large fields:

$$\chi_{\text{iso}}=[kTH_c^2]^{-1}\left[\sum_{i=1}^{4}\langle|\mathcal{H}_1{}^i|^2\rangle_{\text{Av}}\right]. \quad (90)$$

The energy terms can be easily squared and averaged; when this is done the following result is obtained:

$$\chi_{\text{iso}}=\frac{4}{9}g^4\beta^6S^2(S+1)^2(kTH_c^2)^{-1}\sum_{i,j}{}' r_{ij}^{-6}[|F^{ij}|^2+|D^{ij}|^2]. \quad (91)$$

It is necessary to make substitutions similar to (70) in (91), and then average over all directions of λ_1, λ_2, λ_3. The following values then result for the lattice sums:

$$\sum_{i,j}{}'|F^{ij}|^2r_{ij}^{-6}=\sum_{i,j}{}'|D^{ij}|^2r_{ij}^{-6}=2.52. \quad (92)$$

Substitution into (91) then gives

$$\chi_{\text{iso}}=2.24g^4\beta^4S^2(S+1)^2/(kTH_c^2). \quad (93)$$

Use of $\chi_0=Ng^2\beta^2S(S+1)/3kT$ and (15) then results in

$$\chi_{\text{iso}}=0.80\chi_0H_i^2/2H_c^2. \quad (94)$$

L. J. F. Broer has made an independent calculation of the ratio of the adiabatic and isolated susceptibilities for spins in a large field without exchange (private communication). He arrived at the factor of 0.80. This factor is the same as that of (94).

VII. ACKNOWLEDGMENTS

In conclusion, the writer should like to take this opportunity of expressing her deep appreciation to Professor J. H. Van Vleck for suggesting this subject as a research topic. She would like to acknowledge his invaluable counsel and helpful criticism. Also she would like to express her appreciation for the Atomic Energy Commission Predoctoral Fellowship which made this research possible.

PHYSICAL REVIEW VOLUME 76, NUMBER 12 DECEMBER 15, 1949

Precision Wave-Length Measurements of the 1.1- and 1.3-Mev Lines of CO^{60} with the Two-Meter Focusing Curved-Crystal Spectrometer*

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(Received August 15, 1949)

Recent improvements in the two-meter focusing curved-crystal gamma-ray spectrometer are described which have extended its quantum energy range well above 1 Mev and have also yielded much better luminosity and resolving power than were obtained initially. The improved components are (1) the crystal holder whose aperture and resolving power have been nearly doubled and (2) the collimator the new model of which can now discriminate between the reflected and transmitted beams when these differ in direction by only 8 minutes of arc, a threefold improvement over our first model. Our plans for further possible improvements and the factors governing these are also discussed. Wave-lengths of two gamma-rays emitted following β-decay of Co^{60} have been measured with this new equipment using a source of about 50 mc strength and found to have values of $(9.308\pm0.005)\times10^{-11}$ cm and $(10.580\pm0.005)\times10^{-11}$ cm corresponding to quantum energies of 1.3316 ± 0.0010 Mev and 1.1715 ± 0.0010 Mev, respectively. The lines appear to have equal intensities. The integrated reflection coefficient of the (310) planes of the curved-quartz crystal still appears to follow a λ^2-dependence on wave-length down to 9 x.u. the shortest so far observed.

EXTENSION OF THE WAVE-LENGTH RANGE OF THE TWO-METER FOCUSING CURVED-CRYSTAL GAMMA-RAY SPECTROMETER

THE two-meter focusing curved-crystal gamma-ray spectrometer[1–3] has up to the date of the work here described, been applied only to the measurement of nuclear gamma-ray lines of quantum energy equal to or less than 640 kev.[4–7] Many natural and artificial radioactive sources of great interest exist however which have lines in the quantum energy range from 1 to 2 Mev and even far beyond this. Our present experience in measuring these ultra-short wave-lengths by direct crystal diffraction has shown that the upper limit of quantum energy beyond which the precision of the method falls to a value comparable with the precision obtainable with the magnetic β-ray spectrometer is probably fixed by the characteristics of the crystal planes used for the diffraction. In the case of our present two-meter curved-crystal spectrometer utilizing the (310) planes of quartz this limiting precision for the measurement of wave-lengths seems to correspond to an uncertainty of about ±0.005 x.u. This uncertainty is essentially constant independent of the wave-length measured. Thus, at a wave-length of 5 x.u. or about 2.5 Mev, a precision of the order of one part in a thousand in wave-length measurement can still be obtained. It has, therefore, seemed well worth while to try to extend the range of applicatility of the instrument as far as possible above the 1 Mev value. Such an extension requires the following two improvements in the method.

* Assisted by the Joint Program of the AEC and the ONR.
[1] Jesse W. M. DuMond, Rev. Sci. Inst. **18**, 626 (1947).
[2] DuMond, Lind, and Cohen, Rev. Sci. Inst. **18**, 617 (1947).
[3] D. A. Lind, Rev. Sci. Inst. **20**, 233 (1949).
[4] DuMond, Lind, and Watson, Phys. Rev. **73**, 1392 (1948).
[5] Watson, West, Lind, and DuMond, Phys. Rev. **75**, 505 (1949).
[6] DuMond, Lind, and Watson, Phys. Rev. **75**, 1226 (1949).
[7] Lind, Brown, Klein, Muller, and DuMond, Phys. Rev. **75**, 1544 (1949).

(1) The collimator which arrests the transmitted beam but allows the selectively reflected beam to pass unhindered to the multicellular counter, must be improved as regards the minimum angular difference between the directions of these two beams for which it will satisfactorily give the above-mentioned discrimination with adequate contrast.[8] At 3 Mev, for example, and with the (310) planes of quartz, the angular difference between the reflected and transmitted beams is only 8 minutes of arc.

(2) Since the reflecting power of the crystal planes diminishes with dimishing wave-length λ about as λ^2 every means must be directed toward compensating for this rapid loss in intensity without doing so at the expense of increased background. The means at our disposal for this are about five in number as listed below.

(a) We may increase the thickness of the curved-crystal slab. The limit here for quartz with our two-meter radius is at about 2 mm thickness, if the crystal is to have any margin of safety against breakage. Such lamina are now in preparation but to date have not been tried. This will be a twofold gain over the present 1 mm thickness.†

(b) The cross section of the gamma-ray beam may be increased by increasing the window aperture in the curved-crystal clamping holder. Several limitations enter here. Large and perfect specimens of quartz are extremely difficult to obtain beyond a certain size. If the beam has too large an angular opening the resolving power diminishes because of what may be called cylindrical aberration[9] and also vertical divergence. In order to utilize a larger beam opening, the cross section of the multicellular counter or other intensity measuring device must be increased and unless this can be done without a correspondingly great increase in background counting rate, there will be no real gain in contrast to compensate for the loss of crystal reflecting power with decreasing wave-length. The counter background comes from cosmic rays, local radiation from radioactivity of the building and other surroundings, and internal radioactivity of the counter (chiefly alpha-activity in the lead partitions of the counter).

(c) Increased crystal reflection may be sought by using other Miller indices and other crystals. The choice is not wide here, however, because of several restrictions. First, one is limited to small grating constants of the order of one angstrom without which the angular difference between transmitted and reflected beams becomes so small that the construction of a collimator for discriminating between them is practically impossible. Second, the crystal must approach as closely as possible the ideally perfect type in order to give the requisite resolving power. Third, it must be obtainable in large samples free from twinning or distortion with extreme parallelism of the reflecting planes (to within a second or so of arc) over large samples. Fourth, it must be capable of being bent elastically without cleaving or yielding along slip planes. Fifth, it must permit taking high (optical) precision surface figuring and be sufficiently inert chemically to retain its figure and resist deterioration. Preparations are under way to try about five different planes in quartz and a few other crystals such as topaz and sheelite.

(d) Increased counter efficiency (without corresponding increase in background counting rate) may be sought. This can be obtained partly by adjusting the *thickness* of the partitions (from which electrons are ejected by the gamma-ray beam) to an optimum value for the hardness of radiation to be studied. Also, the *number* of partitions should be increased as far as possible. Developments of this sort are under way in the form of a series of multicellular counters of square cross section (to fit more closely the cross section of the beam) in which twice as many partitions per unit length of beam can be introduced as in our present multicellular counters. In the new counter the spacing between partitions is only 0.25 inch and the four-pronged anode spiders between partitions are omitted. Instead of these, four parallel 2-mil tungsten anode wires pass axially through four sets of $\frac{3}{8}$-inch holes in

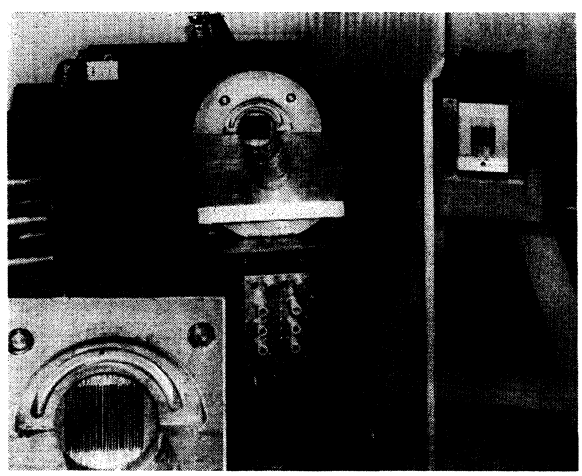

FIG. 1. Comparison of new and old lead collimators for the two-meter focusing curved-crystal gamma-ray spectrometer. To indicate the scale, a horizontal one-foot rule can be seen at the back of the lead shielding on the new collimator. The cylindrical multicellular counter and the upper half of the lead shields that cover it have been removed. The semicircular cavity that can be seen in the end of the first circular lead shield is provided to accommodate the battery of anticoincidence counters which in operation are situated above the multicellular counter to protect it from cosmic rays. A meter stick stands vertically near the old collimator which is at the right. Both collimators are 30 inches long. The insert is an enlarged view of the exit end of the new collimator, the thicker ends of the tapering lead partitions being visible.

[8] The transmitted beam to be suppressed by the collimator may be from 500 to 2000 times as intense as the selectively reflected beam which it is the object of the instrument to measure.

† Since this manuscript was submitted, we have now had a 2-mm thick quartz lamina bent to a radius of two meters for two months without breakage.

[9] For a discussion of this geometrical aberration, see p. 629 and Fig. 3 of reference 1.

all of the partitions. These counters are, however, still under development and study, and have not been used in the presently described measurements on Co^{60}.

(e) Increased source strength may be sought. There is no gain however, if the source covers a wider arc of the focal circle (in the direction of dispersion of the instrument) than that defined by the resolving power of the crystal. With our present two-meter instrument, this calls for source widths of the order of 0.001 inch. Increased source strength therefore implies increased *specific* source activity and this is limited by the bombarding flux available for exciting the activation.

As we have indicated, four of the five means of increasing intensity listed above (namely, a, c, d and e) are still under development. In the work to be described in this article, the chief improvements effected were under headings (b) and (d), increased crystal window aperture and somewhat increased counter efficiency by the use of more partitions. The counters of square cross section however (with four longitudinal wires and no four pronged spiders) had not as yet been developed when the present work on Co^{60} was done. Specifically, the important improvements made for the present Co^{60} work were:

(1) A new crystal holder with much larger aperture than the first, 1.7×2 inches in dimensions, traversed by two thin horizontal ribs to give extra support to the crystal so as in insure accurate curvature throughout the active window surface. This holder has a focal length of 196 cm and by some improvements in mechanical stiffness of the equipment for the initial grinding of the cylindrical surface,[2] it has been possible to obtain a profile so accurate as to give us a resolution of 0.05 x.u. as regards the aberrations from perfect focusing. This new crystal holder represents a twofold gain over our first attempt both in respect to luminosity and resolving power.

(2) A new collimator with twenty-four (instead of seven) die-cast tapering lead alloy partitions thirty inches long whose thickness at the entry end is only 40 mils. To insure no loss in the fraction of radiation transmitted through such a collimator, these partitions must be rigorously straight and true, a very difficult requirement to meet. In our present new collimator, the transmission which was designed theoretically to be 50 percent turned out by measurement to be about 35 percent. The geometry of the new collimator is such that if we ignore scattering upon and penetration of the radiation into the lead plates the least angular difference between transmitted and reflected beams between which the collimator will discriminate is 8 minutes of arc. This corresponds to a calculated theo-

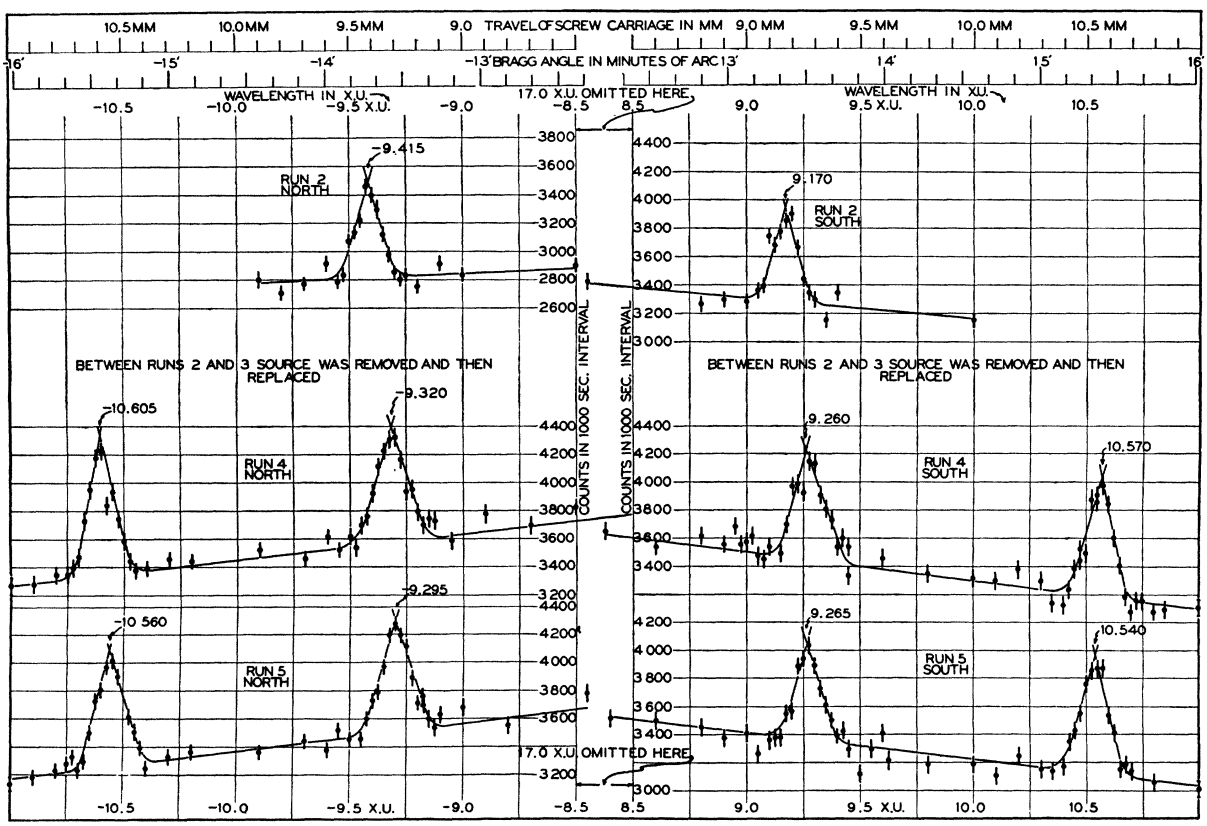

FIG. 2. Three spectral curves corresponding to runs 2, 4, and 5 showing the 1.1 and 1.3 Mev lines of Co^{60}. If the entire wave-length scale down to zero were included for both sides of this plot, it would be about four times as wide.

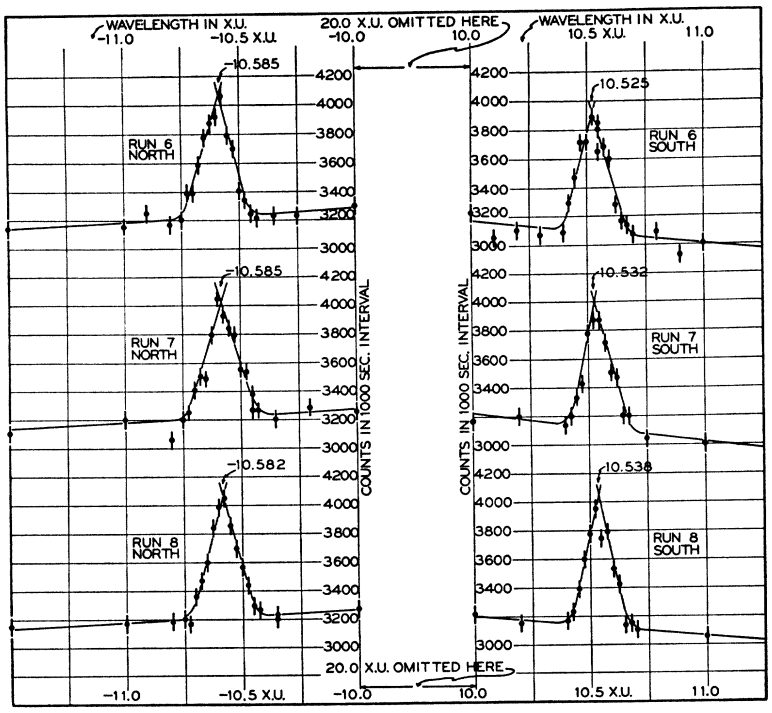

FIG. 3. Three spectral curves, runs 6, 7, and 8, on the 1.1-Mev line of Co^{60}. The scales of this figure are similar in all respects to those of Fig. 2.

retical upper working limit of quantum energy of 3 Mev if the (310) planes of quartz are used. This is a threefold improvement over our first collimator (as regards limiting quantum energy). Figure 1 is a photograph of the two-meter instrument looking from the detector end toward the source end in which the multicellular counter and some of its lead shielding have been removed. The exit end of the collimator is thus exposed to view and the thicker ends of the twenty-four tapering lead partitions are thus visible. The insert in this figure is an enlargement of this rear end of the new collimator in order to show the exit ends of the partitions more clearly. For comparison, the old collimator is also shown just to the right of the new one, the exit end in this case also being the one exposed.[10]

APPLICATION TO THE STUDY OF GAMMA-RAYS FROM CO^{60}

Figure 2 shows three of our spectral curves obtained by reflections from both sides of the crystal planes. Run 1 (not shown) was exploratory in nature to locate the 1.3-Mev line. Run 2 shows the 1.3-Mev line of Co^{60} while runs 4 and 5 show both the 1.1- and 1.3-Mev lines. In run 2, a single line profile of the 1.1-Mev line (not plotted in Fig. 2) was also run on the right-hand side only. The source was taken out of its holder after run 2 to permit temporary study of another much shorter lived source, Ta^{182}. The Co^{60} source was then replaced in the instrument and another exploratory run (No. 3, not shown) was made to relocate the lines. This removal and replacement of the source accounts for the slight shift in the line positions on the instrument scale, a shift corresponding to a displacement of the source in the source holder of about 0.07 mm. The *separation* however between the reflections from the two sides of the crystal planes (which is used as the measure of the wave-length) is very reproducible from run to run. The scales at the top of Fig. 2 show the Bragg angles of reflection in minutes of arc and also the displacement of the wave-length screw carriage in millimeters. The wave-length scale is also shown at both top and bottom in nominal x units.[11] To save space, some 17 x.u., or more than three times the total width of the figure, is omitted from the wave-length scale in the space at the center of Fig. 2 between the right- and left-hand orders. It will be noted also that the spectral lines do not occur at exactly the same nominal scale readings on the two sides. This is because

[10] Plans are under way for a still better collimator which we hope to construct with tungsten partitions. Because of the greater density of tungsten, it will be possible to design this for about 80 percent theoretical transmission. The partitions will not be tapered in thickness but will be retained in converging grooves in heavy steel guide plates at top and bottom. This collimator will have the same theoretical limiting angular discriminating power but in addition will have a truncated or trapezoidal transmission characteristic (because the partitions do not taper in thickness) so that there will be a finite small range of angles over which the maximum 80 percent transmission obtains. This places less rigid retuirements on the mechanical features of the instrument which must hold the direction of the reflected beam nearly invariable in the collimator slots.

[11] Nominal x units on the wave-length drum of the instrument are convertible into true x units (Siegbahn scale) by dividing the instrument reading by the factor 1.00024 which we have established with high precision by measurements on the x-ray lines of the K-spectrum of tungsten.[5] These wave-lengths in x units are then converted from the Siegbahn scale to milliangstroms by multiplying them by 1.00203.

TABLE I. Wave-length measurements of two gamma-ray lines from CO^{60}.

1.3-Mev line			1.1-Mev line	
	Screw reading (double)	Weight factor		Screw reading (double)
Run 1	18.570	0.5	Run 6	21.111
Run 2	18.595	1.0	Run 7	21.118
Run 3	18.578	0.5	Run 8	21.121
Run 4	18.585	1.0		
Run 5	18.573	1.0	Average	21.117
Average	18.582		Corrected results	$(10.578\pm0.005)\times10^{-11}$ cm; 1171.8 kev±1.0
Corrected results	$(9.308\pm0.005)\times10^{-11}$ cm; 1331.6 kev±1.0		Runs 2, 4, 5 Corrected results	$(10.583\pm0.005)\times10^{-11}$ cm; 1171.3 kev±1.0
			Average	
			6, 7, 8 Weight 1; 2, 4, 5 Weight 0.5	$(10.580\pm0.005)\times10^{-11}$ cm; 1171.5 kev±1.0

of a slight decentering of the "β-point"[12] of the instrument relative to the "zero" of the instrument scale which can easily be shown to introduce entirely negligible error. The ordinate scales for the different runs are indicated by numbers giving the total number of counts observed at each setting for a standard counting interval which was uniformly 1000 sec. for all the runs plotted in Fig. 2.

Figure 3 shows three more runs taken on the 1.1-Mev line alone. This gives a good idea of the reproducibility of the measurements, when the instrument is working at its best. The scales on this figure are similar to those of Fig. 2 in all respects.

Table I shows the results of all these measurements on the two lines of Co^{60}. The columns marked "screw reading (double)" give the distance, in terms of the wave-length screw (nominal x units) between line peaks as reflected in the first order to the left and to the right of the (310) planes of the quartz lamina. These screw readings must be (1) halved to obtain nominal x units and (2) converted to milliangstroms by the procedure explained in footnote 11. This procedure gives the wave-lengths shown under "corrected results." The conversion to kev energy units has been effected by means of the conversion factor 12395×10^{-8}, the wave-length associated with one ev.[13] It will be noted in Table I that less weight has been assigned to the 1.3-Mev wave-lengths obtained from the exploratory runs 1 and 3. This is because these curves were taken with a shorter counting interval than the rest, their primary object being to locate the positions of the lines to sufficient accuracy to permit planning an economical schedule of settings for the more careful runs. Also, less weight was assigned to the 1.1-Mev wave-lengths obtained in runs 2, 4, and 5 than in the three later very satisfactory runs 6, 7, and 8. In run 2, the wave-length of the single 1.1-Mev line had to be determined by measuring its distance from the neighboring 1.3-Mev line. Also, the counting interval in this case was only 5 minutes. In run 4, there were counter troubles which were not extremely serious but sufficient to make repetitions of some of the readings desirable. Such repetitions required reversing the direction of travel of the screw carriage to reach a previous setting and then proceeding as before. We have found that such a procedure in the middle of a run may introduce minute but detectable hysteresis effects which may lead to small errors in the wave-lengths readings. There was a suspicion also in run 5 that the β-point of the instrument might have shifted ever so slightly in going from the reflections on one side to those on the other. Checks were therefore made on the 1.1-Mev wave-length in runs 4 and 5 by measuring the *separations* from the adjacent 1.3-Mev lines. In the case of run 5, it was found that a slight β-point shift had indeed occurred and therefore in this run the wave-length *differences* from the adjacent 1.3-Mev lines together with the average value of these latter were used (instead of the separation between right and left hand orders) and only half-weight was assigned to this individual value. All of these deviations are small and give no cause for alarm regarding the reliability of our final average results which are shown at the bottom of Table I.

INTEGRATED REFLECTION COEFFICIENT AT 9 X.U.

The 1.3-Mev line at 9 x.u. is the shortest wave-length we have so far studied and it was of interest to measure the integrated reflection coefficient of the (310) planes of our quartz crystal at this wave-length for comparison with the data at longer wave-lengths. The same method was followed as that used for previous nuclear gamma-ray lines. The total counting rate in the directly trans-

[12] The "β-point" is the point on the focal circle to which the reflecting planes of the quartz crystal would converge if produced. This is therefore the point on the focal circle which corresponds to the true zero of wave-length. The nominal scale of the instrument wave-length screw has a nominal zero point which never agrees exactly with the "β-point" because of slight variations in the way in which the source is adjusted in its holder and in other instrument adjustments. Such deviations are however quite unimportant.

[13] J. W. M. DuMond and E. R. Cohen, Rev. Mod. Phys. **20**, 82 (1948).

mitted beam was determined by setting the instrument at "zero wave-length" so that the primary beam was directly transmitted through the crystal and collimator to the counter. Because this direct beam was far too intense to be measured by the counter directly, absorbing sheets were introduced into the beam and the absorption curve (logarithm of counting rate against thickness of absorber) was extrapolated back to zero absorption thickness to determine the true counting rate. One-half of this direct beam counting rate we associate with each of the two lines since our spectra show that they are quite closely equal in intensity. On this basis, we find for the 1.1 Mev line 4.97 counts in the diffracted beam per 10,000 counts in the direct beam, and for the 1.3-Mev line 4.12 counts in the diffracted beam per 10,000 counts in the direct beam. This is *not* the integrated reflection coefficient but it gives a basis from which the latter can be calculated. Such a calculation[14] is too involved for the scope of the present paper, but it has been shown that the integrated reflection coefficient for wave-lengths from $\lambda=200$ x.u. down to $\lambda=30$ x.u. diminishes quite closely as λ^2. The present measurements at 9 x.u. agree substantially with this law. If anything, they lie slightly higher than the λ^2 line determined at longer wave-lengths but we cannot guarantee that this is a significant deviation.

[14] A paper by D. A. Lind on this question of the integrated reflection coefficient for the (310) planes of quartz as a function of wave-length over a very wide range of wave-lengths is now in preparation.

COMPARISON WITH PREVIOUS WORK

The shortest wave-lengths measured previously by direct crystal diffraction are, we believe, those observed with the photographic crystal spectrometer of Frilley.[15] The shortest of Frilley's lines was 16 x.u. or 770 kev quantum energy. From an examination of Frilley's photograph it seems doubtful whether a precision in this wave-length determination better than ± 2.0 percent could be claimed. We believe it safe, therefore, to say that the present measurements of the 1.1- and 1.3-Mev lines of Co^{60}, quite independent of their high precision, set a new record for the shortest wave-lengths ever measured directly. They surpass Frilley's hardest lines as to quantum energy by a factor of nearly two. The precision is, of course, from 50 to 100 times that of Frilley's spectra.

These results are far from representing the limit of the instrument even in its present state of development. We believe it quite possible with our present collimator and crystal holder to go to 2 Mev and perhaps somewhat beyond this. With the improved tungsten partition collimator and further improvements in the crystal and the detecting system such as we have outlined above, it is probable that the limit can be pushed still further upward very substantially.

[15] Frilley, thesis, Paris, 1928. For reproductions of Frilley's spectra, see Rutherford, Chadwick, and Ellis, *Radiations from Radioactive Substances* (Cambridge University Press, London, 1939), p. 380.

PHYSICAL REVIEW VOLUME 76, NUMBER 12 DECEMBER 15, 1949

Penetration and Diffusion of Hard X-Rays through Thick Barriers. III. Studies of Spectral Distributions*

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(Received August 15, 1949)

Two kinds of spectral distributions of gamma-rays are discussed. These are (1) spectra found in an infinite medium with an uniformly distributed monochromatic source, and (2) equilibrium spectra obtained in an artificial penetration problem. Curves are shown for various media and for various energies of the source photons. The relations of these spectra to the general problem of γ-ray penetration is discussed.

In this note we report some results of a study of two rather simple problems in multiple γ-ray scattering.[1–4] These results may have some interest in themselves and also serve to illuminate certain features of the general problem of γ-ray penetration.

The first problem to be discussed is the following: a time-independent monochromatic source of gamma-rays is distributed uniformly in an infinitely extended, isotropic medium which is capable of absorbing gamma-rays by means of photoelectric effect and pair production, and of scattering them by Compton effect. The problem is to find the steady state spectrum in the medium, i.e., the average number of photons per cm^3 per unit energy range, as a function of the photon energy.

Solutions are to be found for various media and for various energies of the source photons. An experimental study of this problem has recently been described[5] by workers at the Naval Research Laboratory. For this reason we call this the "N.R.L." problem.

* Work supported by an ONR contract.
[1] Bethe, Fano, and Karr, Phys. Rev. **76**, 538 (1949). (Part I of the series "Penetration and diffusion of hard x-rays through thick barriers".)
[2] U. Fano, Phys. Rev. **76**, 739 (1949). (Part II of the series.)
[3] U. Fano, Nucleonics (to be published).
[4] L. V. Spencer and F. A. Jenkins, Phys. Rev. **76**, 1885 (1949).
[5] W. R. Faust and M. H. Johnson, Phys. Rev. **75**, 467 (1949).

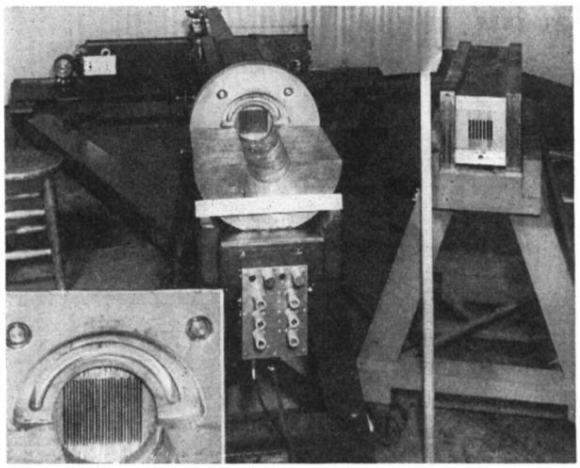

FIG. 1. Comparison of new and old lead collimators for the two-meter focusing curved-crystal gamma-ray spectrometer. To indicate the scale, a horizontal one-foot rule can be seen at the back of the lead shielding on the new collimator. The cylindrical multicellular counter and the upper half of the lead shields that cover it have been removed. The semicircular cavity that can be seen in the end of the first circular lead shield is provided to accommodate the battery of anticoincidence counters which in operation are situated above the multicellular counter to protect it from cosmic rays. A meter stick stands vertically near the old collimator which is at the right. Both collimators are 30 inches long. The insert is an enlarged view of the exit end of the new collimator, the thicker ends of the tapering lead partitions being visible.