SOME NEW EXPERIMENTS ON BUCKLING OF
THIN WALL CONSTRUCTION

by
F. J. Bridget, C. C. Jerome and A. B. Vosseller

ABSTRACT

The first section of this paper describes a series of tests of the
strength of thin walled cylinders under a combination of torsion and axial
compression or tension. Curves are obtained showing the strength of each of
the several types of cylinders tested, under all possible combinations of these
loads. All the curves obtained seem to have the same general form, and the
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1) The tests described in this paper were made at the Guggenheim Aeronautical
Laboratory of the California Institute of Technology. The authors have been
assigned to the Institute by the Navy Department for graduate work in aero-
nautics, and this work represents part of the requirements for the Master's
Degree. The first section is by Lieut. Bridget and the second by Lieutenants
Jerome and Vosseller. The authors wish to thank Dr. Th. von Kármán, director
of the Guggenheim Laboratory of the Institute, for the opportunity for making
these researches. The researches were suggested by and carried out under
the direction of Dr. L. H. Donnell. Acknowledgment is also due to L. Secretan,
who cooperated in the first research, and to Dr. A. L. Klein and E. E. Schler
(all of the staff of the Institute) for numerous helpful suggestions.

2)Lieutenant, U.S. Navy; was graduated from U.S. Naval Academy in 1921 and
served seven years on duty in the fleet with gunnery duties. Completed
flight training in 1929 at Pensacola, Florida. Served in fighting squadron
attached to the U.S.S. Lexington in 1929 and 1930. Inspection duty of naval
aircraft in 1931. Attended Post Graduate School U.S. Naval Academy in 1932,
and upon completion assigned to duty at California Institute of Technology for
post graduate work in Aeronautical Engineering.

3)First Lieutenant, U.S. Marine Corps; graduated from U.S. Naval Academy in 1922.
With various Marine Infantry units 1922-1924. On aviation duty with Marine
Aviation in United States, China, Philippines, Guam and Nicaragua, 1924-1932.
Assigned to duty at California Institute of Technology for post graduate work
in Aeronautical Engineering, 1933.

4)Lieutenant, U.S. Navy; was graduated from U.S. Naval Academy in 1924 and
served in various ships of the fleet and on the staffs of Commander Battleships
and Commander Destroyers, Battle Force until assignment to Aviation duty in
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a designer could determine the buckling strength of a structure under any combination of shear and normal stress, if he knows its strength under pure shear and under pure compressive stress.

The second section describes tests made to test the independence of different possible types of buckling of a structure. A set of L-section struts, identical except for the widths of the sides, were tested in compression. With small widths the struts buckle as Euler columns, but with the wider widths buckling of the sides, as plates hinged on three edges, occurs first. Great care was taken to eliminate the effect of initial eccentricities. The results check the well-known theories for these two types of buckling, and indicate that, for practical purposes, the two types can be considered independently of each other. The results also illustrate how enormously the strength-weight ratio of thin wall construction may be affected by details of design.

Section I.

The stresses produced by the loads in the skin of monocoque construction are usually combinations of compression or tension in one direction, with shear in the same and perpendicular directions. Much theoretical and experimental information is available to the designer on the behavior of thin sheets under compression or tension alone, or under shear alone, but nothing seems to be known about their behavior under combinations of these stresses.

The general nature of the behavior of thin sheet construction under such combination loading can be predicted by some very simple reasoning. Let \( \sigma \) and \( \tau \) be the normal and shear stresses produced in the wall, and \( \sigma_0 \) and \( \tau_0 \) the values of these stresses when failure occurs under pure axial compression or tension, and under pure torsion respectively. Then by plotting \( \sigma/\sigma_0 \) against \( \tau/\tau_0 \) a curve is obtained, passing through the points 0,1 and 1,0, and showing how failure occurs under all possible combinations of these two types of stresses. This curve obviously must be symmetrical about the \( \sigma_0 \) axis, as a change in the sign of the shear (for a symmetrical structure) can make no difference. Hence
the curve must be perpendicular to the $\frac{E}{E_0}$ axis where it crosses it, as it is certainly continuous at this point. On the other hand the curve will not be symmetrical about the $\frac{J}{J_0}$ axis, but must cross it at some angle as shown in Fig. 1(a), as compression will obviously decrease and tension increase the shear which the structure can take before buckling. This immediately suggests some kind of parabolic or power relation as in Fig. 1(b), with an equation:

$$1 - \frac{E}{E_0} = \left(\frac{J}{J_0}\right)^n$$

(1)

where $n$ is greater than 1 (of course $n$ should be an even number to satisfy the condition of symmetry about the $\frac{E}{E_0}$ axis, but this relation can be used as an empirical formula with any value greater than 1 for $n$, provided $\frac{J}{J_0}$ is always considered positive). This reasoning is perfectly general, applying to the stability of any type of symmetrical structure whatever.

To get some experimental information on this subject, series of similar thin walled cylinders have been tested to failure under various combinations of torsion and axial compression or tension. In the most complete of these series, about forty cylinders, made as nearly identical as possible, were tested. Fig. 2 shows the results for this series, (Series G). The experimental values for $\frac{E}{E_0}$ and $\frac{J}{J_0}$ being plotted against each other (it obviously makes no difference whether $\sigma, J, \sigma_0, J_0$ are defined as above, or if $\sigma$ and $J$ are taken to mean the total axial load and torsional moment and $\sigma_0$ and $J_0$ the values of these loads when failure occurs under pure axial compression or tension and under pure torsion respectively). It seemed to make no difference in what order the loads were applied, that is whether a fixed amount of torsion was applied first and the axial load increased until failure occurred, or vice versa.

The full line in Fig. 2 is a cubic parabola, that is the curve given by equation (1) when $n = 3$. This curve seems to fit the points about as well as anything. More definite information as to the nature of the relation given by these experiments is obtained by plotting $1 - \frac{E}{E_0}$ against $\frac{J}{J_0}$ on double logarithmic paper, as has been done in Fig. 3 for Series G. It will be seen that the ex-
Experimental points lie close to a straight line, which proves the validity of equation (1) as an empirical formula for this case. The slope of this straight line gives the value of \( n \) which, when substituted in (1), gives the best empirical relation to fit these experiments. A determination of this slope by eye or the method of least squares gives a value of about 3.3 for \( n \).

The lower portion of the curve in Fig. 2, shown dotted, obviously represents stress combinations for which some portion of the cylinder has reached the elastic limit. The lowest point on the curve represents a pure tension test. Most practical applications do not fall within this region and no particular study has been made of this portion of the curve. Designers will, of course, realize that the relation given by (1) can hold only when the principal stresses as given by elementary mechanics are within the elastic limit (or, according to the maximum shear theory, when the algebraic difference between these principal stresses is less than the elastic limit stress).

Six other less complete series of tests have been made, each series on a radically different type of cylinder. These tests were made with torsion and axial compression only, and with a much smaller number of cylinders. The results are shown in Fig. 4. Fig. 5(a-f) shows the results plotted on double logarithmic paper for these series A-F inclusive. These results are similar to those from the group of tests described above. The value of \( n \) appears to be different for each type of cylinder, the values obtained ranging from about 1.0 to 4.25, apparently indicating that \( n \) is a function of the dimensions or material of the cylinder. Some of the sets of results, when plotted on logarithmic paper, do not seem to give a straight line, apparently indicating that (1) is not exactly suitable as an empirical relation for these cases. However, the number of points is so few and the unavoidable experimental scatter so great, that nothing definite can be said on any of these questions. The results do indicate, however, that the general empirical relation (1) covers all these tests (and hence probably all thin walled cylinders, and possibly all thin walled structures of any kind) suf-
ficiently well for practical purposes. If \( n \) is taken as 3, the deviation between the empirical law and the experimental results is not large compared to the general scatter of the points, for any of the types of cylinders tested. It therefore seems safe to say that designers can use the relation (1) with \( n = 3 \), for designing cylinders, with considerable confidence, until better information is available.

Obviously this work is only a small part of the work which should be done along this same line. It is planned to continue this work at the California Institute of Technology, and it is hoped that this paper will stimulated other research organizations to do work in this field. Some tests have already been started here on long square tubes under axial load and torsion, giving approximately the condition of flat hinged-edge panels under normal and shear stresses; the initial results are similar to those described above. As stated before, there has heretofore been little investigation in this field, and it is felt that further work will result in information of great value to designers of stressed skin structures.

All of the tests were made on small scale models, of steel and brass "shim stock". They were made by a technique and were tested on special testing machines developed by Dr. L. H. Donnell, and described by him in a previous paper\(^5\). The figures mentioned in the following paragraph refer to figures in this paper. As shown in this paper the results from such small scale tests compare quite favorably with tests on a larger scale, but it is necessary to use great care in selecting stock and in making the specimens in order to reduce experimental scatter to a minimum. In spite of this care it is impossible to avoid considerable scatter, especially when the load is mainly compression; much of this is probably due to the fact that it is impossible to obtain such thin stock entirely without initial waviness.

\(^5\)N.A.C.A. Report No. 479.
The thickness of the sheet was measured in the thickness tester shown in Fig. 14a. The modulus of elasticity, E, was obtained with the special testing machine and tensometer shown in Fig. 15a. The proportional limit stress ρ was found at the same time, for purposes of record. The sheets were rolled around rods of the proper diameter to give them the correct curvature. They were then wrapped around a wooden roll, of the diameter of the finished cylinder, which had previously been oiled to enable removal of the cylinder after soldering. The sheet was held against the roll with a metal strap, while a special clamp was used to prevent the edges from warping under the heat of soldering. The edges were soldered with about 1/8 inch overlap. (It has been found that buckling waves seem to form across the joints as freely as elsewhere, so the effect of the double thickness at the joint is probably very small). After removal the cylinders were soldered at each end to rings which fitted inside them, and the cylinder and ring were soldered to heavy base plates with which they were mounted in the testing machine. The shortening of the effective length of the cylinders due to the rings is allowed for in the following table. The cylinders were tested in the special testing machine shown in Fig. 6a. This testing machine is capable of testing specimens in any combination of torsion, bending and axial compression or tension. Figs. 6 to 18 inclusive show typical failures under combined compression and torsion, while Fig. 14 shows a typical failure under combined tension and torsion with the tension cage attached. Fig. 12 shows the cylinders of Series 6 after testing. Table I shows the properties of the cylinders in each of the series tested. The values given for n are the slopes of the straight lines in the logarithmic plots, Figs. 3 and 5(a-f), as previously discussed.

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5) N.A.C.A. Report No. 479.
TABLE I

Properties of cylinders in series tested

<table>
<thead>
<tr>
<th>Series</th>
<th>Mat.</th>
<th>Length</th>
<th>Diam.</th>
<th>Thick.</th>
<th>E x 10^6</th>
<th>σ x 10^5</th>
<th>n</th>
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<tbody>
<tr>
<td>A</td>
<td>steel</td>
<td>5.315</td>
<td>1.88</td>
<td>.00204</td>
<td>31.4</td>
<td>57.7</td>
<td>1.992</td>
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<td>brass</td>
<td>5.315</td>
<td>1.88</td>
<td>.0032</td>
<td>16.5</td>
<td>27.0</td>
<td>4.25</td>
</tr>
<tr>
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<td>steel</td>
<td>5.315</td>
<td>3.75</td>
<td>.00295</td>
<td>30.6</td>
<td>43.6</td>
<td>2.625</td>
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<td>D</td>
<td>steel</td>
<td>11.315</td>
<td>1.88</td>
<td>.00204</td>
<td>27.06</td>
<td>53.3</td>
<td>2.156</td>
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<td>E</td>
<td>steel</td>
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<td>1.88</td>
<td>.00295</td>
<td>30.6</td>
<td>48.6</td>
<td>1.0</td>
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<tr>
<td>F</td>
<td>steel</td>
<td>1.32</td>
<td>3.75</td>
<td>.00295</td>
<td>31.4</td>
<td>57.7</td>
<td>2.781</td>
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<td>G</td>
<td>steel</td>
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<td>1.88</td>
<td>.00295</td>
<td>29.6</td>
<td>36.0</td>
<td>3.33</td>
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</table>

Table II shows the ultimate axial loads σ, and torsion moments τ, for each of the cylinders tested. The values used for σ₀ and τ₀, in plotting Figs. 2 to 5, were the averages of the values of σ and τ found in the pure compression and pure torsion tests. The values chosen for σ₀ and τ₀ have considerable influence on the value found for n, and it is probable that part of the variation in the values of n, found for different types of cylinders, is due to the inaccuracy of these values. In making this type of research it is important to make numerous pure compression and pure torsion tests (especially pure compression, because the scatter is much worse here than for torsion), so as to get a good average value for σ₀ and τ₀; in future research it is planned to make even more of such tests than have been made.

TABLE II

Experimental Results Used in Plotting Curves

Series A

<table>
<thead>
<tr>
<th>σ lbs.</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>170</th>
<th>172</th>
<th>230</th>
<th>192</th>
<th>218</th>
<th>231</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ lb.in.</td>
<td>55</td>
<td>52</td>
<td>49</td>
<td>47</td>
<td>40</td>
<td>39</td>
<td>36</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>0</td>
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Series B

<table>
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<tr>
<th>σ lbs.</th>
<th>20</th>
<th>45</th>
<th>90</th>
<th>110</th>
<th>120</th>
<th>150</th>
<th>175</th>
<th>190</th>
<th>230</th>
<th>250</th>
</tr>
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<tbody>
<tr>
<td>τ lb.in.</td>
<td>70</td>
<td>64</td>
<td>50</td>
<td>64</td>
<td>48</td>
<td>60</td>
<td>56</td>
<td>54</td>
<td>30</td>
<td>0</td>
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<tr>
<td>Series</td>
<td>(\sigma_0)</td>
<td>(\sigma_0)</td>
<td>(\tau_0)</td>
<td>(\tau_0)</td>
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<tr>
<td>C</td>
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<td>(\sigma) lbs.</td>
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<td>(\sigma) lbs.</td>
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<td>50</td>
<td>100</td>
<td>150</td>
<td></td>
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<td>217</td>
<td>240</td>
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<td>178</td>
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<tr>
<td>D</td>
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<tr>
<td>(\sigma) lbs.</td>
<td>(\tau) lb.in.</td>
<td>(\sigma) lbs.</td>
<td>(\tau) lb.in.</td>
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<td>40</td>
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<tr>
<td>(\sigma) lbs.</td>
<td>(\tau) lb.in.</td>
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<td>(\tau) lb.in.</td>
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<td>F</td>
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<td>(\sigma) lbs.</td>
<td>(\tau) lb.in.</td>
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<tr>
<td>(\sigma) lbs.</td>
<td>(\tau) lb.in.</td>
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<td>613</td>
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Beyond elastic limit
Buckling of cylinders under combined torsion and compression or tension.
BUCKLING OF CYLINDERS UNDER COMBINED TORSION AND COMPRESSION OR TENSION

FIG. 4
FIG. 6
BUCKLING OF CYLINDERS
UNDER COMBINED TORSION
AND COMPRESSION OR TENSION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Length</th>
<th>Dia.</th>
<th>Thick.</th>
<th>$E \times 10^{-6}$</th>
<th>$G_y \times 10^{-3}$</th>
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<tr>
<td>A</td>
<td>5.3</td>
<td>1.88</td>
<td>0.020</td>
<td>31.9</td>
<td>57.7</td>
</tr>
<tr>
<td>B</td>
<td>5.3</td>
<td>1.88</td>
<td>0.032</td>
<td>16.5</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>5.3</td>
<td>3.75</td>
<td>0.030</td>
<td>38.6</td>
<td>48.4</td>
</tr>
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<td>D</td>
<td>11.3</td>
<td>1.88</td>
<td>0.020</td>
<td>27.1</td>
<td>53.3</td>
</tr>
<tr>
<td>E</td>
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<td>1.88</td>
<td>0.020</td>
<td>31.4</td>
<td>57.7</td>
</tr>
<tr>
<td>F</td>
<td>1.3</td>
<td>3.75</td>
<td>0.040</td>
<td>24.0</td>
<td>56.0</td>
</tr>
</tbody>
</table>

$G = \text{Compressive stress}$
or tensile
$T = \text{Shear stress}$
$G_y = G$ when $T = 0$
$T_x = T$ when $G = 0$
FIG. 7
BUCKLING OF HINGED END ANGLE-SECTION STRUTS

--- Theoretical curve for buckling as an Euler column.

--- Theoretical curve for buckling of sides as plates hinged on 3 sides.

- Type of buckling predominantly Euler.
- Type of buckling predominantly of sides as plates hinged on 3 sides.

(All struts of dural, E=105000 ksi, $\gamma=40000$ ksi
length = 22 in., thickness = .025 in.)

Cross-section

6
lbs./in.
5000

4000

3000

2000

1000

0

0.5

1.0

1.5

2.0

Width of sides

W, inches
FIG. 1