EFFECTS OF SURFACE AND SUBSURFACE IRREGULARITIES ON THE AMPLITUDES OF MONOCHROMATIC WAVES

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ABSTRACT

Measurements of surface ground motion generated by forced vibration of a nine-story reinforced concrete building at a distance of 2 to 5.5 km are described. Three components of the displacement field were measured at 13 points along a line traversing an elongated canyon underlain by a shallow and dipping alluvial layer. The variations of measured displacement amplitudes have been modeled by (a) a two-dimensional surface topographic feature corresponding to the average cross section of the canyon and (b) by a two-dimensional model of an alluvium valley excited by a line source. Comparison of the observed and computed amplitude variations with distance suggests that for the geometry corresponding to this experiment the effect of the dipping layer of alluvium seems to play a considerably more important role than the canyon.

INTRODUCTION

For some time it has been recognized that inhomogeneities in the Earth's crust and irregularities of surface topography influence the amplitudes of seismic waves recorded on or near the ground surface. Numerous models have been proposed and tested to predict such effects for known input motions, to correct for these effects, and to derive the properties of excitation from instrumental recordings (e.g., Hollis, 1971; Wong and Trifunac, 1974a, 1974b). In the development of such models the solution of the forward problem, i.e., the computation of surface motions for known model geometry and known input excitation, is easy in principle. Such a task usually leads to a unique solution once the model geometry, the boundary conditions, and the excitation function have been idealized and specified. The inverse problem (where one looks for an excitation function, having specified the model and the known output function, or searches for the model when input motions are known or assumed), however, usually has no unique solution. This important fact, therefore, cannot be overlooked when an attempt is made to interpret the characteristics of observed wave motions in terms of one or several simple models. Such studies can, of course, enrich one's physical understanding of such problems and clarify numerous phenomena associated with wave propagation in irregular media; but thus far they do not seem to offer an adequate basis for simple engineering computations which may be needed for design. Before it becomes possible to develop deterministic models which can correctly predict general characteristics of wave motions without being overly dependent on the detailed geometry of the problem, it might be preferable to employ simple empirical models for characterizing ground motions (e.g., Trifunac, 1976a, 1976b) as a preliminary vehicle. The detailed and often expensive computations which cannot be tested critically may not be justified at the present time.

The purpose of this study is to examine the adequacy of two simple two-dimensional models of surface and subsurface irregularities for their ability to predict the amplitude variations of $SH$ waves with distance for a particular geologic setting. The results of the analyses presented are limited to a particular experiment, and to the monochromatic wave excitation which is generated by a nine-story reinforced building.
oscillating in its apparent fundamental mode of vibration. Although such excitation provides well-controlled conditions for full-scale experiments, the fact that we are studying wave amplitudes at only one or more discrete frequencies limits the extent to which the conclusions and findings from this analysis can be generalized and be applied to a continuous frequency band which is of interest in earthquake engineering (generally from 0.05 to 30 Hz). However, by shaking a group of tall buildings at several of their lowest modes of vibration, it may be possible to create a number of discrete frequency sources which cover a broad frequency range. These experiments can then be used to test some aspects of soil-structure interaction and the resulting wave propagation through inhomogeneous and geometrically irregular media. Comparison of the results based on simple theoretical models and experimental measurements, which are both examined in this paper, may illustrate the feasibility and the extent to which such experiments can be extended for the study of more realistic three-dimensional models.

The Experiment

Figure 1 shows the general setting of the experiment which is described in this paper. The Millikan Library, located on the campus of the California Institute of Technology (north of California Boulevard and east of Lake Avenue in Figure 1) was excited to vibrate in its lowest NS mode of vibration at a frequency of about 1.8 Hz. This vibration was produced by one forced vibration generator (Hudson, 1962) which
was mounted on the roof of the Library. Shaker baskets were fully loaded with lead weights and at 1.8 Hz generated a periodic force in the NS direction with a maximum amplitude of about 2750 lbs. The resulting horizontal force and moment on the soil were estimated to be approximately $2.8 \times 10^6$ lb and $2.8 \times 10^7$ lb ft (Wong et al., 1976). This type of wave source generates motions whose NS components in the layered half-space consist of $SH$ and Love waves in the direction west of Millikan Library. This occurs because the radiation pattern of $SH$ and Love waves has a maximum in the direction toward east and west for a point source consisting of a harmonic NS force and a harmonic rocking moment acting in the same direction (Luco et al., 1975).

Approximately 3.5 km west of Millikan Library there is a surface topographic feature which will be referred to in this paper as a "canyon." This canyon, called Arroyo Seco, is located west of Orange Grove Boulevard in Pasadena and runs roughly in the NS direction. Figure 2 shows nine cross sections of surface topography along a pencil of lines emerging in a westerly direction from Millikan Library. This figure also shows the average and the average plus/minus one standard deviation for the nine cross sections. It can be seen that the average width of the canyon is approximately $\frac{1}{4}$ km with the maximum depth being less than 50 m.

Figure 1 shows the 13 locations where the three components of ground displacement (NS, EW and vertical) generated by the vibration of Millikan Library were measured. The recording equipment consisted of three ranger type seismometers (moving coil, velocity-type transducers, with natural period in the vicinity of 1 sec), an Earth Sciences SC-201A Signal Conditioner and an Ampex SP-300 tape recorder. The signal from a ranger type seismometer, proportional to the relative velocity of the transducer mass, was first amplified 350,000 times by the SC-201A Signal Conditioner. The velocity proportional voltage was then attenuated and passed through a filter which had 6 dB per octave slope and 90° phase shift so that the voltage output...
recorded on the tape would be proportional to the relative displacement of the transducer mass.

Analysis of displacement amplitudes at these 13 locations was performed by using an analog band-pass filter centered at 1.8 Hz. Three Krohn-hite filters (two model 3750's and one model 335) were employed to filter the data recorded on magnetic tape. The cutoff frequencies for the band-pass filters were selected to be 1.2 and 2.4 Hz. The amplitudes of the filter transfer function were chosen to have a decay with the slopes of 18 dB/octave for frequencies lower than 1.2 Hz and higher than 2.4 Hz. The filtered voltage output was then used to determine the amplitudes of recorded displacements at all 13 locations.

Table 1 presents the normalized NS displacement amplitudes for the 13 locations where measurements were taken (Figure 1). Displacement amplitudes at point 1 were arbitrarily set to unity. No attempt was made to determine actual displacement amplitudes because we are only concerned with the relative amplitude changes with distance across the canyon cross section.

The signal-to-noise ratio for the measured NS displacements ranged from about 15 for point 1 to about 1.5 to 2 for points 6 through 13 (Figure 1). Recorded displacement amplitudes in the EW and vertical directions at points 1, 2 and 3 were about one order of magnitude smaller than the recorded NS displacements. For points 4 through 13 the signal-to-noise ratio for EW and vertical displacements was equal to one or less, showing that these amplitudes are consistently about one order of magnitude smaller than NS displacements. This is as would be expected, since for the NS excitation of Millikan Library the theoretical radiation pattern for EW and vertical displacement in the direction east from the Library would have zero amplitudes (Luco, et al., 1975). The recorded EW and vertical displacement amplitudes for this particular experiment are not exactly zero, because the general trend of the points where the measurements have been taken is approximately W5N and because of the inhomogeneities waves encountered between the Library and the canyon.

The accuracy of the relative amplitudes as measured for points 1 through 13 (Table 1) depends on the assumption that the source excitation does not change with time,

<table>
<thead>
<tr>
<th>Station</th>
<th>Distance from Source (km)</th>
<th>NS Displacement Amplitudes (normalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>2.3</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>2.6</td>
<td>0.51</td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>3.2</td>
<td>0.12</td>
</tr>
<tr>
<td>6</td>
<td>3.5</td>
<td>0.11</td>
</tr>
<tr>
<td>7</td>
<td>3.6</td>
<td>0.08</td>
</tr>
<tr>
<td>8</td>
<td>3.9</td>
<td>0.10</td>
</tr>
<tr>
<td>9</td>
<td>4.1</td>
<td>0.11</td>
</tr>
<tr>
<td>10</td>
<td>4.3</td>
<td>0.16</td>
</tr>
<tr>
<td>11</td>
<td>4.3</td>
<td>0.17</td>
</tr>
<tr>
<td>12</td>
<td>5.2</td>
<td>0.09</td>
</tr>
<tr>
<td>13</td>
<td>5.4</td>
<td>0.08</td>
</tr>
</tbody>
</table>
i.e., that the frequency and the amplitudes of building vibration do not change throughout the time required to measure displacements at all 13 locations. This uniformity of the source was monitored carefully during this test. One operator continuously checked the shaker controls throughout the experiment and reported no frequency variations of the shaker greater than the normal accuracy (1 to 2 per cent) with which the Monsanto frequency counter (model 100A) can detect the number of shaker revolutions per minute.

**Theoretical Analysis**

The simplicity of the harmonic source, as well as the good quality of the recorded ground-motion signals render this set of experimental data particularly suitable for theoretical modeling. With this and similar types of well-controlled experimental measurements, it is now possible to initiate a series of critical tests which will experimentally examine the usefulness of different theoretical models. An exact three-dimensional analytical solution of the problem presented in this paper is clearly well beyond the theory which is now available; the finite element or finite difference schemes would in all probability prove to be too restrictive and too expensive even for a two-dimensional analysis that satisfies the fundamental principles of the wave propagation approach. For these reasons, we examine two aspects of the two-dimensional analytical wave-propagation models to show what might be learned from such a simplified analysis and to set the stage for more advanced work in the future. In particular, we investigate the influence of surface topography and the influence of a dipping alluvium layer separately to see which of these two effects is more important in governing the observed amplitude variations (Table 1).

1. Analysis for surface irregularities. We first assume that the variations of the recorded amplitudes tabulated in Table 1 are entirely a consequence of the wave scattering in the vicinity of surface irregularities, i.e., that the variable depth of alluvium does not contribute to these amplitude variations. The following analysis can then be developed to take into account only the surface effects.

The canyon geometries shown in Figures 1 and 2 might roughly be assumed to be two-dimensional (cross sections 3 to 8 in Figure 1) over the length of interest. The analysis can then be reduced to a simple one which may be handled by existing two-dimensional theories.

With the assumption that the subsurface soil conditions can be neglected, one can begin by using the wave solutions for a homogeneous medium. For $SH$-type (anti-plane) motion, the harmonic displacement $w e^{i\omega t}$ satisfies the scalar Helmholtz equation in two dimensions,

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + k_1^2 w = 0,$$  \hspace{1cm} (1)

where $k_1 = \omega/\beta_1$ is the wave number and $\beta_1$ is the shear-wave velocity in the soil medium.

The method of solution for equation (1) is generally not difficult, but the number of geometrical configurations for which it is possible to find an exact solution is limited. For wave scattering by more realistic topographic cross sections such as those shown in Figure 2, for example, this problem can be solved only by approximate or numerical methods.

Using Weber's integral formulas (Mow and Pao, 1971), one may obtain an integral
solution for differential equation (1) everywhere in the soil medium once the boundary
value, \( w^b \), and its normal derivative, \( \partial w^b / \partial n_0 \), at the scatter's surface are determined.
For the particular problem which we are dealing with, the boundary values may be
determined numerically by first quantizing the integral equation into an \( N^{th} \)-order
matrix equation of the form

\[
[ A_i ] w^b = w^f_f
\]  

(2)

where \( w^b \) and \( w^f_f \) are the approximate representations of \( w^b \) and the "free-field"
motion at \( N \) discrete points on the scatterer's surface. Clearly, the approximation of
a continuous integral equation by a discrete matrix equation improves as the order \( N \)
increases. Details for obtaining matrix equation (2) for this analysis may be found in
Appendix A and in previous publications, e.g., Banaugh and Goldsmith (1963), and
Wong and Jennings (1975).

After the numerical representation of \( [ A_i ] \) has been determined, the only quantity
left to be specified is the free-field displacement, \( w^f_f \). In several previous studies
(Wong and Trifunac, 1974a, b; Trifunac, 1973), the assumption of plane incident
waves was frequently used because the curvature of the wave front becomes negligible
when it is relatively far from the source. Since the distance between the Millikan
Library and the Arroyo Seco is about 3.5 km, we will assume that the incident \( SH \)-
wave motion generated near the canyon may also be modeled by plane waves.

In the half-space, the "free-field" motion consists of the incident plane wave and
its reflection from the half-space surface; hence,

\[
w^f_f(x, y, \theta) = 2 \exp \left( i \eta \left( \frac{x}{l} \right) \cos \theta \right) \cos \left[ \eta \left( \frac{y}{l} \right) \sin \theta \right],  \]  

(3)

where \( \theta \) is the angle measured from the positive \( x \) axis (Figure 3, a and b). The dimen-
sionless parameter \( \eta \) used in equation (3) may be defined as \( 2 \kappa o / 2 \pi \), where \( 2l \)
is the width of the canyon surface \( \Gamma \). Physically, \( \eta \) represents the ratio of the canyon width
\( 2l \) to the incident wavelength \( \lambda \).

For the present analysis, the following values will be used:

- \( 2l \approx 0.7 \) km
- \( \beta_0 \approx 2 \) km/sec (or less)
- \( \omega \approx 11.6 \) rad/sec (the lowest NS frequency of the Millikan Library).

Hence, \( \eta \) for this particular experiment is approximately 0.65 (or more). Because of
the uncertainties which exist in \( \beta_0 \) (it changes with depth) and \( \theta \) (the incident wave
energy may come from several directions because of the reflective boundary between
the bedrock and alluvium), a range of numerical values of \( \eta \) and \( \theta \) were chosen for the
calculation of theoretical surface displacements.

In Figure 3, the two cases for \( \eta = 0.4 \) and \( \eta = 1.0 \) are presented along with the
experimental data. The theoretical amplitudes of surface displacements were plotted
with different dashed lines for different angles of incidence, \( \theta = 0^\circ, 30^\circ, \) and \( 60^\circ \).
The spectral amplitudes for vertical incidence (\( \theta = 90^\circ \)) are presented by a solid line.
The normalized raw experimental data are represented in Figure 3 by squares; the
data corrected for geometric spreading in terms of \( (r / r_0)^n \), where \( r_0 \) is the distance
between Millikan Library and point 1 in Figure 1, are also plotted with various other
symbols as indicated in the figure.
Fig. 3. Computed surface displacement amplitudes (a) for $\eta = 0.4$ and (b) for $\eta = 1.0$. 
Each set of the normalized displacement data after being multiplied by $(r/r_0)^n$ was least-squares fitted to the theoretical amplitudes for $\eta = 0.4$ and $\eta = 1.0$ shown in Figure 3. This scaling was performed to maximize the chances for finding similarity of relative amplitude variations with distance and by disregarding the actual amplitudes of both experimental data and theoretical models. Variations of surface displacement amplitudes which are predicted by the two-dimensional theoretical model of the canyon in Figures 1 and 2 are typically less than 20 per cent for $\eta = 0.4$ and 1.0, for plane $SH$ waves in homogeneous elastic half-space when incidence angle, $\theta$,

![Fig. 4. Depth of alluvium in west Pasadena (after Gutenberg, 1957).](image)

varies from $0^\circ$ to $90^\circ$. On the other hand, the variations of measured displacement amplitudes are much larger and, for all values of $n$ considered in Figure 3, a and b, do not even resemble the amplitudes predicted by the theoretical model. This comparison of theoretical and measured amplitudes in Figure 3, a and b, thus indicates that the observed displacement amplitudes in this experiment cannot be explained by the two-dimensional representation of surface topography of the Arroyo Seco (Figures 1 and 2) alone and that there may exist some other more prominent factor that governs the observed variations of surface displacement amplitudes at points 1 through 13. We hypothesized that this factor may be associated with the alluvium crystalline rock interface beneath the test area.

II. Analysis for subsurface irregularities. The central and western portions of Pasadena are underlain by alluvium whose depth ranges from zero to about 1200 ft north-
west of Millikan Library. Figure 4 (from Gutenberg, 1957) shows the variations of the alluvium depth there and in the canyon area studied in this paper. Figures 4 and 5 show that the depth of the alluvium underneath the Library is approximately equal to 900 ft and becomes shallower toward the west and southwest. Approximately 4.5 km west of the Library and north of the Raymond fault (Gutenberg, 1957; Figure 4) the crystalline basement rocks and deep tertiary become exposed on the ground surface. The Raymond fault, which is roughly parallel to sections AA and BB (Figures 4 and 5) and located about 1.5 km south of Millikan Library, introduces an abrupt increase of alluvium depth toward Huntington Drive. North of Millikan Library, alluvium depth first increases and then decreases toward the north and northwest.

The detailed three-dimensional geometry of the body of alluvium and the periodic NS vibration of Millikan Library cannot be modeled by available analytical methods. Finite element or finite difference schemes are difficult to apply for a steady-state source in this problem. Thus, we choose a method which is similar to the two-dimensional analysis used to study surface topographies. We will examine a simplified two-dimensional model of an alluvium layer overlying a homogeneous half-space, but will allow its depth to vary with distance as indicated in Figures 4 and 5 (sections AA and BB). Such simplification is adequate because we intend to focus our attention on a small area in the vicinity of the Arroyo Seco, the total extent of this zone being less than 3 km. Also, considering that the waves generated by the Library vibrating at its lowest NS mode are less than about 1 km long, we expect that the two-dimensional geometry in the immediate vicinity of the experimental stations might control the amplitude variations.

In the analysis which follows, the assumption will be made that the top surface of the alluvium layer is plane, and any possible influences of the topography in and near the canyon will be neglected. This is equivalent to the assumption that the wave scattering and diffraction caused by surface and subsurface irregularities may be considered separately. Although this is not correct, the analysis of the previous section has indicated that the overall contribution to the amplitude variation caused by the surface irregularities does not exceed about 20 per cent. This suggests that the influence of surface topography can be temporarily ignored during the analysis of subsurface inhomogeneities.

Following the assumptions made above, we can again obtain a numerical solution to the problem by the integral equation method. However, unlike analysis 1, two integral equations are now required because the displacements \( v_1 e^{i\omega t} \) in the semi-infinite medium and \( v_2 e^{i\omega t} \) in the layer satisfy the Helmholtz equation (1) with wave numbers
\[ [A_{II}]\mathbf{w}^b = \mathbf{w}_{II}^f, \]

in which \( \mathbf{w}_{II}^f \) is the incident-wave motion caused by the vibration of the Millikan Library.

Since the analysis and the geometry described above have been derived for a two-dimensional problem, the input waves \( \mathbf{w}_{II}^f \) have also been taken as two-dimensional. The input waves generated by the Library are of a spherical rather than cylindrical nature. But since the wave scattering and diffraction take place far from the source and for a narrow range of azimuth angles, the spherical waves might be approximated well by a cylindrical wave front in the vicinity of Arroyo Seco, the major difference being in the rate of amplitude attenuation with distance. Therefore, allowing for a later correction for this rate of amplitude decay, one may assume the input waves from the Library to be of the form

\[ \mathbf{w}_{II}^f(|\mathbf{r} - \mathbf{r}_s|) = \frac{H_0^{(2)}(k_2 |\mathbf{r} - \mathbf{r}_s|)}{H_0^{(3)}(k_2 a)}; \quad |\mathbf{r} - \mathbf{r}_s| \geq a, \]

which represents a cylindrical wave generated by an embedded foundation with a semi-circular cross section (Luco, 1969). \( \mathbf{w}_{II}^f \) of equation (5) is arbitrarily scaled to 1 at the location of the foundation; the radius of the foundation cross section, \( a \), is approximately 25 meters. Since the characteristic dimension of the foundation is much smaller than that of the alluvial valley, the detailed configuration of the Library foundation is important only in the vicinity of Millikan Library and the wave form in the far-field is usually not altered (Wong, 1975).

Figure 6 presents the results which are based on the above two-dimensional theory. It shows NS displacement amplitudes for three models (A, B and C) which differ only in the assumed variations of alluvium depth with distance. Models A and B

![Graph showing comparison of computed and measured displacement amplitudes](image-url)
(Figures 4 and 5) have symmetric cross sections, while model C coincides with model B west of Millikan Library and has considerably greater depth east of the Library. Alluvium depths for models A and B were derived from Figure 4 for a distance corresponding to the full length of sections A and B in Figure 4. For distances greater than about 2.5 km east of the Library, alluvium depth for models A and B was assumed arbitrarily. To test whether this arbitrary extension of alluvium layer has some significant influence on the computed displacements west of the Library we considered model C.

For the two-dimensional line source moving as $1 e^{\omega t}$ (for $\omega = 11.6$ rad/sec), Figure 6 shows surface displacement amplitudes east and west of the source for models A, B and C. For the excitation and the geometry considered here, it appears that the shape of the alluvium valley east of the Library has little or no effect on the amplitudes in the western portion of the valley. A comparison of displacement amplitudes for models B and C, for example, shows no significant difference for motions west of the Library and considerable difference for motions east of the Library. Although one case study is not enough to conclude that the shape of alluvium east of the Library has little or no effect on the motions west of the Library, in general, these results suggest that, for the particular experiment studied in this paper, the manner in which the alluvium valley is terminated east of the Library probably influences motions near Arroyo Seco by only a negligible amount.

Measured displacements at 13 points traversing Arroyo Seco (Figure 1) are shown with triangles in Figure 6. The relative amplitudes of these data points were scaled to have amplitudes comparable to the computed amplitudes. As we have already pointed out, no attempt was made to correct for overall amplitudes because of the discrepancy between the three-dimensional experiment and the two-dimensional model. Rather, the effort of this work was concentrated on the relative variations of amplitudes with distance. As can be seen in Figure 6, the variations of measured displacements with distance appear to be well characterized by the computed variations of displacement amplitudes for model A. All measured amplitudes do not fall exactly on the full line corresponding to model A but follow its general trend quite well.

Minor departures between the measured and computed displacement amplitudes might be interpreted as resulting from more complicated layer geometry than that represented by model A, but could also be attributed to the interaction of incident waves with the irregular surface topography (Figure 2).

If we consider relative variations of the displacement amplitudes only for the four points in the immediate vicinity of Arroyo Seco, then, as Figure 7 indicates, the relative changes of recorded displacements for this short distance become comparable to the changes predicted by the model for the effects of surface topography only. Vertical error bars for the five measurements in Figure 7 show estimated total errors in recording and processing this data set.

The above discussion, of course, cannot be interpreted to mean that the simple model consisting of a variable layer of alluvium (model A) underlain by a homogeneous elastic half-space uniquely explains the observed displacement amplitudes. However, the characteristic pattern of the observed variations of displacement amplitudes and a virtually identical pattern predicted by model A suggest that it is very likely that the approach considered in this paper is pointing in the right direction.

One of the most important elements in numerous engineering studies which deal with the effects of alluvium layers on the variations of strong earthquake ground motion is associated with the assumption concerning the direction and the manner in
which seismic waves enter into the model. Whether seismic energy arrives from below, from a side, outside or from within, the body of alluvium layer can lead to remarkably different surface displacement amplitudes for a given model. Patterns of constructive and destructive interference as well as the fraction of energy scattered by the material discontinuities in the model, can change dramatically as a function of these different inputs. An initial buildup and then a rapid decay of observed displacement amplitudes probably results from the focusing effect of progressively shallower layer as wave energy propagates west from the source (Figures 5 and 6). The sloping bottom of the layer turns the incident rays to reflect off the free surface with progressively larger angles of incidence. When this angle exceeds 90°, instead of propagating further on and away from the source, the ray turns back toward the source. For an ensemble of different rays the net effect would thus be to focus a considerable amount of energy in the vicinity of the turning point. Here the input energy is partly reflected through the bottom of the layer, radiated away into the underlying half-space, and partly reflected back toward the source. Only a fraction of total input energy, which is associated with essentially horizontal rays, is transmitted farther on past the focusing point where rays are turned back toward the source. Behind this turning point, one would expect to find relatively low displacement amplitudes and then a gradual buildup as the depth of alluvium reaches zero. This gradual build up of amplitudes would result from the waves traveling through the underlying half-space. The foregoing qualitative interpretation of what might be caused by a dipping layer of alluvium seems to be quite consistent with the results presented in Figure 6. On the other hand, if one were to assume that all input energy arrives from below through the underlying half-space, as it is commonly done for some simple models of local site conditions, the focusing of incident waves would be significantly diminished or would be eliminated altogether.

\[ \eta = 0.60 \]

![Diagram](image.png)

**Fig. 7.** Computed surface displacement amplitudes for \( \eta = 0.6 \), and the experimental data measured inside the Arroyo Seco Canyon.
Conclusions

The findings of the above analysis may be summarized as follows:

1. It appears that the monochromatic waves generated by a large building vibrating at one of its natural frequencies are suitable for study of shallow wave propagation through soil and alluvium layers. The steady-state nature of such excitation and the stability of the wave source in time offer a rare opportunity for full-scale low-amplitude wave propagation experiments.

2. Because the modeling of the variations of surface displacement amplitudes resulting from the waves propagating through irregular alluvium layers and scattering from surface topography is associated with a high degree of nonuniqueness, the well-controlled monochromatic full-scale tests provide an invaluable tool for critical examination of simple theoretical models.

3. Unless it can be demonstrated that simple one- or two-dimensional models are capable of displaying the overall characteristics which can be observed during well-controlled experiments, it will be difficult to justify the use of such models in routine seismological and engineering applications.

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Appendix A

The following is a brief description of the integral equation method used in this paper for the study of surface and subsurface irregularities. As shown by Wong and Jennings (1975), the method of images can be used to analyze two-dimensional SH-wave problems with the half-space as a basic configuration. Here, we shall extend the method to cover the case of a layer overlying a semi-infinite soil medium as illustrated in Figure 8. Because of the arrangement of the images, the soil surface is assumed to be flat; but the shape of the interface, Σ, between the layer (medium 2) and medium 1 below can be arbitrary.

Integral and approximate matrix equations for the boundary values. By first defining
the Green's function, $G$, for the half-space as

$$
G(k_j | \mathbf{r} - \mathbf{r}_0) = \frac{i}{4} \left[ H_0^{(2)}(k_j \sqrt{(x - x_0)^2 + (y - y_0)^2}) + H_0^{(2)}(k_j \sqrt{(x - x_0)^2 + (y + y_0)^2}) \right] \quad (A1)
$$

in which $H_0^{(2)}$ is the Hankel function of the second kind; $k_j$ is the wave number of the $j^{th}$ medium; $\mathbf{r} = (x, y)$ and $\mathbf{r}_0 = (x_0, y_0)$ are the position vectors for the "observation" and "source" points, respectively, the Weber's Integral equations for the boundary values at $\Sigma$, $w_j$, and $\partial w_j / \partial n_0$, $j = 1, 2$; can be written as (Mow and Pao, 1971)

$$
\frac{1}{2} w_j(\mathbf{r}) = S_j \int_\Sigma \left[ w_j(\mathbf{r}_0) \frac{\partial G(k_j | \mathbf{r} - \mathbf{r}_0 |)}{\partial n_0} - G(k_j | \mathbf{r} - \mathbf{r}_0 |) \frac{\partial w_j(\mathbf{r}_0)}{\partial n_0} \right] dS_0
$$

$$
+ w_j^{hf}(\mathbf{r}), \mathbf{r} \text{ on } \Sigma, \quad (A2)
$$

the notation, $\mathbf{r}$ on $\Sigma$, indicates that the observation point $\mathbf{r}$ is taken on the boundary surface $\Sigma$. The sign $S_j$ in equation (A2) is defined as: $S_1 = +1$ (for exterior diverging wave problems), and $S_2 = -1$ (for interior wave problems). Finally, the term $w_j^{hf}(\mathbf{r})$ on the right-hand-side of equation (A2) represents the "free-field" motion for medium $j$ in the absence of the diffracted waves from the irregular surface $\Sigma$.

To numerically determine the solution of $w_j$ and $\partial w_j / \partial n_0$, $j = 1, 2$; one can quantize the continuous integral equations (A2) into $N \times N$ matrix equation of the form

$$
\frac{1}{2} [I] w_j = S_j [D_j] w_j - [G_j] \dot{w}_j + w_j^{hf} \quad (A3)
$$

where $w_j$, $\dot{w}_j$, and $w_j^{hf}$ are approximate representations of $w_j$, $\partial w_j / \partial n_0$, and $w_j^{hf}$ at $N$ chosen points on the boundary. The matrices $[D_j]$ and $[G_j]$ are numerical decompositions of the integrals in equations (A2) such that

$$
[D_j] w_j \cong \{ \cdots , \int_\Sigma w_j(\mathbf{r}_0) \frac{\partial G(k_j | \mathbf{r}_k - \mathbf{r}_0 |)}{\partial n_0} dS_0 , \cdots \}, \quad (A4)
$$

and

$$
[G_j] \dot{w}_j \cong \{ \cdots , \int_\Sigma G(k_j | \mathbf{r}_k - \mathbf{r}_0 |) \frac{\partial w_j(\mathbf{r}_0)}{\partial n_0} dS_0 , \cdots \}, \quad (A5)
$$

in which $\mathbf{r}_k$ is the $k^{th}$ discrete point on $\Sigma$. The detailed method for numerically evaluating $[G_j]$ and $[D_j]$ can be found in Banaugh and Goldsmith (1963).

**Boundary conditions.** Along with the two integral equations, two boundary conditions are necessary for determining the total of four variables, $w_1$, $\partial w_1 / \partial n_0$, $w_2$, and $\partial w_2 / \partial n_0$. They are (a) continuity of displacements at $\Sigma$, i.e., $w_1 = w_2$; and (b) continuity of stresses at $\Sigma$, i.e., $\mu_1 (\partial w_1 / \partial n_0) = \mu_2 (\partial w_2 / \partial n_0)$, where $\mu_1$ and $\mu_2$ are the shear moduli of medium 1 and 2, respectively. In discrete vector form, the boundary conditions are simply

$$
w_1 = w_2, \quad (A6)
$$
and

\[ \mu_1 \dot{w}_1 = \mu_2 \dot{w}_2, \quad (A7) \]

**Solution to analysis I.** For analysis I, only a local canyon topography is considered; thus, we can take \( \mu_2 \) to be zero and \( \Sigma \) as the irregular surface of the canyon. Since \( w^b \) is zero from the boundary condition (A7), equation (A3) simplifies to a matrix equation for the displacement \( w_1 \),

\[ \{ \frac{1}{2}[I] - [D_2] \} w_1 = w_2'' , \quad (A8) \]

where \( w_1'' \) is the free-field motion in medium 1.

**Solution to analysis II.** For analysis II, the incident-wave excitation \( w_1'' \) is zero because all the waves are generated within medium 2. Hence, the matrix equations (A3) may be rearranged as

\[ \dot{w}_1 = -[G_2]^{-1} \{ \frac{1}{2}[I] - [D_1] \} w_1, \quad (A9) \]

and

\[ \{ \frac{1}{2}[I] + [D_2] \} w_2 - [G_2] \dot{w}_2 = w_2''. \quad (A10) \]

Using boundary conditions (A6) and (A7), equation (A9) may be expressed as

\[ \dot{w}_2 = -\frac{\mu_1}{\mu_2} [G_1]^{-1} \{ \frac{1}{2}[I] - [D_1] \} w_2, \quad (A11) \]

which, upon substitution into equation (A10) yields an \( N^{th} \)-order matrix equation for the unknown variable \( w_2 \)

\[ \left( \{ \frac{1}{2}[I] + [D_2] \} + \frac{\mu_1}{\mu_2} [G_2][G_1]^{-1} \{ \frac{1}{2}[I] - [D_1] \} \right) w_2 = w_2''. \quad (A12) \]

The numerical solution of \( w_2 \) can then be used to obtain \( \dot{w}_2 \), \( w_1 \), and \( \dot{w}_1 \) by equations (A11), (A6), and (A9), respectively.

**Integral formulas for surface amplitudes.** The above derivation is concerned mainly with the determination of the boundary values at \( \Sigma \). For most analyses, however, these boundary values are not as important as the amplitude on the soil surface. For such cases, the displacements at the flat surface or anywhere in medium 1 and 2 may be calculated from the known boundary values by Weber's integral formulas

\[ w_j(r) = S_j \int_\Sigma \left[ w_j(r_0) \frac{\partial G}{\partial n_0} (k_j | r - r_0 |) - G(k_j | r - r_0 |) \frac{\partial w_j(r_0)}{\partial n_0} \right] dS_0 \]

\[ + w_j''(r), \quad j = 1, 2, \quad (A13) \]

in which \( r \) is located in medium \( j \).
References


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