Abstract. We estimated the amplitudes of Jovian normal modes excited by a near-surface impact. The planet is modeled as a self-gravitating fluid sphere, and the impact source is approximated by a single force or a point pressure source. The effects of rotation are not included. An impact of 700 km at a distance of about 25 occurs near the antipode. This estimate is not sensitive to the assumptions on the density distribution above the surface. The amplitude in the higher altitude can be significantly higher. At shorter periods (100 to 200 s), the amplitudes depend on the source depth. For an impact with $Mv=10^{23}$ g cm/s, and for a source depth of about 20 to 50 km, the amplitude and the particle velocity at a distance of 90° are of the order of 100 m, and 3 m/s, respectively. The pressure change at a depth of 20 to 50 km is of the order of 20 dyne/cm², or about $2\times10^9$ of the ambient pressure at a depth of 40 km.

Introduction

In view of the expected impact of the Shoemaker-Levy 9 comet on the Jovian atmosphere in July, 1994 [Chapman, 1993], it is of interest to estimate the amplitude of seismic excitation of Jovian normal modes by an impact source. Although the periods of Jovian normal modes have been computed by many investigators [e.g. Vorontsov et al. 1976; Bercovici and Schubert, 1987; Mosser, 1990; Lee, 1993], their excitation by an impact source has not been investigated.

Although normal-mode excitation in the solid planets like Earth is a straightforward problem, there is some question regarding how the excitation problem should be treated for the planet Jupiter because of the very rapid decrease in density, incompressibility, and sound velocity from the planet interior to the surface. Here, we treat the problem in essentially the same way as for Earth, except that the planet is modeled as a compressible self-gravitating fluid sphere. We assume, somewhat arbitrarily, the 1 bar isobaric surface to be the surface of the planet, and compute the displacement on this surface. Although what type of kinematic source is appropriate for a comet impact is not obvious, here we consider excitation by a single force and an isotropic pressure source as a simple extension of the treatment for Earth.

Since the rotation period of Jupiter is about 10 hours, the effects of rotation on normal modes are more significant for Jupiter than Earth. Vorontsov and Zharkov [1981] used a perturbation method to investigate the effects of rotation and ellipticity on low order modes. Their results show that the spectral peaks of low order modes split as much as ±30%. Mosser [1990] used an asymptotic theory to estimate the effects of rotation on high order modes. He showed that the effects on high order modes are mainly due to the oblateness caused by rapid rotation, rather than the effects of Coriolis force itself. Lee [1993] also investigated the effects of rotation and internal phase boundaries on normal modes. All these studies clearly demonstrated that rotation causes significant spectral splitting.

It is thus clear that the effects of rotation cannot be ignored if we are to use the normal-mode periods to determine the internal structure of Jupiter. However, since the main purpose of this paper is to obtain order-of-magnitude estimates of the normal-mode amplitudes due to an impact, we did not include the effects of rotation in this study. The amplitude of spectral singlets estimated for a non-rotating Jovian model is probably a good estimate of the representative amplitude of split multiplets.

However, we note that the rapid rotation may excite a suite of inertial normal-modes which are not included in computations for a non-rotating model.

Method

The excitation of free oscillations of a self gravitating fluid sphere can be easily formulated from the fully developed theories presented by Love [1911], Pekeris and Jarosch [1958], Alterman et al. [1959], Takeuchi et al. [1961], Bolt and Dorman [1961], Saito [1967] and Takeuchi and Saito [1972].

Using the standard spherical coordinate $(r, \theta, \phi)$, the displacement field $(u_r, u_\theta, u_\phi)$ and the perturbation of the gravitational potential, $\psi$, in a fluid sphere can be written as follows:

$$u_r = y_1(r) S^*(\theta, \phi) e^{i\omega t}, \quad u_\theta = y_2(r) \frac{\partial}{\partial \theta} S^*(\theta, \phi) e^{i\omega t},$$

$$u_\phi = y_3(r) \frac{\sin \theta}{\sin \theta} \frac{\partial}{\partial \phi} S^*(\theta, \phi) e^{i\omega t},$$

where $y_1(r), y_2(r)$ and $y_3(r)$ are functions of $r$, and $S^*(\theta, \phi)$'s are the spherical harmonics with the angular and azimuthal order numbers $l$ and $m$. We introduce two more functions $y_4(r)$ and $y_5(r)$ defined by

$$p = -k\Delta = -y_4(r) S^*(\theta, \phi) e^{i\omega t},$$

$$\psi' - 4\pi G\rho_0 y_5 = y_5(r) S^*(\theta, \phi) e^{i\omega t},$$

where $p$ is the pressure, $k$ is the incompressibility, and $\Delta$ is dilution; the symbol $'$ denotes differentiation by $r$.

The functions $y_1, y_2, y_3, y_4$, and $y_5$ satisfy the following set of differential equations:

$$y_1' = \frac{2y_1}{r} + \frac{2y_2}{k} + \frac{l(l+1)}{r} y_3,$$

$$y_2' = -\alpha^2 p_0 + \frac{4\rho_0 g_0 y_5}{r} + \frac{l(l+1)\rho_0 g_0}{r} y_2 - \rho_0 y_6,$$

$$y_3' = -4\pi G\rho_0 y_5 + y_6,$$

$$y_4' = -\frac{4\pi G\rho_0}{r} l(l+1) y_5 + \frac{l(l+1)}{r^2} y_3 - \frac{2y_5}{r},$$

where $G$, $\rho_0(r)$, and $g_0(r)$ are the universal constant of gravity, unperturbed density and gravity inside the planet, respectively.
The function \( y_3(r) \) above is given by
\[
y_3 = \frac{1}{r a^2} (g_0 y_2 - y_3)
\]
(4)

Solving (3) with the "free surface" boundary condition and the condition for continuity of gravitational potential at the surface \( r=a \):
\[
y_2 = 0, \quad \text{and} \quad y_3 + [(l+1)/l] y_3 = 0
\]
(5)
results in a set of eigen values \( \omega_n^2 \) and eigen functions \( u_n^2(r) \), each with radial factors \( \psi_n, \psi_2, \ldots, \psi_n \). The azimuthal order number \( m \) for \( \omega_n^2 \) can be dropped when rotation, ellipticity and lateral heterogeneity are ignored.

When a step function single force \( f \) is applied at \( (r_0, 0, 0) \) in the direction of \( \theta = \pi/2 + \delta \), and \( \phi = \phi_0 \), the displacement at \( r, \theta, \phi \) is given by
\[
u(r,t) = \sum_y y_1(r) \frac{f}{\psi(r)} K_2 \left[ P^n_1 y_1(r) f \sin \delta - P^n_1 y_1(r) f \cos \delta \cos \phi \right]
\]
\[
u_\phi(r,t) = \sum_y y_1(r) \frac{f}{\psi(r)} K_2 \left[ P^n_1 y_1(r) f \cos \delta \cos \phi \right]
\]
\[
u_\theta(r,t) = \sum_y y_1(r) \frac{f}{\psi(r)} K_2 \left[ P^n_1 y_1(r) f \sin \delta \cos \phi \right]
\]
(6)
where
\[
K_2 = \frac{2l + 1}{4 \pi \omega^2 (l + 1)(l + 2)} \frac{y_3(r)}{r}
\]
(7)

Similarly, for a step function isotropic source with a dipole moment \( M_0 \), the displacement is given by
\[
u(r,t) = \sum_M M_0 y_1(r) N_0 \phi^2 \cos \phi
\]
\[
u_\phi(r,t) = \sum_M M_0 y_1(r) N_0 \phi^2 \sin \phi
\]
\[
u_\theta(r,t) = 0,
\]
(8)
where
\[
N_0 = \frac{2l + 1}{4 \pi \omega^2 (l + 1)(l + 2)} \frac{y_3(r)}{k}
\]
(9)

An isotropic source with a dipole moment \( M_0 \) is equivalent to a pressure source in a small cavity of radius \( \varepsilon \) with pressure \( p = M_0 / 4 \pi \varepsilon \).

In (7) and (9), \( I_1 \) and \( I_2 \) are the energy integrals defined by
\[
I_1 = \int_0^a \rho_0 v^2 y_1^2(r) \, dr, \quad \text{and} \quad I_2 = \int_0^a \rho_0 v^2 y_3^2(r) \, dr
\]
(10)

The excitation functions \( K_2 \) and \( N_0 \) were used in Kanamori and Cipar [1974], Okal [1978] and Kanamori and Given [1982]. In (6), (7), (8), and (9), the order numbers \( n \) and \( m \) for eigen frequencies and eigen values are omitted. Expressions (6) to (9) hold for each radial overtone number \( n \) which is omitted for clarity.

The pressure \( p(r,t) \) can be obtained by replacing \( y_1(r) \) by \( -y_3(r) \) in \( u_\phi(r,t) \) in (6) and (8).

To compute short-period motions, \( l \) must be increased. However, to compute the motion at a period of \( 100 \) s, normal modes with \( l \) up to \( 10,000 \) must be computed, and the mode summation is impractical. It is more convenient, then, to use high-frequency asymptotic forms of equations (6) and (8) using the asymptotic form of the Legendre functions. Then (6) can be written as
\[
u_\phi(r,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{C}(\omega) \exp(i\omega \delta) \, d\omega
\]
(11)
where
\[
\tilde{C}(\omega) = \frac{1}{\sin \theta} \exp(-i\omega \theta / (C + \pi / 4) y_1(r) L_1 \sqrt{-K_1^2 f)} \left[ y_1(r) \sin \delta \cos \phi \phi_{i} \right]
\]
(12)
where \( C \) and \( U \) are the phase and group velocities respectively.

Results

Vorontsov et al. [1976] used equation (3) to compute the periods of the Jovian normal modes. We used the structure given in Vorontsov et al. [1976] to compute the normal modes. We used the density and pressure curves given in Vorontsov et al. [1976] to compute the normal modes. We smoothed the density and pressure curves given in Vorontsov et al. [1976] to compute the normal modes.

The eigen frequencies, \( f \), thus computed for the fundamental mode \( (n=0) \) and the first overtone \( (n=1) \) are shown in Figure 1 as a function of the angular order number \( l \).

Fig. 1. Jovian normal modes. Eigen frequencies are shown as a function of the angular order number \( l \), for the fundamental mode \( (n=0) \) and the first overtone \( (n=1) \). The skin depths are shown for a few modes.
Figure 2 is dominated by the wave with a period of about the exact mechanism of seismic excitation by the impact of the Shoemaker-Levy 9 comet is uncertain, the time scale of the impact would be much shorter than the period of the normal modes, and the source can be approximated by a delta function. Since we included the modes only up to \( f = 10 \, \text{Hz} \) in this computation, the waveform shown in Figure 2 is dominated by the wave with a period of about 700 s. The density and the entry velocity of the comet are 1 g/cm\(^3\) and 60 km/s, respectively, this force is equivalent to a momentum transfer, \( M_v \), of a comet with a radius of 1.6 km.

Figure 2 shows the displacements at a distance of \( \Delta = 90^\circ \) and \( 180^\circ \) (antipode of the source) computed for a vertical single force applied at a depth of 100 km. The peak-to-peak amplitudes are 31.5 and 824 cm at \( \Delta = 90^\circ \) and \( 180^\circ \), respectively. An amplification of a factor of about 25 occurs for a single force shown in Figure 2. The results are summarized in Table 1, which includes the estimates for comets with larger sizes. Also, at short period, the modes are strongly affected by the density and temperature distributions near the surface. Although detailed computations cannot be made, the effects can be expressed in terms of a cut-off frequency for normal modes. At frequencies higher than a certain frequency, the acoustic cut-off frequency, the mode energy propagates into the atmosphere, which prevents development of normal modes in the planet's interior. Bercovici and Schubert [1987] and Vorontsov et al. [1989] suggest that the acoustic cut-off frequency for Jupiter is about 3 mHz. Thus, the results at periods shorter than 300 s obtained here are subject to large uncertainties.

Since we included the modes only up to \( f = 200 \) (period of about 700 s) in this computation, the waveform shown in Figure 2 is dominated by the wave with a period of about 700 s. Since the skin depth at this period is 380 km (Figure 1), the excitation at short period diminishes very rapidly as the source depth increases to 50 km. Also, at short period, the modes are strongly affected by the density and temperature distributions near the surface. The pressure vanishes at the surface, increases gradually as the depth increases, and begins to decrease at a certain depth. The results are shown in Table 2, and Figure 4 shows the variation of pressure as a function of depth. The pressure change is about \( 2 \times 10^5 \) dyne/cm\(^2\) at a depth of 40 km.

<table>
<thead>
<tr>
<th>Source</th>
<th>( M_v = 10^{23} ) g-cm/s</th>
<th>( M_v = 6.8 \times 10^{23} ) g-cm/s</th>
<th>( M_v = 2.5 \times 10^{23} ) g-cm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta = 90^\circ )</td>
<td>32 cm</td>
<td>2.1 m</td>
<td>79 m</td>
</tr>
<tr>
<td>( \Delta = 180^\circ ) (antipode)</td>
<td>8.2 m</td>
<td>56 m</td>
<td>2.1 km</td>
</tr>
<tr>
<td>( (a_i = 1.6 \text{ km}) )</td>
<td>0.28 cm/s</td>
<td>1.9 cm/s</td>
<td>0.71 m/s</td>
</tr>
<tr>
<td>( (a_i = 3 \text{ km}) )</td>
<td>0.28 cm/s</td>
<td>1.9 cm/s</td>
<td>0.71 m/s</td>
</tr>
<tr>
<td>( (a_i = 10 \text{ km}) )</td>
<td>0.28 cm/s</td>
<td>1.9 cm/s</td>
<td>0.71 m/s</td>
</tr>
</tbody>
</table>

Table 2. Peak-to-peak amplitudes of displacement, velocity and pressure of short period (about 100 to 200 s) waves at \( \Delta = 90^\circ \) for an impact with \( M_v = 10^{23} \) g-cm/s.

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>Disp. (m)</th>
<th>Vel. (m/s)</th>
<th>Pressure (dyne/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>at 20 km at 50 km</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>890</td>
<td>15</td>
<td>51</td>
</tr>
<tr>
<td>20</td>
<td>210</td>
<td>4.1</td>
<td>22</td>
</tr>
<tr>
<td>50</td>
<td>98</td>
<td>1.2</td>
<td>6.7</td>
</tr>
<tr>
<td>100</td>
<td>12</td>
<td>0.2</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Fig. 3. The spectrum of the seismogram at \( \Delta = 90^\circ \) computed for a single force shown in Figure 2. The amplitude does not significantly vary for a range of source depth from 0 to 300 km. As the frequency increases beyond 2.5 mHz (400 sec), the amplitude of the eigen function decreases significantly at a depth of 100 km. As a result, excitation by a source at a depth of 100 km decreases and \( A(f) \) rolls off beyond this frequency. However, for a very shallow source the trend given by (13) continues to higher frequencies. For a source within a depth range of 0 to 300 km, the amplitude of the fundamental modes is about four times larger than that of the first overtone at a given period (not shown here). Hence a comet impact will excite primarily the fundamental modes.

Short-period oscillation

We computed short-period waves using the asymptotic form (11) with \( \ell \) up to 10,000 (\( T = 100 \) s). As suggested by the skin depths shown in Figure 1, the excitation at short period is highest for a source at a depth of 100 km. As a result, the excitation at short period diminishes very rapidly as the source depth increases to 50 km. The pressure vanishes at the surface, increases gradually as the depth increases, and begins to decrease at a certain depth. The results are shown in Table 2, and Figure 4 shows the variation of pressure as a function of depth. The pressure change is about \( 2 \times 10^5 \) dyne/cm\(^2\) at a depth of 40 km.

Fig. 2. Synthetic seismograms at \( \Delta = 90^\circ \) and \( 180^\circ \), computed for a vertical single force applied at a depth of 100 km. The detail of the first wavetrain is shown with expanded time scale on the figure at the bottom.
depth is varied from 0 to 100 km.

Discussion and Conclusion

In the above, we assumed the 1 bar surface to be the Jovian "free surface", and computed the displacements on this surface. In the real Jovian atmosphere, $p_0$ and $k$ do not vanish completely above the 1 bar surface. If we assumed small $p_0$ and $k$ at higher altitudes and computed the displacement there, the amplitude would be significantly larger. However, since the total normal-mode energy contained in the higher altitude is small because of the low density, the energy integrals $I_1$ and $I_2$ in the excitation function $K_2$ (equation 7) would remain approximately the same. As a result, $rK_2y(r_j)$ in $u_1$ and $u_2$ in (6) would also remain unchanged. Then it is evident from (6), that the displacement at $r$ is determined by the values of the eigen functions at $r$ and the source depth $r_s$. This means that the displacement at the 1 bar surface is insensitive to what we assume above this surface. Thus we can conclude that, despite the uncertainties in the details of the atmospheric structure near the Jovian surface, the amplitude at the 1 bar surface computed above is constrained well at periods longer than 700 s. At very short periods, however, the "free surface" boundary condition would be no longer valid.

The detectability of Jovian normal modes using Doppler shift of infrared and ultra-violet absorption lines has been discussed by Schmider et al. [1991] and Mosser et al. [1992]. According to these investigators, the detection threshold in the displacement at the 1 bar surface is insensitive to what we assume above this surface. Thus it is evident from (6), that the displacement at $r$ is determined by the values of the eigen functions at $r$ and the source depth $r_s$. This means that the displacement at the 1 bar surface is insensitive to what we assume above this surface. Thus we can conclude that, despite the uncertainties in the details of the atmospheric structure near the Jovian surface, the amplitude at the 1 bar surface computed above is constrained well at periods longer than 700 s. At very short periods, however, the "free surface" boundary condition would be no longer valid.

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