SU(5) and the Invisible Axion

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Dine, Fischler, and Srednicki have proposed a solution to the strong CP puzzle in which the mass and couplings of the axion are suppressed by an inverse power of a large mass. We construct an explicit SU(5) model in which this mass is the vacuum expectation value which breaks SU(5) down to SU(3)\otimes SU(2)\otimes U(1).

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The standard SU(3)\otimes SU(2)\otimes U(1) gauge theory appears to be adequate to describe all of the phenomenology of the strong, electromagnetic, and weak interactions. Moreover, much of the structure of these interactions is explained by the theory in the sense that it follows directly from the form of the gauge interactions. However, there are a number of features which can be described in the context of SU(3)\otimes SU(2)\otimes U(1) but which are in no sense explained. Some of these features, such as charge quantization and the observed value of the weak mixing angle, are explained by the extension of SU(3)\otimes SU(2)\otimes U(1) to the grand unifying group SU(5).\textsuperscript{1} The rest comprise the fundamental puzzles of contemporary particle physics: Why SU(3)\otimes SU(2)\otimes U(1) [or SU(5)] and not some other gauge group? How many generations of quarks and leptons exist and why? Why do the quark masses and mixing angles take their observed values? Why is the CP nonconservation in the SU(3) strong interactions so small? Finally, in the context of grand unified theories, there is the hierarchy puzzle. Why are the mass scales associated with the electroweak and strong interactions so small compared to the unification mass scale \(M_u \approx \text{10}^{15}\) GeV?\textsuperscript{2} Some or all of these questions may not have answers. The world may just be the way it is.

Our penultimate question, the puzzle of the smallness of strong CP nonconservation, is particularly tantalizing. Several different mechanisms have been proposed to explain the smallness. Soft CP nonconservation\textsuperscript{3} or a massless up quark\textsuperscript{4} might do it at a price in elegance. The Peccei-Quinn\textsuperscript{5} symmetry would do it, but the predicted axion\textsuperscript{6} is not seen.\textsuperscript{7} Some workers\textsuperscript{8} have suggested scenarios in which the axion is heavy and hard to see. Dine, Fischler, and Srednicki\textsuperscript{9} (DFS) have recently suggested a clever variant of the Peccei-Quinn scheme in which the axion mass and its coupling to normal matter are inversely proportional to a large and arbitrary vacuum expectation value (VEV) of an SU(2) singlet scalar field. If this VEV is large enough, their axion is invisible.

In this paper, we comment on the DFS idea. We first note that the singlet VEV must be greater than \(10^9\) GeV to satisfy astrophysical constraints.\textsuperscript{10} In the SU(3)\otimes SU(2)\otimes U(1) theory, such a large mass scale is unnatural. Thus, in the context of SU(3)\otimes SU(2)\otimes U(1), the DFS idea is a trade-off. It explains the smallness of strong CP nonconservation at the cost of introducing a hierarchy puzzle.

In a grand unified theory, it seems reasonable to imagine that the singlet VEV is of order \(M_u\). Our main purpose in this paper is to describe a model in which it is more than reasonable, it is automatic, because the DFS singlet field is precisely the field whose VEV breaks SU(5) down to SU(3)\otimes SU(2)\otimes U(1). In our model, the hierarchy puzzle is still with us, but the strong CP puzzle is solved at no additional cost.

The astrophysical constraints on a light axion have been discussed by Dicus, Kolb, Teplitz, and Wagoner.\textsuperscript{11} They find that for a light axion with conventional couplings, the power radiated in axions by the helium core of a red supergiant star would exceed the power in photon emission by about \(10^{33}\). Consistency with the usual stellar models can only be achieved if the axion couplings are reduced by at least \(10^{-6}\). In the DFS model, the axion coupling is reduced by the ratio of the usual Higgs VEV, \(\mu \approx 250\text{ GeV}\), to the singlet VEV. Thus the singlet VEV must be of order \(10^9\) GeV or larger.

Our main concern is the construction of an ex-
plicit SU(5) model which solves the strong CP puzzle. The fermion fields are the usual left-handed $10'$s ($T_L$) and right-handed $5'$s ($F_R$). The spinless fields are two 5's, represented by column vectors $H_1$ and $H_2$, and a complex 24, represented by a traceless $5 \times 5$ matrix $\Sigma$. The Yukawa couplings are (schematically)

$$g_1 \bar{T}_L c T_L H_1 + g_2 \bar{T}_L F_R H_2,$$

where $c$ denotes charge conjugation. These are invariant under the Peccei-Quinn symmetry

$$T_L \rightarrow e^{-i\alpha} T_L, \quad F_R \rightarrow e^{i\alpha/2} F_R,$$

$$H_1 \rightarrow e^{i\alpha/2} H_1, \quad H_2 \rightarrow e^{-i\alpha} H_2,$$

we demand that this be a symmetry of the scalar meson self-interactions with the addition of the following transformation law for the $\Sigma$ field:

$$\Sigma \rightarrow e^{-i\alpha} \Sigma.$$

Then the most general potential for the scalars is

$$V(H_1, H_2, \Sigma) = V_1(\Sigma) + V_2(\Sigma) + V_3(\Sigma),$$

where

$$V_1(\Sigma) = -\frac{1}{2} \mu^2 \text{Tr}(\Sigma^\dagger \Sigma) + \frac{1}{2} \alpha \left[ \text{Tr}(\Sigma^\dagger \Sigma) \right]^2 + \frac{1}{2} \alpha \text{Tr}(\Sigma^\dagger \Sigma \Sigma^\dagger \Sigma) + \frac{1}{4} \alpha \left[ \text{Tr}(\Sigma^3) \right]^2 + \frac{1}{2} \alpha \text{Tr}(\Sigma^2 \Sigma^\dagger \Sigma^\dagger \Sigma) + \frac{1}{2} \alpha \text{Tr}(\Sigma \Sigma^\dagger \Sigma^\dagger \Sigma^\dagger) + \frac{1}{2} \alpha \text{Tr}(\Sigma^\dagger \Sigma^\dagger \Sigma^\dagger \Sigma^\dagger),$$

$$V_2(\Sigma) = -\frac{1}{2} \mu^2 (H_1^\dagger H_1) - \frac{1}{2} \mu^2 (H_2^\dagger H_2) + \frac{1}{2} \alpha (H_1^\dagger H_1)^2 + \frac{1}{2} \alpha (H_2^\dagger H_2)^2,$$

$$V_3(\Sigma) = \gamma_1 (H_1^\dagger H_1) \text{Tr}(\Sigma^\dagger \Sigma) + \gamma_2 (H_2^\dagger H_2) \text{Tr}(\Sigma^\dagger \Sigma) + \beta_1 H_1^\dagger \Sigma H_1^\dagger \Sigma H_1 + \beta_2 H_2^\dagger \Sigma H_2^\dagger \Sigma H_2 + \delta_1 H_1^\dagger \Sigma H_1^\dagger \Sigma H_1 + \delta_2 H_2^\dagger \Sigma H_2^\dagger \Sigma H_2 + g H_1^\dagger \Sigma H_1 + g H_2^\dagger \Sigma H_2 + h H_1^\dagger H_1 + h H_2^\dagger H_2,$$

where all constants except $g$ and $h$ are real.

For a range of parameters, the VEV's will take the form

$$\langle \Sigma \rangle = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 - \epsilon & 0 \\ 0 & 0 & 0 & 0 & 3 + \epsilon \end{bmatrix} \lambda_0/2,$$

$$\langle H_{1,2} \rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \lambda_{1,2}/\sqrt{2} \\ 0 \end{bmatrix}.$$

The SU(3) $\otimes$ SU(2) $\otimes$ U(1) singlet component of $\Sigma$ is the DFS singlet field in this model. Its VEV, $\lambda_0$, must be of order $M$, while

$$|\lambda_1|^2 + |\lambda_2|^2 = \mu^2.$$  

It follows that $\epsilon$ is very small:

$$|\epsilon| = O(\mu^2/M^2).$$

The axion is primarily the antithermetically part of the singlet component of $\Sigma$. But, it contains a small admixture (of order $\lambda_1/\lambda_0$) of the neutral components of $H_1$ through which it couples to fermions.

One might worry that by enlarging the Higgs structure of our SU(5) theory we may have made the hierarchy puzzle more severe than in the standard SU(5) model. We can quantify this worry by counting the number of unnatural con-
to $M_a$ is the invisible axion.

The invisible axion is a curious beast. Although it is very light, it does not really belong to the effective low-energy field theory that describes our world. Because it is a pseudo-Goldstone boson associated with symmetry breaking at $M_a$, all of its interactions are suppressed by inverse powers of $M_a$. This solution to the strong $CP$ puzzle simply has no other consequences in low-energy particle physics. However, there may be cosmological implications of this idea.

Guth and Pi\textsuperscript{13} point out a cosmological problem of conventional SU(5) with no trilinear coupling of the 24. It is associated with the discrete symmetry $\Sigma \rightarrow -\Sigma$ which leads to a twice degenerate vacuum. Our model has no trilinear couplings; however, the discrete symmetry $\Sigma \rightarrow -\Sigma$ is embedded within the continuous Peccei-Quinn symmetry of the Higgs potential.

The invisibility of our axion is established by the following order-of-magnitude estimates of its properties: axion mass $f_a M_a \sim 10^{-8}$ eV; lifetime for $2\gamma$ decay $(M_\gamma f_\gamma)^{1/4} \sim 10^{50}$ yr; pseudoscalar couplings $f_p M_p \sim 10^{-13}$; scalar couplings $f_s M_s \sim 10^{-23}$.

$CP$-nonconserving scalar couplings of the axion are induced by the nonperturbative breaking of the Peccei-Quinn symmetry. In principle, $\theta$ is calculable in our model, and we estimate it to be about $10^{-15}$. The scalar couplings will lead to a "long-range" attraction of baryons by axion exchange.\textsuperscript{14} The effect is about $10^{-24}$ of the universal gravitational attraction. The contribution of $\theta$ to the electric dipole moment of the neutron is about $10^{-23}$ $e \cdot cm$.\textsuperscript{15} The most disquieting aspect of this predicted solution to the strong $CP$ problem is the predicted existence of an almost massless particle which is in practice unobservable.\textsuperscript{15}

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Note added.—Our argument that only a single unnatural condition is needed to produce the hierarchy is rather general. It applies, for example, to the SU(5) model in Ref. 10, where a real 24, a complex singlet, and two $\bar{2}$'s of Higgs are used. The astrophysical constraints\textsuperscript{10} on the SU(2) $\otimes$ U(1) singlet VEV are also mentioned in Ref. 9.

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