SPECTROSCOPIC SIGNATURES OF CRYSTAL MOMENTUM FRACTIONALIZATION: SUPPLEMENTAL MATERIAL

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A. Relations for square lattice space group and time reversal symmetry

Following Ref. [8] of the main text [Phys. Rev. B 87, 104406 (2013), arXiv:1212.0593], we list the 10 relations that define the symmetry group consisting of square lattice space group and time reversal operations, in terms of the generators $T_x$, $P_x$, $P_y$, and $T$ defined in the main text (recall that $T_y = P_{xy}T_xP_{xy}$). The relations are:

\begin{align}
T_xT_yT_x^{-1}T_y^{-1} &= 1, \\
T_xP_yT_x^{-1} &= 1, \\
T_yP_yT_y^{-1}P_x^{-1} &= 1, \\
T_xT_y^{-1}T_x^{-1} &= 1, \\
P_x^2 &= P_y^2 = T^2, \\
(P_xP_y)^4 &= 1, \\
T_xP_yT_y^{-1}P_x^{-1} &= 1, \\
T_yP_y^{-1}P_x^{-1} &= 1.
\end{align}

B. No subtypes of Type D spectral periodicity

Here, we consider the four choices of $\sigma_i$ satisfying $\sigma_{txty} = 1$, $\sigma_{txpz} = -1$, with Type D spectral periodicity. We show that each of these four cases has the same spectral periodicity, and thus there are no subtypes of Type D.

Since $\sigma_{txsz} = -1$, single-spinon eigenstates can be grouped into degenerate doublets with crystal momenta $k$ and $k + (\pi, \pi)$. Therefore, each two-spinon state with single-spinon crystal momenta $k$, $k'$ is part of a degenerate quadruplet, in which the crystal momenta $q$ and $q + (\pi, \pi)$ each appear twice. Making a shift $k \rightarrow k + (\pi, \pi)$ has no effect on the quadruplet.

The four choices of the $\sigma_i$ under consideration can be specified by the ordered triple $(\sigma_{Tzx}, \sigma_{txpz}, \sigma_{txpz})$, and are

\begin{align}
D_A &= (-1, -1, -1), \\
D_B &= (1, 1, -1), \\
D_C &= (-1, 1, 1), \\
D_D &= (1, -1, 1).
\end{align}

First, we note that $D_A$ and $D_B$ can be mapped into one another by shifting the origin of $k$ by $(\pi/2, \pi/2)$, as discussed in the main text, and thus have the same spectral periodicity. The same holds for $D_C$ and $D_D$. It is thus sufficient to focus on $D_A$ and $D_C$.

We now consider the effect of acting with $P_x^s$, $P_{xy}^s$ and $T^s$ on one spinon in the quadruplet described above. This action is given in Eq. (17) of the main text, and we see that for both $D_A$ and $D_C$, the only effect of the non-trivial $\sigma_i$’s is to augment the ordinary symmetry transformation of $k$ with a shift of $(\pi, \pi)$ in some cases. For $D_A$ this shift is present for both $P_x^s$ and $T^s$, while for $D_C$ it is only present for $T^s$. Because such shifts have no effect on the quadruplet, and because the presence/absence of the $(\pi, \pi)$ shift for $P_x^s$ is the only difference between $D_A$ and $D_C$, the spectral periodicity is the same in both cases.

C. Parton mean-field $\mathbb{Z}_2$ spin liquid states

Here, we use fermionic parton theory to exhibit mean-field, gapped $\mathbb{Z}_2$ spin liquid states with Type D and D1d spectral periodicity. We dub these State-D and State-D1d, respectively. The PSG is specified by the action of symmetry generators on the two-component fermionic spinon field $\psi = (\psi_1, \phi_1)^T$ [X.-G. Wen, Phys. Rev. B 65, 165113 (2002), arXiv:cond-mat/0107071], which can be chosen to be

\begin{align}
T^s \psi(x, y) &= \sigma_{txty}^y \psi(x + 1, y), \\
T^s \psi(x, y) &= \sigma_{txty}^x \psi(x, y), \\
P_x^s \psi(x, y) &= \sigma_{txpz} \sigma_{txpy} \psi \mathcal{P}_{zy}^{-1} \psi(-x, y), \\
P_x^s \psi(x, y) &= \sigma_{txty} \mathcal{P}_{xy}^{-1} \sigma_{txpy} \psi(x, y).
\end{align}

Here $\mathcal{P}_{xy}$, $\mathcal{P}_{zy}$, and $\mathcal{P}_{ty}$ are $2 \times 2$ matrices and $\tau^{x,y,z}$ are the $2 \times 2$ Pauli matrices. It is straightforward to construct Hamiltonians quadratic in $\psi$ that are invariant under these transformations.

To construct State-D, we set $\sigma_{txty} = 1$, $\sigma_{txpz} = \sigma_{txpy} = -1$, and construct a Bogoliubov–de Gennes Hamiltonian $\psi^\dagger \mathcal{H}_D \psi$ that involves hopping and pairing terms to next-nearest-neighbor, with

\begin{align}
\mathcal{H}_D(k) &= (u_0 + u_2 \cos k_x \cos k_y) \tau^x \\
+ u_1 (\cos k_x + \cos k_y) \tau^z + u_2 \sin k_x \sin k_y \tau^y.
\end{align}

In this PSG, $g_{P_y} = \pi^0$ (the $2 \times 2$ identity matrix), $g_{P_z} = i\pi^x$, and $g_T = i\pi^y$. The physical (two-spinon) spectrum is shown in Fig. 2(a) of the main text for parameters $u_1 = u_0/4$, $u_2 = -u_0/2$, and $u_2' = u_0/4$. As expected, it has Type D spectral periodicity, with equivalent minima at $(0, 0)$ and $(\pi, \pi)$. As discussed in the main text, when we include fluctuations about this mean-field state, we arrive at a $\mathbb{Z}_2$ spin liquid which we term State-D.

To construct State-D1d, we modify the above PSG by setting $\sigma_{txty} = 1$. Only pairing terms appear now, and a generic Hamiltonian with the same range is

\begin{align}
\mathcal{H}_{D1d}(k) &= (u_0 + u_2 \cos k_x \cos k_y) \tau^x \\
+ [u_1 (\cos k_x + \cos k_y) + u_2' \sin k_x \sin k_y] \tau^y.
\end{align}

An example spectrum is shown in Figs. 2(b) and 2(c) of the main text, with the same parameters as for State-D. As expected, the global minima at $(0, 0)$ and $(\pi, \pi)$ are degenerate but inequivalent, and the projection of the density of states to $q_x$ is periodic under $q_x \rightarrow q_x + \pi$. 