the fault. When the stress S reaches the level $\bar{\sigma}$, the fault is primed and further slip can take place catastrophically. Once the fault is primed, slip can commence anywhere on it. However the center of a source region is the most likely place for the following reason. More

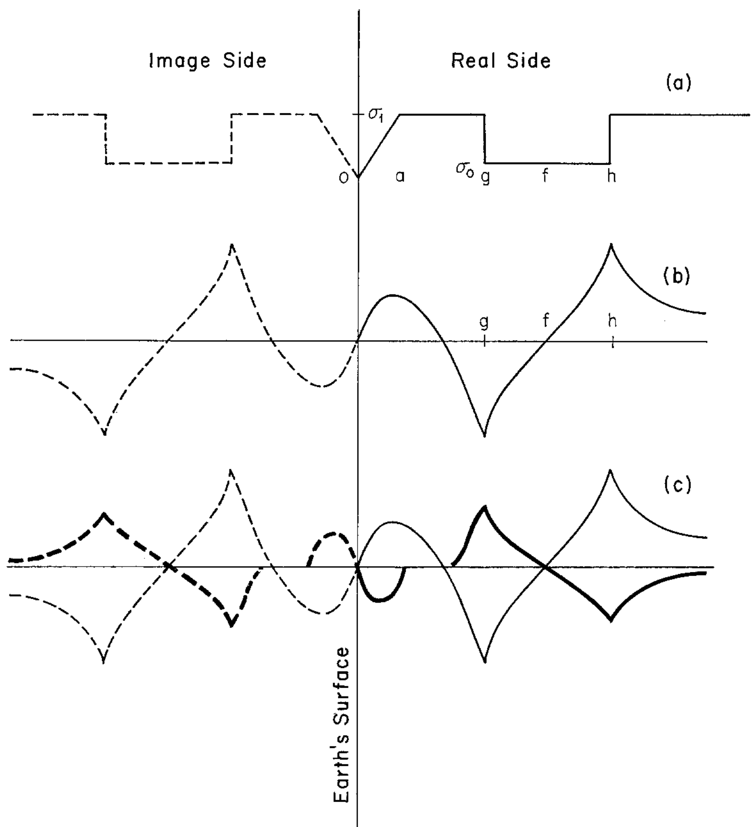

FIG. 9. Model of a vertical fault in the earth's crust. (a) The frictional stress on the fault, (b) The dislocation distribution (schematic) at the moment before the fault slips catastrophically. (c) Additional dislocation distribution (heavy curves) arising from relaxation of the load after catastrophic slipping.

slip takes place there prior to S reaching the level $\bar{\sigma}$. Therefore the horizontal variation of the frictional stress on the fault is more effectively removed at this particular depth.

The new dislocations that are created on a fault during catastrophic slippage cannot be slow moving. If they were the additional stress produced by them would lead to a total shear stress on the fault in excess of the frictional stress, a situation which is physically impossible. This difficulty is removed if the new dislocations are fast moving. Screw dislocations which move at a speed equal to the transverse sound velocity produce zero shear stress on their slip planes (Leibfried and Dietz, 1949;

Frank, 1949; Eshelby, 1949). Edge dislocations which move at the Rayleigh sound velocity (the Rayleigh wave velocity $\approx 0.9 \times$ transverse sound velocity) likewise produce zero shear stress on their slip planes (Weertman, 1961, 1963). Therefore when dislocations move at these velocities a fault experiences no additional shear stress and the frictional stress is not exceeded. Dislocations traveling at these speeds do not interact with each other. It is probable that the catastrophic slippage which

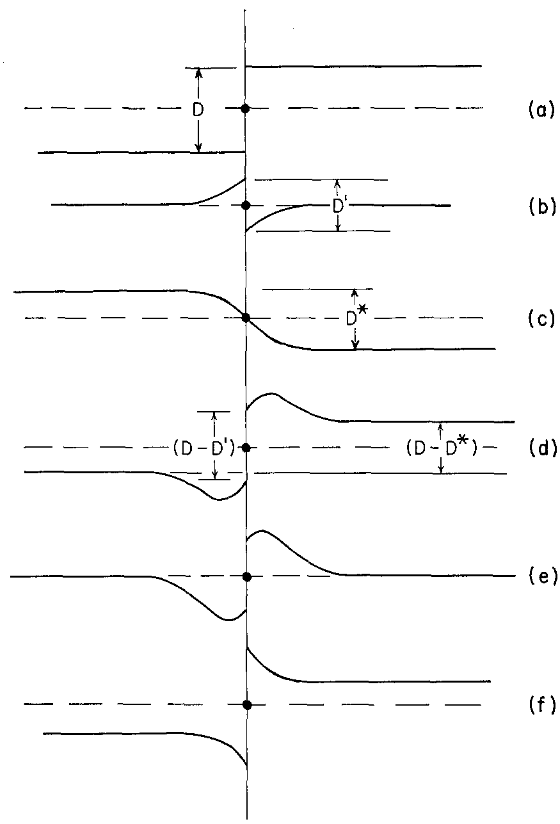

FIG. 10 (a) The displacement of a line on the surface of the earth, immediately after catastrophic slipping and before any relaxation of the dislocation distributions occur. This figure also represents the displacement after the fault is reloaded. (b) The displacement due to reverse slip in the source region at the earth's surface. (c) The displacement due to reversed slip in the source region centered at f in Figure 9. (d) The sum of the displacements of Figures 10 a, b, and c. (e) The displacement on a fault which has slipped only in the region $0 < x < f$. (f) The displacement of figure d when D' is negligible. (Note: The displacement produced by the applied stress itself is not shown in these figures.)

occurs when $S = \bar{\sigma}$ takes place by the movement of such fast moving dislocations. [Slippage on the fault of the great Chilean Earthquake appears to have propagated at a velocity close to the transverse and Rayleigh wave velocities (Press *et al.*, 1961).]

Suppose the dislocations created during catastrophic slippage move completely out of the fault, both at the earth's surface and at the base of the continent. If the stress S is not relaxed so that the (static) dislocation distribution of figure 9b remains unchanged, the fault trace at the earth's surface will be displaced a total distance D equal to the length of the Burgers vectors of the fast moving dislocations which have left the fault. A line drawn on the earth's surface perpendicular to the fault immediately before the catastrophic slippage will be displaced as shown in figure 10a. (We consider only screw dislocations in what follows.) The dashed line indicates the original marking, and the solid lines, the mark after slippage. The material on either side of the fault is displaced as rigid blocks.

A stress relaxation will occur across a fault after catastrophic slippage. It was learned in earlier sections that if the load is removed from a fault, slip in the reverse direction can take place in source regions. This reverse slip leads to a new dislocation distribution. The heavy lines in figure 9c represent the new dislocation distribution that must be added to the old in order to obtain the dislocation distribution in the unloaded state. In the region $0 < x < a$ the new dislocation distribution $B'(x)$ (the heavy line of figure 9c) introduced by reverse slip in the source region centered at $x = 0$ is similar to the distribution function of equation (22). An approximate value for the total strength D' of the Burgers vectors of the dislocations created during reverse slip may be obtained from equation (23). Thus

$$D' = \int_0^{b'} B'(x)\, dx \approx \frac{\pi\alpha\bar{\sigma}^2 a}{2\sigma_1\mu} \tag{29a}$$

This distribution of dislocations produces an effect roughly equivalent to that of a discrete dislocation of Burgers vector D' which enters the earth's surface and descends to a depth given by:

$$\text{depth} \approx \frac{\pi a\bar{\sigma}}{8\sigma_1} \tag{29b}$$

Figure 10b shows schematically the displacement to be expected around a fault at the earth's surface if a dislocation of Burgers vector D' moves to the depth given by (29b). The dashed line in this figure is drawn immediately prior to the introduction of this dislocation.

Reverse slip also may take place in the source region centered around point f of figure 9. The new dislocation density $B'(x)$ around this source is shown in figure 9c. The total length D^* of the dislocations in this distribution function which move from f up towards the earth's surface is given approximately by equation (19b), if a of that equation is set equal to $(f - g)$. If the load is completely relaxed,

$$D^* \approx \frac{8\alpha(f-g)\sigma_1}{\pi\mu} \log\left(\frac{b'}{f-g} + \sqrt{\frac{b'^2}{(f-g)^2} - 1}\right) \tag{30}$$

provided σ_0, the average frictional stress in the source regions, is small compared to σ_1. In this expression for D^*

$$b' \approx \frac{(f-g)}{\sin\left\{\frac{\pi}{4}\left(\frac{2\sigma_1 - \bar{\sigma}}{\sigma_1}\right)\right\}}$$

The dislocation distribution that arises during reverse slip of the source centered at f produces an effect roughly equivalent to that of a discrete dislocation of Burgers vector D^* which moves from f to g and another discrete dislocation of opposite Burgers vector which moves from f to h. Figure 10c shows schematically the displacement at the earth's surface around a fault produced by the movement of these dislocations from f to g and h. (It is assumed in this figure that the dislocation at h has moved to so great a depth that its effect on displacements at the surface is practically the same as if the dislocation had moved to the base of the continent.) The movement of the two dislocations D^* and $-D^*$ has the effect of partially or wholly cancelling the slippage produced in the region from g to h during catastrophic slip.

The total displacement of a straight line drawn on the surface prior to catastrophic slip is shown in figure 10d. This figure contains the sum of the displacements shown in the preceding figures 10a, b, and c. Figure 10e shows schematically the displacements that may result if the catastrophic slipping occurs only from f up to the surface and does not extend to the base of the continent. In this situation the dislocations that leave the fault at the surface of the earth will have left behind them in the region around f equal numbers of dislocations of opposite sign. When the load S is removed these dislocations which remain behind will be pushed up towards the regions around g by the same stress that produces reverse slip in figure 10c. The slip produced during catastrophic slippage will be cancelled in the region from f to g by this dislocation motion.

If D' of figure 10b is small, the displacements of figure 10d will look more like those shown in figure 10f. These latter displacements approximate those that have been observed to be produced on faults by earthquakes (Byerly and DeNoyer, 1958; Chinnery, 1961). (It should be noted that the depth of the dislocation D^* can be determined from the displacements at the surface. It can be shown by dislocation theory that the distance from a fault at which the displacement due to D^* changes by a factor of two is identical to the depth at which the dislocation resides.)

Figure 10f shows the displacement at the surface after the stress causing an earthquake has relaxed. If this stress builds up again the dislocations created during reverse slip will be pushed back towards the center of the source regions and eliminated. (See the section on the source unloaded and reloaded.) In other words, all the reverse slip that occurred in the source regions during the relaxation of the applied stress will be recovered. Therefore as the stress within the crust builds up the displacements shown in figure 10f slowly approach the displacements of figure 10a. As soon as the displacements become equal to those of figure 10a the fault is primed again and able to slip catastrophically. Thus in this ideal situation, it is possible to predict when the next earthquake will occur. Once it takes place the fault will go through the same cycle just described. This cycle can be repeated indefinitely, thereby leading to an infinite amount of slip across the fault.

Semi-Quantitative Calculation

We would like to show in this section that the fault model of figure 9 can lead to reasonable values for the Burgers vectors of dislocations produced by earthquakes (Byerly and DeNoyer, 1958; Chinnery, 1961).

The depth f at the center of the source region in the fault model of figure 9 was estimated as 15 kilometers on the basis of measurements of the depth of earthquake foci. The depth g can be estimated as being of the order of 7 kilometers since this is about the depth at which dislocations have been determined to exist from surface displacement measurements. From the results of the previous section g should approximate this depth. Since h and g lie symmetrically about f, the depth h is about 23 kilometers. The depth a of the source region at the earth's surface should be small. (It will be seen in a moment that this source region is relatively unimportant.) If the coefficient of friction is taken to be $n = 0.1$ and if σ_1 is of the order of 50 bars, a is approximately equal to 2 kilometers.

We will assume that the average frictional stress $\bar{\sigma}$ on the fault is equal to 26 bars, which is the stress level calculated by Benioff (1955) as having produced one particular earthquake. If σ_0 is small compared to σ_1, the average frictional stress of the fault model of figure 9 is $\bar{\sigma} = \sigma_1(L - a - h + g)/L$, where L is the thickness of a continental block. If L is taken to be 35 kilometers, σ_1 is equal to 50 bars.

If these values of $\bar{\sigma}$ and σ_1 are substituted into equations (29), it is found that the Burgers vector D' of the dislocation introduced into the fault during reverse slip at the surface of the earth is $D' \approx 7 \times 10^{-5}a = 14$ cm. (It is assumed that $\mu = 3 \times 10^{11}$ dynes/cm^2.) This is a very small dislocation. The average depth to which this dislocation penetrates is $0.2a = 400$ meters. It would be very difficult to detect the presence of this dislocation from displacements produced around a fault. This result shows that the surface source is relatively unimportant and that figure 10d must approximate closely figure 10f.

Equation (30) gives the length of the Burgers vector D^*. When $\bar{\sigma} = 26$ bars and $\sigma_1 = 50$ bars are placed into this equation it is found that $D^* \approx 1.5$ meters. This Burgers vector is about the same as those of the actual dislocations produced by earthquakes (Byerly and DeNoyer, 1958; Chinnery, 1961). Therefore the fault model of figure 9, or modifications of it, can account for the observations on displacements around faults and can resolve the difficulty that dislocations appear to be located at shallower depths than the foci of earthquakes.

We have accomplished what we set out to do, namely, to show that it is possible to explain the observation that dislocations produced by earthquakes lie above the earthquake foci. This explanation depends on the existence below the surface of the earth of a region on the fault which is characterized by an anomalously low frictional stress. The reader will appreciate that the support for the assumption of a low friction region is weak. We ourselves would prefer to have our analysis inverted to the following: given the fact that dislocations lie at shallower depths than earthquake foci, it is to be concluded that at some depth there must exist a region where the frictional stress is anomalously low.

Acknowledgment

This work was done while I was visiting professor at California Institute of Technology. I appreciate the kindness of Professor Robert Sharp and his invitation to be a guest of that school. I thank Professors Barclay Kamb and Don Anderson for making me aware that shallow dislocations present an interesting problem. I am indebted to Professor Frank Press for his comments on the first draft of this paper.

References

Benioff, H.
1955. "Mechanism and Strain Characteristics of the White Wolk Fault as Indicated by Aftershock Sequence", in *Earthquakes in Kern County California During 1952*, Bull. 171, Dept. of Natural Resources, Division of Mines, San Francisco, 199–202.
Bilby, B. A., A. H. Cottrell, and K. H. Swinden
1963. "The Spread of Plastic Yield from a Notch", *Proc. Roy. Soc.* (*London*), 272A: 304–314.
Byerly, P. and J. DeNoyer
1958. "Energy in Earthquakes as Computed from Geodetic Observations", in *Contributions in Geophysics in Honor of Beno Gutenberg*, H. Benioff *et al.*, eds., New York, Pergamon Press, pp. 17–35.
Chinnery, M. A.
1961. "The Deformation of the Ground around Surface Faults", *Bull. Seism. Soc. Amer.*, 51: pp. 355–372.
1963. "The Stress Changes that Accompany Strike-Slip Faulting", *Bull. Seism. Soc. Amer.* 53: 921–932.
Cottrell, A. H.
1953. *Dislocations and Plastic Flow in Crystals*, Oxford, Clarendon Press.
Eshelby, J. D.
1949. "Uniformly Moving Dislocations", *Proc. Phys. Soc.* (*London*), 62A: 307–314.
Frank, F. C.
1949. "On the Equations of Motion of Crystal Dislocations", *Proc. Phys. Soc.* (London) 62A: 131–134.
Head, A. K. and N. Louat
1955. "The Distribution of Dislocations in Linear Arays", *Australian J. Phys.* 8: 1–7.
Leibfried, G.
1951. "Verteilung von Versetzungen im Statischen Gleichgewicht", *Z. Phys.* 130: 214–226.
1954. "Versetzungsverteilung in Kleinen Plastisch Verformten Bereichen", *Z. angewandte Phys.* 6: 251–253.
Leibfried, G. and H. D. Dietz
1949. "Zur Theorie der Schraubenversetzung", *Z. Phys.* 126: 790–808.
Leonov, M. Ia. and N. Iu. Shvaiko
1961. "Introduction to the Dislocation Theory of Elasto-Plastic Torsion", in *Problems of the Continuum Mechanics*, Philadelphia, Soc. for Industrial and Applied Mathematics, pp. 260–266.
Mikhlin, S. G.
1957. *Integral Equations*, A. H. Armstrong, translator, New York, Pergamon Press.
Muskhelishvili, N. I.
1953a. *Singular Integral Equations*, J. R. M. Radok, translator, Groningen, P. Noordhoff.
1953b. *Some Basic Problems of the Mathematical Theory of Elasticity*, J. R. M. Radok, translator, Groningen, P. Noordhoff.
Orowan, E.
1960. "Mechanisms of Seismic Faulting", in *Rock Deformation,* Memoir 79, D. Griggs and J. Handin, eds. New York, Geological Soc. Amer. pp. 323–345.
Press, F., A. Ben-Menahem, and M. N. Toksöz
1961. "Experimental Determination of Earthquake Fault Length and Rupture Velocity", *J. of Geophys. Research*, 66: 3471–3485.
Read, W. T. Jr.
1953. *Dislocations in Crystals,* New York, McGraw Hill.
Scheidegger, A. E.
1963. *Principles of Geodynamics*, 2nd Edition, New York, Academic Press.
Steinbrugge, K. V. and E. G. Zacher
1960. "Creep on the San Andreas Fault: Creep and Property Damage", *Bull. Seism. Soc. Amer.*, 50: 389–396.

Tocher, D.

1960. "Creep on the San Andreas Fault: Creep Rate and Related Measurements at Vineyard California," *Bull. Seism. Soc. Amer.*, 50: 396–404.

Weertman, J.

1961. "High Speed Dislocations" in *Response of Metals to High Velocity Deformation*, P. G. Shewmon and V. F. Zackay, eds. New York, Interscience, pp. 205–247.

1963. "Dislocations Moving Uniformly on the Interface Between Isotropic Media of Different Elastic Properties", *J. Mech. Phys. Solids*, 11: 197–204.

Weertman, J., and J. R. Weertman

1964. Elementary Dislocation Theory, New York, MacMillan (in press).

Whitten, C. A. and C. N. Claire

1960. "Creep on the San Andreas Fault: Analysis of Geodetic Measurements along the San Andreas Fault", *Bull. Seism. Soc. Amer.*, 50: 404–415.

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Manuscript received April 17, 1964.