In Situ Acoustic Attenuation Measurements in Glacial Ice

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Abstract. The attenuation coefficient of ice in a temperate valley glacier was measured by spectral analysis of the pressure pulse, directly transmitted through the ice from a small explosion. Values varied from 0.014 ± .002/m at 2.5 kc/s to 0.215 ± .002/m at 15 kc/s. The attenuation function follows a form $\alpha = A + Bf^4$ closely, suggesting Rayleigh-type scattering as the dominant source of attenuation of high-frequency acoustic waves in glacial ice. Scattering from ice crystal boundaries is compatible with the observed scattering coefficient.

Introduction. The attenuation of acoustic waves in polycrystalline materials has been studied by many investigators; however, most of these studies have been in metals [*Mason and McSkimin*, 1947; *Zener*, 1948] or in fine-grained rocks [*Knopoff and MacDonald*, 1958]. The only published study of acoustic attenuation in ice above 2 kc/s that was found is that of *Kneser et al.* [1955], who studied single-crystal tap-water ice by torsional resonance methods.

During a study of techniques to improve seismic sounding results in valley-glacier environments, it became desirable to acquire knowledge of ice attenuation values at high frequencies, since the spatial resolution of such soundings of very rough ice-rock interfaces is greatly improved by the utilization of short wavelengths.

Technique of measurement. The present measurements were made on the Blue Glacier, 30 km south-southwest of Port Angeles, Washington, during August 1963. The general configuration of this glacier has been reported by *Allen et al.* [1960], and special studies of the crystalline nature of the ice have been made by *Kamb* [1959]. The present investigation was conducted in bare glacial ice, of about 300-m total depth, near the pressure melting point. The surface near the measuring location was reasonably level, with a surface topography rough on the 10-cm scale. There were a few open crevasses in the general area, but none was closer than 50 m.

A 61-m hole, 6 cm in diameter, was drilled vertically into the ice with an electrically heated 'hot point.' This hole was filled with water as the detector cable was moved for each measurement, ensuring a known hydrostatic load on the detector.

A 0.5-m hole was drilled 2.5 m from the detector hole for the explosive charges. This hole was adjacent to a running stream of melt water, so that a constant 'tamp' was always available for the explosion. Standard electric 'seismic' blasting caps were used for sources, providing more than adequate signal-to-noise ratios for the recording.

A radially polarized, cylindrical piezoelectric pressure detector was attached to 65 m of shielded two-conductor cable loaded with a 27,000-ohm resistor. A Textronics type 502 oscilloscope was photographed with a Polaroid camera attachment to produce permanent recordings of the transmitted pulses. The detector and cable capacitance, the oscilloscope input capacitance, and the 27,000-ohm load combined to produce a 15-kc/s low pass filter. The recording system was therefore flat from about 3 cps to 15 kc/s and then attenuated at about 3 db/octave toward higher frequencies.

Data. The reproducibility and high signal-to-noise ratio of the transmitted pulses are shown in Figure 1, where two successive shots are recorded from a detector depth of 59.5 m. Spectral analysis of these two pulses yield attenuation values identical within ±0.002/m, which is therefore considered the random error in the data.

Figure 2 shows the transmitted pulse after it had traveled 8.9 m from the source. The spectrum of this pulse is illustrated in Figure 3. The high-frequency roll-off is due, in part, to the

[1] Contribution 1293.

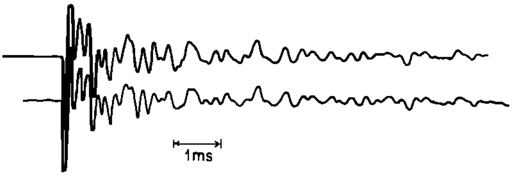

Fig. 1. Repeatability of the recorded pulses. Hydrophone depth 59.5 m.

15-kc/s low-pass filter. The hydrophone was lowered into the detector hole to depths of 9.0, 17.9, 32.4, and 59.5 m. Several records were taken at each depth to insure reproducibility of the source and system. No evidence of 'fatigue' of the 6-cm-diameter source hole during several dozen shots was observed.

The photographic records were digitized with a measuring engine and processed on an IBM 7094 computer. The spectral analysis program, identified as 'COQUAD MOD4' (furnished by S. W. Smith of California Institute of Technology) follows a procedure described by *Blackman and Tukey* [1960].

Since the full length of the pulses could not be conveniently recorded and digitized, a truncation occurred at the end of the digitized pulse. Figure 3 also illustrates the difference in the power spectrums obtained by either ignoring this truncation or by apodizing the pulse with a Gaussian multiplier of the form e^{-kt^2} so as to produce a pulse which is essentially complete within the digitized range. Amplitude spectrums from the apodized pulses were used in the calculations.

Reduction of the data. The attenuation coefficient, α, is normally defined by

$$A = A_0 e^{-\alpha r}$$

where A is the pressure amplitude of a plane acoustic wave, A_0 is the pressure amplitude at some initial point, α is the attenuation coefficient, and r is the distance from the initial point. Thus, $\alpha = \ln(A_0/A)/r$ and has dimensions of L^{-1}.

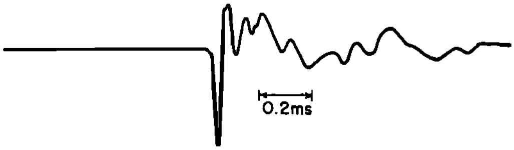

Fig. 2. Pulse transmitted through 8.9 m of ice.

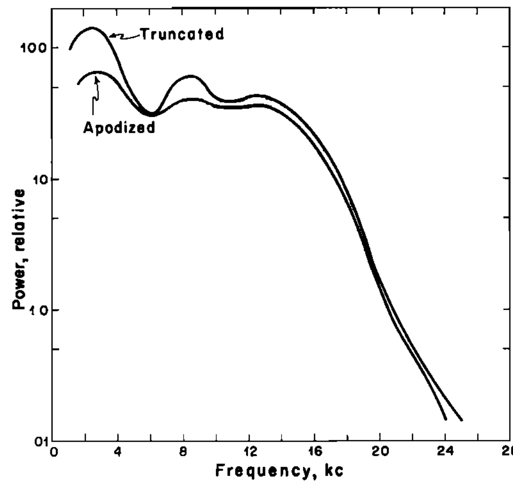

Fig. 3. Power spectrum of 8.9-m pulse with and without apodization.

Since the pressure amplitude decays as r^{-1} from the source, the value of α may be computed differentially from data from two detector positions located at distances r_1 and r_2 from the source.

$$\alpha = \frac{\ln(A_1 r_1 / A_2 r_2)}{(r_2 - r_1)} \tag{1}$$

The data were reduced differentially using (1) with the recording from 9.0 m as an initial value. For each range of frequency, the record from a depth that gave the most accurate data was chosen. This procedure ensures that acoustic nonlinearities near the explosion are not differentially reflected in the data. The variation of α with frequency is illustrated in Figure 4.

Discussions. The data illustrated in Figure 4 display two distinct attenuation regimes, that from 2 to 5 kc/s where the value of α is about constant and that from 5 to 15 kc/s where the attenuation increases rapidly with frequency. A fit of these data to the form

$$\alpha(f) = A + Bf^4 \tag{2}$$

yields $A = 0.0140$/m and $B = 7.7 \times 10^{-18}$/m sec^4. Values of $S = \alpha(f) - A$ are shown in Figure 5 along with a line corresponding to $7.7 \times 10^{-18} f^4$. The fit suggests that Rayleigh-like scattering is a significant source of attenuation above 5 kc/s. The deviation toward lower values

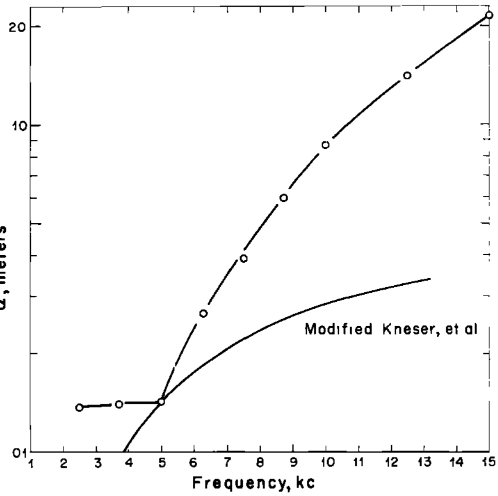

Fig. 4. Measured attenuation coefficient α and calculated values of α modified from Kneser et al. [1955].

of S above 10 kc/s is significant and will be examined below.

The use of (2) to fit the data assumes a frequency-independent attenuation coefficient A. *Kneser et al.* [1955] have studied the decay of torsional vibrations of a single tap-water ice crystal. Unfortunately, as Kneser et al. point out, the relaxation time derived from their studies depends greatly on the water purity, and therefore the data are not directly useful for the more nearly pure ice encountered in a glacier. If one utilizes, however, the relaxation time measured in quite pure ice by *Auty and Cole* [1952], using dielectric techniques, and fits an attenuation curve, of the form derived by Kneser et al., to the present data at 5 kc/s, the resultant run of attenuation with frequency is shown by the solid line in Figure 4. By using this modication of their data in place of A in (2), assuming that their values more nearly represent the non-scattered attenuation, I plotted a run of data

$$S' = \alpha(f) - A'$$

in Figure 5. These data lead to the value $B' = 5.6 \times 10^{-19}$. The values derived by this modification of the data of Kneser et al., however, yield a very poor fit for the attenuation below 5 kc/s, suggesting that the modification used here is inappropriate at least below 5 kc/s and that the Kneser experiment should be rerun with pure ice.

Two possible sources of scattering are obvious for glacial ice. The ice is coarsely crystalline; thus the lattice anisotropy of the elastic constants will cause elastic discontinuities at each interface and, therefore, scattering. On the other hand, much of the ice is filled with bubbles of 1- to 2-mm diameter, forming up to 10% of the volume (W. B. Kamb, private communication, 1964). Since the elastic properties of air are very different from those of ice, a marked elastic discontinuity exists at each bubble boundary which should cause scattering.

The intercrystalline scattering coefficient can be calculated from the elastic properties of the ice if we make assumptions as to the effective crystal size and the mean orientation of the individual crystals.

Mason and McSkimin [1947] give an expression

$$B = \frac{2\pi^4}{3c^4}\left[\left\langle\frac{\Delta k}{k}\right\rangle^2\right]D^3 \qquad (3)$$

where c is the velocity of propagation of acoustic waves in ice, k is the compression modulus, and D is the effective crystal diameter. The value of $\langle\Delta k/k\rangle^2$, the mean value of the variation in modulus, may be computed for hexagonal ice crystals by [*Mason,* 1950]

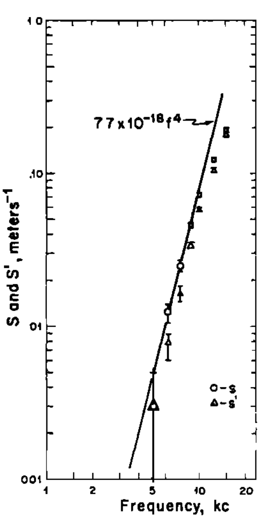

Fig. 5. Values of S and S', the scattered components of the attenuation, plotted with $S = 7.7 \times 10^{-18}f^4$.

$$\left\langle\frac{\Delta k}{k}\right\rangle^2 = \left\langle\frac{c_{11}' - c_{11}'}{c_{11}'}\right\rangle^2$$

$$= \frac{4}{1575}\left\{\frac{48c_{11}^2 - 64c_{11}c_{33} + 28c_{33}^2 - 16c_{11}(2c_{13}+4c_{44}) + 4c_{33}(2c_{13}+4c_{44}) + 3(2c_{13}+4c_{44})^2}{[\frac{8}{15}c_{11} + \frac{3}{15}c_{33} + \frac{2}{15}(2c_{13}+4c_{44})]^2}\right\} \tag{4}$$

If we solve (3) for D, the effective scattering diameter, using the elastic constants of *Bass et al.* [1957] and the observed value of B, we obtain a value of about 8 cm. *Allen et al.* [1960] and *Kamb* [1959] have described the grain size distribution for the ice in the locality of these measurements. The ice consists of three general types, coarse-bubbly ice in crystals 1 to 6 cm in diameter, coarse-clear ice crystals up to 20 cm long, and fine ice from 0.5 mm to 2 mm in diameter. These ice types occur in layers and lenses with dimensions of up to a few meters. Accordingly, we may conclude that intercrystalline scattering by randomly oriented ice crystals appears to be of the appropriate magnitude to explain the observed attenuations.

The bubbly ice should also be a good scatterer; however, the wavelength at 15 kc/s is about 265 mm, which is more than 100 times the average bubble diameter. To evaluate the theoretical scattering coefficient due to spherical air bubbles we may follow *Roney* [1950], who gives an expression for Rayleigh scattering from one particle as

$$\frac{W_s}{W_I} = \frac{4\pi^3 T^2}{A\lambda^4}\left[\left(\frac{\Delta k}{k}\right)^2 + \frac{1}{3}\left(\frac{\Delta\rho}{\rho}\right)^2\right]$$

where

- w_s is the scattered intensity.
- W_1 is the incident intensity.
- A is the area of the scattering particle.
- T is the volume of the scattering particle.
- k is the compression modulus (equal to c_{11} in solid bodies) of the medium.
- ρ is the density of the medium.
- $\Delta\rho, \Delta k$ are the differences of ρ and k between the scattering body and the medium.

Retaining $\Delta\rho/\rho$, we may obtain, neglecting multiple scattering,

$$B = \frac{2\pi^4}{3c^4}\left[\left\langle\frac{\Delta k}{k}\right\rangle^2 + \frac{1}{3}\left(\frac{\Delta\rho}{\rho}\right)^2\right] \tag{5}$$

The computed value of B from (5) for bubbles of 2 mm diameter spaced on the average 4 mm apart is about 2×10^{21}. This value is more than three orders of magnitude less than that due to intercrystalline scattering, and it is therefore unlikely that the bubbles are a significant source of scattering below 15 kc/s.

A further possibility exists: the scattering may be due to larger pods of bubbly ice contrasting with the clear-coarse ice. These lenses may be large enough, meter size, to contribute significantly to the scattering around 5 kc/s. This and other more complicated circumstances of possible significance will not be considered further here.

The values for S and S' deviate significantly from Bf^4 at frequencies above 10 kc/s. *Roney* [1950] has derived a theoretical expression for the value of S, under the assumption that the values are small and that multiple scattering is insignificant, for values of D of the order of a wavelength. This work, though not directly applicable since metals have a slightly different attenuation function, indicates a slow decrease of the exponent of f from 4 in the Rayleigh region to zero in the extreme case, where D is very large with respect to a wavelength. We should therefore expect, qualitatively, the observed deviation in α from fourth-power variation when the wavelength falls below the Rayleigh criterion of

$$\lambda \geq 3D \tag{6}$$

Summary. In situ studies of the propagation of compressional acoustic waves in glacial polycrystalline ice in the frequency range from 2.5 to 15 kc/s have shown the presence of attenuation due to Rayleigh scattering. For frequencies above 5 kc/s this scattering is apparently due to the elastic inhomogeneity at crystal interfaces. Both the scattering coefficient and the deviation from a strict f^4 law at high frequencies suggest true scattering diameters of a few centimeters. The major population of the ice crystals range from 1 to 6 cm in diameter. The presence of scattering will probably preclude use of frequencies above about 7.5 kc/s for seismic sounding through thick temperate glaciers.

Acknowledgments. I would like to thank Drs. H. Dix, B. Murray, and D. Anderson for valuable discussions and Dr. C. Allen for help with part of the field work.

This research was supported by NSF grant GP-745.

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(Manuscript received November 28, 1964; revised January 20, 1965.)