SCALING RELATIONS FOR EARTHQUAKE SOURCE PARAMETERS AND MAGNITUDES

By Robert J. Geller

Abstract

A data set of 41 moderate and large earthquakes has been used to derive scaling rules for kinematic fault parameters. If effective stress and static stress drop are equal, then fault rise time, τ, and fault area, S, are related by $\tau = 16S^{1/2}/(7\beta^{3/2})$, where $\beta$ is shear velocity. Fault length (parallel to strike) and width (parallel to dip) are empirically related by $L = 2W$. Scatter for both scaling rules is about a factor of two. These scaling laws combine to give width and rise time in terms of fault length. Length is then used as the sole free parameter in a Haskell type fault model to derive scaling laws relating seismic moment to $M_s$ (20-sec surface-wave magnitude), $M_s$ to $S$ and $m_b$ (1-sec body-wave magnitude) to $M_s$. Observed data agree well with the predicted scaling relation. The “source spectrum” depends on both azimuth and apparent velocity of the phase or mode, so there is a different “source spectrum” for each mode, rather than a single spectrum for all modes. Furthermore, fault width (i.e., the two dimensionality of faults) must not be neglected. Inclusion of width leads to different average source spectra for surface waves and body waves. These spectra in turn imply that $m_b$ and $M_s$ reach maximum values regardless of further increases in $L$ and seismic moment. The $m_b$ versus $M_s$ relation from this study differs significantly from the Gutenberg-Richter (G-R) relation, because the G-R equation was derived for body waves with a predominant period of about 5 sec and thus does not apply to modern 1-sec $m_b$ determinations. Previous investigators who assumed that the G-R relation was derived from 1-sec data were in error. Finally, averaging reported rupture velocities yields the relation $v_R = 0.72\beta$.

Introduction

The purpose of this paper is to examine empirical relations between gross fault parameters and the agreement of these relations with theoretical models of seismic sources. The gross parameters to be studied are fault length, width, and rise time, rupture velocity $m_b$ and $M_s$, and seismic moment. Data from other investigators’ studies of individual earthquakes are used to study scaling of source parameters in an approximate way. In general, the data are consistent with fault width scaling proportionately to fault length and rise time scaling proportionately to the square root of fault area. This scaling can then be used to find $m_b-M_s$, $\log M_s-M_s$ and $\log S-M_s$ relations. Some of those relations have been studied by Kanamori and Anderson (1975b), who provided a theoretical basis for many of the empirical relationships used in seismology.

Tsuboi (1956) was the first investigator to utilize similarity, the concept of relating earthquakes of different sizes by a one-parameter model. By assuming that the horizontal dimensions of the earthquake source volume were three times larger than the vertical dimension, Tsuboi derived from the relation $E = \mu e^2 A^{1.5}/6$, where $E$ is released energy, $\mu$ is average rigidity, $e$ is average strain drop and $A$ is aftershock area. Such approximate scaling relations, as first pointed out by Tsuboi, require that the physics of material failure be identical for large and small earthquakes. If that assumption is generally true and if earthquakes tend to be geometrically similar, then it follows that fault length and width,
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<th>$W$ (km)</th>
<th>$D$ (m)</th>
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and final dislocation all will scale together. Differences in material properties will weaken the exactness of the similarity when earthquakes from two different regions are compared, but, in an approximate sense, similarity, as is shown by the data presented below, is a valid concept.

The first paper to systematically relate observed gross seismic source parameters to the source spectrum was the now classic work of Aki (1967). Although the results presented in this paper modify his results, the methodology and basic outlook are similar to Aki’s. Later Brune (1970, 1971) contributed to the understanding of seismic source spectra.

Similarity between earthquakes is a dynamic as well as a static concept. Not only the final static parameters, but also the spectral shape of the equivalent source-time function, scale with fault length. Spectral similarity can best be demonstrated by comparing two earthquakes with identical location and focal mechanism, but different magnitude. Such a comparison ensures that seismograms from both events will be affected equally by the medium response, so that all differences between the records will be from the source effects.

Observational studies of similar pairs of earthquakes have been made by Berckhemer (1962) whose results were interpreted by Aki (1967) to support Aki’s $\omega^{-2}$ model. Tsujiura (1973) studied groups of events from various regions, concluding that most data were in accord with Aki’s $\omega^{-2}$ model, but that some were better fit by an $\omega^{-1}$ or $\omega^{0}$ model.
One cannot directly compare spectral characteristics of source mechanisms from different regions without first correcting the seismograms for transmission effects. Removing the effects of medium response will usually require use of synthetic seismogram methods. We assume however that one can compare logarithmic fault parameters, such as $m_b$, $M_s$ or $\log L$, for events in different regions. These comparisons are made with the intention of looking at order of magnitude relationships rather than details.

In this paper, we will look at scaling relations between five sets of such logarithmic parameters: $\log L$ versus $\log W$ (fault length versus width), $\log \tau$ versus $\log S$ (fault rise time versus area), $M_s$ versus $\log M$ (surface-wave magnitude versus seismic moment), $m_b$ versus $M_s$, and $\log S$ versus $M_s$. What will be shown are not exact correlations, but rather trends which appear to be applicable to most earthquakes. Agreement between the simple model used in this paper and the data are quite good. (Long narrow transform faults, such as the San Andreas, are a separate class of faults which are not considered in this paper.)

**Observational Data Set**

The earthquake data shown in Table 1 are from the same 41 shallow events used by Kanamori and Anderson (1975b). All values for $M_s$ are from their paper; the sources for all other observational parameters are given in the table of references. Except for minor differences which are primarily due to the use of slightly different references, data for $M_s$ and $S$ are equivalent to that of Kanamori and Anderson (1975b). Each numbered entry in the table of references corresponds to the earthquake with the same number in Table 1. All but two of the columns are observational data; $\tau^*$, predicted rise time, and $\Delta\sigma$, calculated stress drop, will be discussed below. Length and width have been taken from the references, or in some cases estimated. Length always refers to length along the strike, regardless of focal mechanism; width refers to width along the dip. Average dislocation comes either from field measurements or from dividing the moment (determined from seismograms) by the area and an assumed value of the shear modulus.

For all events since August 1963 the $m_b$ value is either taken directly from the PDE Monthly Summary, or calculated from the data in Earthquake Data Reports. As reported by L. M. Murphy in Bâth (1969), USCGS (later NOAA and now USGS) asks for the amplitude of the largest pulse (with period less than 3 sec) in the first five cycles of the teleseismic $P$ or $P_n$ arrival. The values of $A$ and $T$ are then used in the Gutenberg-Richter formula

$$m_b = \log_{10} \left( \frac{A}{T} \right) + Q,$$

(1)

to derive $m_b$ for each station. Values more than 0.7 magnitude units from the mean are deleted and the final average is then taken.

Estimates of rise times typically were made by fitting the first upswing on long-period local records to synthetic seismograms calculated using the Haskell (1969) whole-space model at one or two stations. Clearly it would be desirable to use synthetics made for more realistic models of earth structure, but they have not yet been calculated for these events. Uncertainties due to the tradeoff between rise time and rupture velocity and due to the model may combine to cause errors which cannot be estimated. In some cases, such as the Tottori earthquake (Kanamori, 1972b), rupture velocity and rise time are independently constrained.

**Length versus Width**

Fault length is plotted against fault width in Figure 1. It can be seen that (with considerable scatter) observational data demonstrate that $L=2W$. In Figure 1, the
numbers refer to earthquakes in Table 1. Intraplate events are plotted as open circles and interplate events as solid circles. (This convention is also used in all later figures.) There is not any clear difference between the interplate and intraplate groups. Abe (1975c) has independently found $L = 2W$ for a data set of mainly Pacific earthquakes.

The Haskell model uses $L$ as the direction in which rupture propagates, while $L$ was measured along the direction of the strike for Figure 1. It is implicitly assumed, then, that for these 41 events, rupture propagated parallel to the strike. This almost certainly is false for some thrust events such as San Fernando (Boore and Zoback, 1974; Trifunac, 1974; Mikumo, 1973b), and may well be false for events like Nemuro-Oki with $L < W$. In spite of these exceptions it seems that rupture usually propagates parallel to the strike, especially for strike-slip faults.

![Fig. 1](image)

**Fig. 1.** Plot of fault length (along strike) versus fault width (along dip) for earthquakes in Table 1. Open circles are intraplate events; closed circles are interplate events. Numbers refer to Table 1. These conventions are used for all plots of earthquake data.

**Rise Time versus Theoretical Predictions**

Kanamori (1972b) showed that

$$
\dot{D} = \frac{\bar{D}}{\tau} \approx \frac{\sigma_{e,o} \beta}{\mu}, \tag{2}
$$

where $\dot{D}$ is dislocation velocity, $\bar{D}$ is average dislocation, $\tau$ is rise time, and $\sigma_{e,o}$ is effective stress. If one assumes that effective dynamic stress is equal to static stress drop, $\Delta\sigma$, this assumption can be tested by comparing observed rise times to the theoretically predicted rise time

$$
\tau^* = \frac{\mu \bar{D}}{\beta \Delta\sigma}. \tag{3}
$$

One can obtain stress drop in closed form for only a few simple models. The most straightforward of these is the circular crack with constant stress drop discussed by Keilis-Borok (1959). For that model stress drop is given by

$$
\Delta\sigma = 7\pi^{3/2}\mu \bar{D}/(16\sqrt{S}) = 7M_0/(16(LW/\pi)^{3/2}). \tag{4}
$$
Although this formula does not give the exact stress drop for the rectangular fault model, it follows from the work of Sato (1972) that this is a good approximation. If we substitute (4) in (3), where $S$ is the area of the rectangular fault and $\bar{D}$ is average dislocation, we obtain

$$\tau^* = 16S^{1/2}/(7\pi^{3/2}\beta).$$

(5)

The values of $\Delta\sigma$ and $\tau^*$ in Table 1 were calculated using (4) and (5), respectively.

Figure 2 is a plot of observed versus predicted [from (5)] rise times for a number of earthquakes. It can be seen that, again with considerable scatter, observational and theoretical rise times are in agreement. Abe (1975b) reached a similar conclusion from a data set of five Japanese earthquakes.

The agreement between theoretical and observed rise times has important implications for engineering seismology. The only observational parameter required in (5) is fault area, which frequently can be estimated from geological data. If total dislocation can also be determined from geological field work, then particle velocity near the fault, an important parameter in engineering seismology, can be reliably estimated. This is potentially of great value in areas lacking in historical seismicity or good instrumental data.

Rupture Velocity

Table 1 lists rupture velocities reported by various investigators. These values were determined from matching synthetic seismograms to local records or from surface-wave analysis. To a certain extent then, these values are model-dependent. Some also may be affected by the difficulty in resolution between rise time and rupture velocity. Nevertheless, these measurements probably represent a good average sample of rupture velocity measurements. If one picks values of $\beta$ ranging from 3.5 km/sec for shallow crustal events to 4.5 km/sec for events breaking the entire lithosphere, one then can calculate that the average value of $(v_R/\beta)$ is 0.72.
Archuleta and Brune (1975) found $v_R/\beta = 0.7$ in experiments on fracture of prestressed foam rubber. Their measured value was for the surface of the foam rubber, but if one assumes infinite rupture velocity along the dip, their result agrees very well with the result $v_R/\beta = 0.72$ observed for earthquakes. (Their minimum possible value for $v_R/\beta$ at depth is $0.63\beta$.) Agreement between the earthquake and foam rubber rupture velocities may be fortuitous or may be caused by a common physical friction mechanism.

**Choice of Fault Models**

All “deterministic” source models specify some (nearly always kinematic) conditions at the source, which then fix via the representation theorem of de Hoop (1958) and Burridge and Knopoff (1964), the complete time history at every point in the medium. [Aki (1967, 1972) and Haskell (1966) proposed “statistical” models in which only the amplitude spectrum at the source function is specified. Since we will be looking at dislocation rise times, these statistical models are not appropriate choices.] Typically the source theory papers calculate seismograms for an isotropic homogeneous whole space. Since our interest is in logarithmic source parameters, we will assume the whole-space models are adequate.

In most deterministic source models, either fault dislocation (e.g., Haskell, 1969, and Mikumo, 1973b) or stress drop [and therefore fault dislocation, e.g., Burridge and Willis (1969), Richards (1973), Sato and Hirasawa (1973)], is specified, which in turn gives displacement at other points in the medium. Other authors, e.g., Hanson et al. (1974) and Andrews (1975), have studied numerical models with friction between the fault surfaces.

All of these models predict far-field pulses which scale linearly with fault dimensions. Also they all yield flat spectra at low frequencies and $\omega^{-n}$ high-frequency asymptotes ($n \geq 2$). Thus all of the models have at least one “corner frequency” (and some have several). For these models the static or low-frequency level, which is proportional to seismic moment, grows as $L^3$.

We will continue to use the Haskell (1964, 1969) model of a rectangular fault (shown in Figure 3) in this paper. Most studies have used this model in the determination of rise times from local seismograms. The basic Haskell model is a fault with length $L$, width $W$, rise time $\tau$ (linear ramp time function), final dislocation $D$ and rupture velocity $V_R$. Rupture is instantaneous in the width direction and propagates (starting at one end) along the length with velocity $v_R$. Some investigators have made the natural extension to bilateral rupture propagation.

Haskell's (1964) expressions are for a “one-dimensional” model in which width is included only as a weighting factor in the moment. Hirasawa and Stauder (1965) and
Mikumo (1969) included the complete effect of the width to obtain an expression for spectral source amplitude.

\[ \mu_c(\omega) = \frac{M_o R_{s\theta}^c}{4\pi \rho r c^2} \left| \frac{\sin(\omega \chi_t)}{\omega \chi_t} \right| \left| \frac{\sin(\omega \chi_L)}{\omega \chi_L} \right| \left| \frac{\sin(\omega \chi_W)}{\omega \chi_W} \right| . \]  

In (6) \( M_o \) is moment, \( \rho \) is density, \( r \) is distance, \( c \) is either \( P \) or \( S \) velocity and \( R_{s\theta}^c \) is the radiation pattern (given by Haskell, 1964). \( \chi_L \) and \( \chi_W \) are duration times associated with length and width, respectively, and determined by fault geometry and position of the observer.

\[ \chi_L = |L(1/v_R - \cos \theta/c)/2| \]  

\[ \chi_W = |W(\cos \phi \sin \theta)/(2c)| \]  

\[ \chi_t = \tau/2. \]

### Spectral Characteristics

For the present, let us adopt (in slightly modified form) the similarity relations given by Kanamori and Anderson (1975b).

\[ \frac{W}{L} = C_1 = \text{const} \]  

\[ \frac{D}{L} = C_2 = \text{const} \]  

\[ \frac{\beta \tau}{L} = C_3 = \text{const} \]

(10) is the condition of geometrical similarity; (11) and (12) imply constant stress drop and constant effective stress.

We will select values of the constants which seem to be good averages of observational data. We found that

\[ L = 2W \]

seemed to be the approximate average of the empirical data. When we substitute \( L = 2W \) into (5) and set \( \beta = 4.0 \text{ km/sec} \) we get

\[ \tau = \left[ 16 \sqrt{L^2/2/(7\pi^{3/2} \cdot 4)} \right] = 0.0726L \]

where \( \tau \) is in seconds and \( L \) is in kilometers.

We could use (11) directly to get a scaling relation between fault displacement and length. In practice though, most estimates of \( D \) in Table 1 come from dividing \( M_o \) by \( \mu S \), so it seems better to relate moment directly to length. Setting \( L = 2W \) in (4) gives moment in terms of fault length and stress drop.

\[ M_o = L^3 \Delta \sigma \cdot 16/(7(2\pi)^{3/2}) = (1.45 \times 10^{20})L^3 \Delta \sigma \]

where \( M_o \) is in dyne centimeters, \( L \) is in kilometers and \( \Delta \sigma \) in bars.

From (6) we can isolate a spectral factor, dependent only on fault parameters and frequency.

\[ A(\omega) = L^3 \left| \frac{\sin(\omega \chi_t)}{\omega \chi_t} \right| \left| \frac{\sin(\omega \chi_L)}{\omega \chi_L} \right| \left| \frac{\sin(\omega \chi_W)}{\omega \chi_W} \right| . \]
The $L^3$ term follows from the similarity relation $M_o \sim L^3$. When $A(\omega)$ is multiplied by stress drop and the constant in similarity equation (15) we get the source moment rate spectrum.

Equation (16) and the factors (7) to (9) are well-known results for the case of a rectangular fault in a whole space. These expressions can also be applied directly to the case of a rectangular fault with horizontal rupture propagation in the Earth. The form of the expressions remains identical, but $c$ now should be interpreted as the apparent velocity along the Earth's surface.

For body waves $c = v_c / \sin i$, where $v_c$ is the near-field $P$ or $S$ velocity and $i$ is the takeoff angle of the teleseismic ray from the focal sphere. This can be understood physically by invoking reciprocity. Signals from a source at the position of the teleseismic receiver would be picked up $(L \cos \theta)/(2c)$ sooner at the end of the fault than at the center. Thus for the case of infinite rupture velocity, this is the difference between arrivals at the receiver from the end and center of the fault. This type of geometrical interpretation can be applied to both (7) and (8), so that these factors are seen to be the difference in arrival times obtained from geometrical optics. Ben-Menahem (1962) gives a more rigorous derivation of this result.

For surface waves (7) is the well-known directivity factor first given by Ben-Menahem (1961). If we neglect the variation of the excitation function with depth, (8) is the factor for the effect of fault width on the surface-wave spectrum. In both (7) and (8), $c$ is the (frequency-dependent) surface-wave phase velocity. The geometrical interpretation of (7) and (8) as phase delay between “arrivals” from the center of the fault and the ends is the same as for body waves.

Typical values of $c$ for teleseismic $P$ waves might be 14 km/sec, while for surface waves 4 km/sec is appropriate. If rupture velocity, $v_R = 2.5$ km/sec, then for surface waves $1/c$ will be of the same order of magnitude as $1/v_r$. As $\theta$ varies from 0 to $2\pi$, $\chi_L$ will range from 0.15 $L$ to 0.65 $L$. Thus a horizontally propagating rupture will cause a large directivity effect. On the other hand, for body waves from the same source, $\cos \theta = \sin i$, so because of the relatively steep takeoff angles of teleseismic rays, it is reasonable to assume $|\cos \theta| < 0.5$.

This assumption leads to the conclusion that $\chi_L$ will vary only from 0.36 $L$ to 0.44 $L$. There will be only a small azimuthal dependence (i.e., directivity effect) of the teleseismic body-wave pulse. This implies that one can infer the nature of a horizontal rupture propagation much more easily from surface waves than teleseismic body waves.

Because $c$ is the apparent velocity, rather than the near-field $P$ or $S$ velocity, it is inadequate to present only a single spectrum representing the effect of the seismic source, as was done, for example, by Aki (1967, 1972). There is a separate “source spectrum,” $A(\omega)$, for each body-wave phase or surface-wave mode, because of the different values of $c$. The source spectrum depends on azimuth, source dimensions, and the phase velocity, $c$. Both the length factor, (7), and the width factor, (8), are different for each mode. In the next section we will average $\chi_L$ and $\chi_w$ over all azimuths. In these averages, the value of $c$ will affect only $\chi_w$, even though both factors are affected at nearly every particular azimuth.

Both (6) and (16) completely neglect the effect of the Earth's transfer function on observed seismic waves. If one were to calculate synthetic seismograms for an individual earthquake, it would be necessary to consider the Earth's response and the earthquake source parameters, e.g., Langston and Helmberger (1975) for body waves or Harkrider (1964, 1970) for surface waves. Langston and Helmberger (1975) have demonstrated that $sP$ and $pP$ phases play a crucial role in the “$P$ wave” from shallow earthquakes. Similarly, one must consider the surface-wave excitation functions and the source mechanism to calculate accurate Rayleigh and Love amplitudes.

In this paper we consider trends among events, rather than accurate determination of parameters of particular events. Therefore, we assume that the effect of the earth structure
averages out when we construct scaling relations. Thus we will use (16) to get relations between \( m_b \) and \( M_S \), \( \log L \) and \( M_S \), \( \log M_o \) and \( M_S \).

### Average Spectra

We now want to find average asymptotic forms for \( \log A(\omega) \) from (16). In particular we require expressions for teleseismic \( P \) phases (from which \( m_b \) will be determined) and for 20-sec Rayleigh waves (from which we find \( M_S \)). For both cases we will find average values of \( \chi_L \) and \( \chi_W \) which take the direction of radiation into account. In making our approximation, we will replace \( |(\sin X)/X| \) by one for \( X < 1 \) and by \( X^{-1} \) for \( X \geq 1 \).

Takeoff angles of teleseismic body waves are nearly vertical. We will adopt the approximation that the rays take off straight down. Thus, for body waves, we set \( \theta = \pi/2 \) in (7) and (8). Also, in that case, \( |\cos \phi| = \sin \delta \), where \( \delta \) is dip angle of the fault plane. Using these values, average spectral factors for body waves are

\[
\langle \chi_L \rangle_{\text{body}} = \frac{L}{2v_R} \quad (17)
\]

and

\[
\langle \chi_W \rangle_{\text{body}} = W \sin \delta/(2c). \quad (18)
\]

For surface waves we will average \( \chi_L \) and \( \chi_W \) for \( \theta = 0 \) to \( \theta = 2\pi \). On the Earth’s surface we get \( |\cos \phi| = \cos \delta \). Thus we get

\[
\langle \chi_L \rangle_{\text{surf}} = \frac{L}{2v_R} \quad (19)
\]

and

\[
\langle \chi_W \rangle_{\text{surf}} = W \cos \delta/(\pi c). \quad (20)
\]

Comparison of (17) and (18) with (19) and (20) shows that the average corner frequency due to fault length will be the same for body waves and surface waves, but that the corner frequencies due to width will be different. This difference affects the high-frequency spectrum only since the average corner frequency for width is higher than that for rise time or length. Note that we have assumed that rupture propagates parallel to the Earth’s surface to obtain (17) to (20).

Before calculating numerical values for (17) to (20) we must fix \( v_R \), \( \delta \) and \( c \). Also we will use (13) to relate \( L \) to \( W \). We will set \( v_R = 2.88 \text{ km/sec} \), \( \delta = 45^\circ \), \( c = 14 \text{ km/sec} \) for body waves and \( c = 3.9 \text{ km/sec} \) for surface waves. (For the Earth, \( c \) must be the appropriate phase velocity, not the S-wave velocity. Neglecting the frequency dependence of surface-wave phase velocity is a reasonable approximation.)

From (9) and (14)

\[
\chi_L = 0.0363 L = C_r L \quad (21)
\]

(17) and (19) both give

\[
\langle \chi_L \rangle = 0.174 L = C_L L. \quad (22)
\]

The width factor for body waves is

\[
\langle \chi_W \rangle_{\text{body}} = 0.0126 L = C_{\text{wb}} L. \quad (23)
\]

For surface waves we get

\[
\langle \chi_W \rangle_{\text{surf}} = 0.0289 L = C_{\text{ws}} L. \quad (24)
\]
We now can approximate the logarithm of $A(\omega)$ from (16).

\[
\begin{align*}
\log A(\omega) &= 3 \log L & \text{for } \omega < (C_L L)^{-1} \\
\log A(\omega) &= 2 \log L - \log \omega - \log C_L & \text{for } (C_L L)^{-1} < \omega < (C_L L)^{-1} \\
\log A(\omega) &= \log L - 2 \log \omega - \log (C_L C_L) & \text{for } (C_L L)^{-1} < \omega < (C_L L)^{-1} \\
\log A(\omega) &= -3 \log \omega - \log (C_L C_L C_L) & \text{for } (C_L L)^{-1} < \omega.
\end{align*}
\]

(25)

The spectra from these relations are plotted in Figure 4. Note that the body-wave spectra have a much longer interval of $\omega^{-2}$ decay than the surface-wave spectra. Calibration of these curves with $M_S$ and $M_o$ is discussed below. Also, note that $\omega$ is in radians in (25), but in hertz in Figure 4.

The asymptotic spectral amplitudes given by (25) are very similar to the results obtained by Kanamori and Anderson (1975b). They used the same asymptotic approximation for

\[
\sin X/X \text{ in conjunction with Haskell's (1964) spectral expression. Since Haskell's expression ignores the effect of width on the spectrum, the results of this paper differ from those of Kanamori and Anderson (1975b) only at frequencies above the corner frequency for width. For example, the model in this paper predicts constant 20-sec spectral amplitude for faults longer than 110 km while that of Kanamori and Anderson (1975b) predicts amplitudes which increase linearly with $L$. As a result, their model predicts $M_S \sim \log L$ for large events, while this paper predicts $M_S = \text{const}$.}

Even though both spectra in Figure 4 have an eventual $\omega^{-3}$ asymptote, they are quite different than Aki's (1967) $\omega$-cube model. Aki's models, as a result of his assumption that $v k_L = k_T$, had only a single corner frequency. His $\omega$-cube model makes a fairly abrupt transition from $\omega^0$ to $\omega^{-3}$ behavior. The spectra presented here, particularly the body-wave spectrum, show a gradual transition from $\omega^0$ to $\omega^{-3}$ asymptotes.
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$m_b-M_s$ relation. Changes in the definition of the body-wave magnitude scale have resulted in a large amount of confusion today. Gutenberg and Richter (1942) extended the body-wave magnitude scale from local events to fairly distant events which were recorded on Wood-Anderson and strong-motion torsion instruments.

Gutenberg (1945) introduced a scheme for $m_b$ differing only in minor details from the summary in Richter (1958). He determined $m_b$ from the instruments available in 1945, which were mostly broad-band mechanical types. Gutenberg (1945) stated that "the average period of $P$ waves in teleseisms is about 4 to 6 sec." In general, Gutenberg did not publish the period of the $P$ waves he used in determining $m_b$, but from a preliminary examination of his unpublished data it seems that many of his amplitudes were obtained at periods of 4 to 10 sec.

Gutenberg and Richter (1956) published their final version of the relation between $m_b$ and $M_s$

$$m_b = 0.63 M_s + 2.50 \quad (26a)$$

$$M_s = 1.59 m_b - 3.97. \quad (26b)$$

Their primary reason for deriving this relation was to facilitate the construction of a "unified magnitude scale." Investigators at that time apparently viewed the discrepancy between the two magnitude scales as an experimental error, rather than a fundamental effect of the seismic source spectrum. This view was not unreasonable at the time because $m_b$ was measured at periods differing only by a factor of 2 to 5 from $M_s$ and modern source theories had not yet been developed. In any case, Gutenberg and Richter found the (body wave) magnitude $m_s$, corresponding to a given $M_s$, by using (26a). They then took a weighted average of $m_b$, the actual body-wave magnitude, and $m_s$ to obtain $m$, the unified magnitude. Later Richter (1958) published values of unified (surface wave) magnitude, $M$, which he obtained by converting $m$ to $M$ using (26b). In retrospect, unified magnitude was inappropriate, since it now is clear that for all seismic source theories $m_b$ and $M_s$ represent different parts of the spectrum which are not related by a factor independent of fault length.

$m_b$ determinations by the USCGS (later NOAA and now the USGS) differ markedly from those used by Gutenberg and Richter. USCGS values for $m_b$ use (1), but $A$ and $T$ are measured on the WWSSN short-period instrument, which is sharply peaked at 0.5 sec. $T$ nearly always is about 1 sec in WWSSN magnitude determinations. Thus WWSSN magnitudes are based roughly on 1-sec spectral amplitude. On the other hand, Gutenberg and Richter determined $m_b$ for many events at about 5 sec, with even larger $T$ for the largest events. Therefore, it is wrong to take the Gutenberg-Richter $m_b$ as being related to spectral amplitude at any one particular period.

Modeling $m_b-M_s$. Aki (1967) proposed two statistical models of seismic sources, an \textit{\$\omega\$-square} model (which decayed as $\omega^{-2}$ at high frequencies) and an \textit{\$\omega\$-cube} model, after Haskell (1966) (which decayed as $\omega^{-3}$). Aki compared these two models by calculating spectral ratios for similar events and by calculating the relation of $m_b$ to $M_s$ predicted by each model.

Aki calculated $M_s$ by adding a constant to the logarithm of spectral amplitudes at 20 sec. The constant was chosen to give the best agreement between theoretical and observational spectral ratios of pairs of similar earthquakes studied by Berckhemer (1962). After fixing the additive constant for $M_s$, Aki then defined a similar relation for $m_b$. He set $m_b = \text{const} + (0.71 \sim 0.83) \log A (1 \text{ sec})$ and found the constant which would make $m_b = M_s$ when $M_s=6.75$. The coefficient of $\log A (1 \text{ sec})$ comes from a correction for duration.

Aki (1967) calibrated his curves for the $\omega$-square and $\omega$-cube models in this way. He
then compared the $m_b$-$M_s$ curves predicted by the models to the Gutenberg-Richter $m_b$-$M_s$ relation (26). He suggested that the excellent agreement of the $\omega$-square model with (26) strongly supported it, over the $\omega$-cube model. Unfortunately, his theoretical $m_b$-$M_s$ curve was based on 1-sec spectral amplitudes, while (26) was derived from mostly 4- to 10-sec data. Actually it seems that Aki’s support for the $\omega$-square model was incorrect. The WWSSN $m_b$-$M_s$ data (based on 1-sec $m_b$), discussed below, disagree with the $\omega$-square model.

The approach in this paper is to match $m_b$-$M_s$, $\log S$-$M_s$, $\log M_s$-$M_s$ and spectral ratio data simultaneously, adjusting the two free parameters to get good overall agreement with the data. A least-squares solution is not particularly appropriate because of the large number of parameters and the lack of similarity (e.g., different stress drops) found when earthquakes are examined in detail.

$m_b$ is approximated by a constant plus $\log A$ (1 sec) and $M_s$ by another constant plus $\log A$ (20 sec). $A(\omega)$ was found using (25) with the constants in (21) to (24). After several trials the additive constants for $m_b$ and $M_s$ were determined to be $C_{m_b} = 4.30$ and $C_{M_s} = 2.97$. To obtain seismic moment as a function of $L$, it was necessary to assign $\Delta \sigma$ for use in (15). Kanamori and Anderson (1975b) found that stress drops are 10 to 30 bars for most interplate earthquakes and 30 to 100 bars for most intraplate earthquakes, so $\Delta \sigma = 50$ bars was used.

Clearly it is not exactly correct to get $m_b$ and $M_s$ directly from spectral amplitudes. A more accurate approach would be computing synthetic seismograms and then measuring $m_b$ and $M_s$ as it is done for data. For this study, using spectral amplitudes seems to be an acceptable approximation.

Archambeau (1975) thoroughly discussed the differences between time-domain and frequency-domain estimates of $m_b$ and $M_s$. His study stressed the very small differences which are crucial in the context of seismic discrimination. In general though, his study supports the applicability of using spectral amplitudes to estimate $m_b$ of small and

![Figure 5](image-url)
moderate events. Probably any discrepancy between spectral amplitudes and time-domain amplitudes is most severe for larger events.

\( m_b-M_S \) data. Two kinds of \( m_b-M_S \) data are plotted in Figure 5. Points below the solid line midway up the figure are from a study of almost one thousand events by Evernden (1975). Each point is the average value of \( M_S \) for all earthquakes with that \( m_b \) value. Because Evernden's \( m_b \) values are an average of 0.3 lower than the USGS values, 0.3 is added to the \( m_b \) values before plotting them. Points above the line are values for individual events since mid-1963 as listed in Table 1.

Data shown in Figure 5 are in general agreement with a study of the \( m_b-M_S \) relation by Nagamune (1972). Nagamune fitted two straight lines to 2 years of WWSSN \( m_b-M_S \) data. He found

\[
M_S = 1.89 m_b - 4.62 \quad M_S > 5.73
\]

and

\[
M = 1.05 m - 0.02 \quad M < 5.73.
\]

The latter equation comes from a study of small events mostly in Hokkaido. Magnitudes in the latter equation are very similar to \( m_b \) and \( M_S \).

\( m_b-M_S \) curves from two models are plotted in Figure 5. The curve on the right is the \( m_b-M_S \) relation predicted by Aki (1972), which is based just on \( \log A(1) \), without any correction for duration. It can be seen that all of the data lie substantially to the left (smaller \( m_b \)) of the \( \omega \)-square curve. Inclusion of a duration correction for large events would not affect the basic conclusion that the \( \omega \)-square model does not agree at all with the data.

The left-hand curve is derived from the Haskell model presented in this paper. It can be seen that the predicted \( m_b-M_S \) curve is generally in good agreement with the data. It would have been better to have averaged the value of \( m_b \) for all earthquakes with a particular \( M_S \) for all WWSSN events for several years, rather than present just a few data points.

The Evernden (1975) data have a slope of one for events with \( m_b \) smaller than 5 1/2 while the predicted \( m_b-M_S \) curve has a slope of 3/4 for \( m_b > 4.2 \), which clearly disagrees with the data. The large events are too scattered to warrant a definite conclusion, but the predicted maximum \( m_b \) of 6.0 is probably 0.3 or 0.4 too small. This discrepancy may be due to use of spectral amplitudes instead of time-domain amplitudes. Also, it was assumed above that \( m_b \) was always based on 1-sec observations, but this is not strictly true. The Portuguese earthquake of 1969 (number 34 in Table 1) has \( m_b \) of 7.3, the largest of any of the events in Table 1. The average \( T \) for this event was 1.77 sec; the Haskell model predicts that if \( m_b \) had been determined at 1 sec, it would be 0.5 smaller. A systematic variation of \( T \) as a function of \( m_b \) could account for part of the difference between the theoretical curve and observations. Another possible explanation of the difference may be heterogeneity of the source mechanism. This possibility is discussed later in more detail.

**High-Frequency Spectra**

The Haskell model, which has \( \omega^{-3} \) high-frequency decay, moment proportional to \( L^3 \) and “corner frequency”, \( \omega_c \), (for fixed source-receiver geometry, source similarity and source mechanism) proportional to \( L^{-1} \), is a particular member of a general class of models having those properties. Following an argument first suggested by Savage (1972), note that for many source models the area radiating energy to a far-field observer will appear to grow as \( t^2 \) and dislocation from that area will grow linearly with \( t \). The far-field
pulse, which is the time derivative of the moment function will grow as \(t^2\), giving (Bracewell, 1965, p. 144) \(\omega^{-3}\) high-frequency spectral decay (assuming the \(t^2\) onset is the most abrupt discontinuity). Many models will also have a "corner frequency" proportional to \(L^{-1}\), where \(L\) is some characteristic source dimension of that model. Finally, most models give \(M_s \sim L^3\), where \(L\) is a source dimension. For all models meeting the above three requirements, high-frequency spectral amplitudes will behave as

\[
A(\omega) \sim M_s(\omega/\omega_c)^3 \sim L^3 (L^{-1}/\omega)^3 \sim \omega^{-3}.
\]

Thus, all events with fixed geometry and source type will share a common high-frequency asymptote which is independent of \(L\). Therefore, the conclusion that \(m_b\), and for much larger events, \(M_s\), will have a maximum value, applies to a more general class of models. For example, Minster (1973) derived \(m_b-M_s\) curves (with a similar shape to the curve from the Haskell model in Figure 5) from an Archambeau type (volume) source (also \(\omega^{-3}\) falloff), although he did not calibrate them against \(m_b-M_s\) data. Minster’s results also predict maximum values of \(m_b\) and \(M_s\).

Many investigators, such as Richards (1973), Dahlen (1974), Sato and Hirasawa (1973) and Madariaga (1975) have outlined crack models for which the initial rupture contributes an \(\omega^{-3}\) high-frequency spectrum while a "stopping phase" caused by simultaneous cessation of fracture everywhere on the fault contributes \(\omega^{-2}\) or \(\omega^{-5/2}\) and therefore dominates the high-frequency spectrum. If such models are applicable, \(m_b\), which is based on the initial rupture, would still have a maximum value, but \(M_s\) would not.

\(M_s\) versus fault area. Figure 6 is a plot of fault area, \(S\) (taken from Table 1), and \(M_s\). The predicted \(M_s\)-log \(S\) curve derived from (21) to (25) agrees quite well with the data. Note that the theoretical curve has four different segments. For small earthquakes, up to \(M_s = 6.76\), the slope is \(\frac{2}{3}\). From \(M_s = 6.76\) to \(M_s = 8.12\), the region in which most moderately

![Figure 6. \(M_s\) versus log \(S\) data with theoretical curve from Haskell model.](image-url)
large earthquakes are clustered, the slope is 1. There is a small section for which the slope is 2, from \( M_S = 8.12 \) to \( M_S = 8.22 \). After \( M_S = 8.22 \), the largest value of \( M_S \) for this calibration of the Haskell model, the slope is infinite (e.g., \( S \) increases with no further increase in magnitude).

There is a systematic difference between the interplate (closed circles) and intraplate (open circles) in Figure 6. Half of the intraplate events fall below the predicted \( M_S - \log S \) curve, while nearly all interplate earthquakes are above the curve. Kanamori and Anderson (1975b) showed that, at least in the region with slope one, this meant intraplate events had a higher apparent stress.

Utsu and Seki (1954) found the empirical relation \( \log S = 1.02 M - 4.01 \). Their \( M \) is Japan Meteorological Agency (JMA) magnitude which is roughly equivalent to \( M_S \), and \( S \) is in square kilometers. For the unit slope part of the \( M_S - \log S \) curve in Figure 6 (\( M_S > 6.76 \)) the Utsu-Seki relation predicts about five times the fault area. This may be due to the way Utsu and Seki apparently determined fault area. They used an area encompassing nearly all of the aftershocks, rather than the one-day aftershock zone which seems to give much better agreement with observed fault dimensions for earthquakes on continents. Båth and Duda (1964) proposed the relation \( \log S = 1.21 M - 5.05 \), based on a study of six earthquakes from different regions. Båth and Duda used \( S \) as aftershock area (in square kilometers) not fault-plane area, so this is basically similar to Utsu and Seki's result. Cinnery (1969) summarizes a number of efforts to find a single linear relation between \( M_S \) and the logarithm of other fault parameters.

**\( M_S \) versus moment.** The data of Kanamori and Anderson (1975b) show that \( \Delta \sigma = 50 \) bars is a good average, about halfway between values for interplate and intraplate events. Using \( \Delta \sigma = 50 \) and (15), we find moment (in dyne centimeters) is related to fault length (in kilometers by \( M_o = (7.26 \times 10^{21}) L^2 \) or \( \log M_o = 21.9 + 3 \log L \). It was shown above that \( M_S \sim n \log L \), where \( n \) varies between 0 and 3, as can be seen from the surface-wave spectra.

![Fig. 7. \( M_S \) versus \( \log M_o \) data with theoretical curve from Haskell model.](image)
in Figure 4. Therefore, for small earthquakes the $M_S:\log M_o$ slope is one: for very large events ($M_S \sim \text{constant}$) the slope is infinite ($M_o$ increases but $M_S$ is already at a maximum).

The $\log M_o-M_S$ data from Table 1 are plotted in Figure 7. Most of the moderate-sized events ($M_S$ from 6.76 to 8.11) fall on the slope 1.5 portion of the curve ($\log M_o \sim \frac{3}{2} M_S$). This part of the curve corresponds to cases where 20-sec spectral amplitudes are measured on the $\omega^{-1}$ part of the spectrum. Because the corner frequency for width is only slightly greater than that for rise time, the slope 3 ($M_o \sim 3 \log M_S$) region is very small, extending only from $M_S = 8.12$ to $M_S = 8.22$. Beyond that, slope is infinite. The data agree quite well with the theoretical curve. As in Figure 6, intraplate events tend to have smaller $M_o$ for a given $M_S$, corresponding to higher apparent stress.

Aki (1972) showed that his $\omega$-square model also agreed well with $M_S$ versus $\log M_o$ data. Brune and co-workers (Brune and King, 1967; Brune, 1968; and Brune and Engen, 1969) presented a magnitude scale based on 100-sec surface-wave amplitude. They then assumed $\log M_o \sim \log A(100)$ and fit two segments, each with slope 1, to the data. Because of their different definition of $M_o$, their results cannot be directly compared to this paper.

Data in this section show that when $M_o$ is larger than about $10^{28}$ dyne cm, $M_S$ reaches its maximum value. It is important to consider this when discussing the "maximum credible earthquake" likely to occur in a particular area. The earthquake size may be specified in terms of $M_S$ for most earthquakes, but when the moment approaches $10^{28}$, magnitude no longer is a valid parameter for specifying earthquake size. Whenever the maximum credible earthquake is in this range, e.g., as is probably the case in discussing the Alaskan pipeline, moment, not $M_S$, should be the parameter used.

**Spectral Ratios of Similar Events**

Berckhemer (1962) studied spectral ratios of earthquakes with roughly the same location and mechanism. In theory, the spectral ratio method eliminates the effect of earth structure and leaves only effects due to the difference in source spectra. Aki (1967) used Berckhemer's data to determine the relation between $M_S$ and corner period for the $\omega$-square and $\omega$-cube models.

Berckhemer's (1962) original data and Aki's (1967) theoretical curves are shown in Figure 8 together with the theoretical curve from the model in this paper. Both models seem to agree fairly well with the data. Perhaps Aki's fits slightly better. Berckhemer presented six pairs of spectra, of which only four are presented here. The remaining two pairs used smaller earthquakes, involving mostly short-period data, which probably are less reliable. No attempt at fitting these two pairs was made.

Tsujiura (1973) published spectral ratio data for many pairs of earthquakes. Most of his events could be fit by both Aki's (1967) $\omega$-square model or Aki's (1972) version of Brune's "$\omega$-model", although usually one model or the other fit somewhat better. There were, however, two pairs of events from the Aleutians which had spectral ratios that were unusually flat and could not be fit by either model. Tsujiura's spectral ratio data have not yet been compared to the model in this paper.

**Discussion**

The Haskell model with parameters (21) to (24) is in general agreement with $m_b-M_S$ data (Figure 5), $M_S - \log S$ data (Figure 6), $M_S - \log M_o$ data (Figure 7) and spectral ratio data (Figure 8). The most serious discrepancy between the data and the model comes in Figure 5. On one hand, the maximum value of $m_b$ is probably several tenths too small. On the
other, the data seem to have a slope of about one up to $m_b = 5\frac{1}{4}$, while the curve from the model has slope one only up to $m_b = 4.19$.

This phenomenon could be explained if most earthquakes are complex sources with the first burst of energy coming from a smaller, substantially higher stress drop source, than the average of the whole earthquake. If this is the case, then $m_b$ would be measured on a flat or flatter part of the spectrum than one would expect for the earthquake as a whole.

Burdick and Mellman (1976) have suggested that for the Borrego earthquake of 1968 most of the body-wave energy came from a source region with radius of 8 km, giving about half the area shown in Table 1. Since they also found a higher moment, $0.112 \times 10^{27}$ dyne cm, their stress drop, 96 bars, is about 4 times the value in Table 1, taken from Hanks and Wyss (1972). Tucker and Brune (1975) also suggested that sources showed a smaller high stress drop event superimposed on the overall average event. The $m_b-M_S$ data in Figure 5 agree with the possibility of the initial fracture having higher stress drop than the bulk event, but certainly do not prove that this happens. Other explanations are equally admissible.

**CONCLUSIONS**

The following scaling relations relating width and rise time to length and fault area have been given

$$L = 2W$$

$$\tau = 16S^{1/2}/(7\pi^{3/2}\beta).$$

The relation for rise time was derived from the assumption that static stress drop and
dynamic effective stress are equal; agreement of theoretical rise times with the data tends to support that assumption.

Averages of observed rupture velocities show that \( v_R = 0.72 \beta \).

The Haskell model predicts that magnitude will reach an upper limit regardless of further increases in fault length and seismic moment. Moment, rather than magnitude should be used to discuss the possible size of great earthquakes.

The “source spectrum” from any source model is a function of apparent (phase) velocity of the mode or phase being considered, as well as of azimuth and source parameters. It is incorrect to speak of a single “source spectrum.”

Theoretical relations between \( m_b \) and \( M_s \) from the Haskell model are

\[
\begin{align*}
  m_b &= M_s + 1.33 & M_s < 2.86 \\
  m_b &= \frac{2}{3} M_s + 2.28 & 2.86 < M_s < 4.90 \\
  m_b &= \frac{1}{3} M_s + 3.91 & 4.90 < M_s < 6.27 \\
  m_b &= 6.00 & 6.27 < M_s.
\end{align*}
\]

\( M_s \) and fault area (in square kilometers) are related by

\[
\begin{align*}
  \log S &= \frac{-2}{3} M_s - 2.28 & M_s < 6.76 \\
  \log S &= M_s - 4.53 & 6.76 < M_s < 8.12 \\
  \log S &= 2 M_s - 12.65 & 8.12 < M_s < 8.22 \\
  M_s &= 8.22 & S > 6080 \text{ km}^2
\end{align*}
\]

if \( L = 2W \) is used.

If we assume a stress drop of 50 bars, then \( \log M_o \) (in dyne centimeters) and \( M_s \) are related by

\[
\begin{align*}
  \log M_o &= M_s + 18.89 & M_s < 6.76 \\
  \log M_o &= \frac{2}{3} M_s + 15.51 & 6.76 < M_s < 8.12 \\
  \log M_o &= \frac{1}{3} M_s + 3.33 & 8.12 < M_s < 8.22 \\
  M_s &= 8.22 & \log M_o > 28.
\end{align*}
\]

These scaling relations fit observed data quite well. They should not be used to determine the value of a parameter for any individual earthquake, since these “averages”, and the assumptions made to derive them, are not exactly correct for any single event.

A review of work by Gutenberg and Richter reveals that their \( m_b-M_s \) relation was derived from \( m_b \) data at mostly 5- or 10-sec period. Models such as Aki’s (1967) \( m \)-square model which fit theoretical 1 sec \( m_b \) data to the Gutenberg-Richter relation are probably in error.

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References

SCALING RELATIONS FOR EARTHQUAKE SOURCE PARAMETERS AND MAGNITUDES


