A Group Membership Algorithm with a Practical Specification

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Abstract—This paper presents a solvable specification and gives an algorithm for the Group Membership Problem in asynchronous systems with crash failures. Our specification requires processes to maintain a consistent history in their sequences of views. This allows processes to order failures and recoveries in time and simplifies the programming of high level applications. Previous work has proven that the Group Membership Problem cannot be solved in asynchronous systems with crash failures. We circumvent this impossibility result building a weaker, yet nontrivial specification. We show that our solution is an improvement upon previous attempts to solve this problem using a weaker specification. We also relate our solution to other methods and give a classification of progress properties that can be achieved under different models.

Index Terms—Distributed agreement algorithms, group membership, asynchronous systems.

1 INTRODUCTION

Distributed systems consist of groups of processes that cooperate in order to complete specific tasks. A Group Membership Protocol is of particular use in such systems, providing processes in a group with a consistent view of the membership of that group. In this way, when a membership change occurs, processes can agree on which of them must complete a pending task or start a new task. The problem of reaching a consistent membership view is very similar to the one of achieving common knowledge in a distributed system, commonly referred to as the Consensus Problem [39]. The Consensus Problem has been proven insolvable in asynchronous systems with crash failures [28].

Group Membership differs from Consensus in that the value to be agreed upon, namely, the current membership view, may change due to asynchronous failures. Moreover, while Consensus requires all nonfaulty processes to reach the same decision, Group Membership usually allows the removal of nonfaulty processes from the group when they are erroneously suspected to have crashed, thus requiring agreement only on a subset of the processes in the system. Despite these differences, Chandra et al. [17] recently adapted the impossibility result for the Consensus Problem to the Group Membership Problem. At the same time, they conjectured that techniques used to circumvent the impossibility of Consensus can be applied to solve the Group Membership Problem. Such techniques include using randomization [8], probability assumptions on the behavior of the system [13], and using failure detectors that are defined in terms of global accuracy and completeness system properties [15].

It must be understood, however, that the impossibility result in this context really means “not always possible,” as opposed to “never possible.” As a matter of fact, the Chandra et al. result states that any algorithm that tries to solve the Group Membership Problem cannot always make progress; there are cases (although very unlikely) in which the algorithm blocks forever.

The above idea is the basis of Neiger’s [48] approach. He suggests redesigning the problem by using a specification that is weak enough to be solvable—allowing the algorithm to block in some cases—but strong enough to prevent trivial implementations. His specification uses a weak progress requirement, allowing executions in which even a single process crash and its attempted removal from the membership may forever block all processes.

Our approach follows Neiger’s intuition. We propose a specification that requires processes to install a new membership whenever they share a new view of the system connectivity. This requirement is weak because, if no set of processes agrees on the connectivity, no progress is made. The requirement is, however, stronger than the one proposed by Neiger in that it implicitly states that removal and rejoining of any process must be allowed.

We summarize our contributions as follows: We identify the main assumptions required for proving the impossibility of Group Membership in asynchronous systems and relax one of them, namely the progress requirement, to break this result. Thus, we propose a weak, but not trivial, specification and a corresponding algorithm that solves this specification. Both specification and algorithm have the advantage of being simple. The specification we propose requires agreement on a sequence of views, that we call the consistent history requirement. Our consistent history definition requires that processes agree on the order in which the membership changes. This turns out to be a useful feature for many applications (see, for example, the discussion in [22]). The specification also allows partitions to occur and requires maintaining a consistent history within such partitions. Possible
2 Historical Background

The Group Membership Problem first originated in a paper by Birman and Joseph [10]. Since then, a large amount of work has been done in the context of synchronous [18], asynchronous under both a primary partition [4], [17], [29], [33], [41], [45], [48], [50] and a partitionable model [1], [2], [7], [24], [31], [34], and timed-asynchronous [19], [20], [21], [22], [26], [27] systems. A recent survey of these works appears in [54].

In general, specifications developed under asynchronous models are usually complex and many are difficult to understand. It appears that different protocols provide significantly different guarantees about their services and are based on different assumptions about the system’s behavior and failure detectors. Some specifications deal with more than just membership and views and also consider message services with different ordering and reliability properties, as in [6] [7], [10], [25], [27], [34]. Such specifications are known as Group Communication Services. This paper does not consider such extensions, but focuses on Group Membership alone.

As indication of how challenging specifying and implementing asynchronous membership algorithms is, widely cited research articles that attempt to give formal specifications for the primary partition and partitionable asynchronous membership problem [50] and [24], as well as their updated versions [25], [49], [51], contain some flaws in their formalisms. Anceaum et al. [3] showed that algorithms in [49], [50] and [51] allow undesirable executions and the specifications in [24] and [25] can be satisfied by trivial protocols.

Most of the difficulties in building a specification for the Group Membership Problem arise from the impossibility results in [17] and [28]. These results built a gap between group membership protocols that work in real systems—some of which have been around for many years, anticipating more theoretical results—and the formal specifications that they satisfy. Examples of such real systems that run group membership protocols are Amoeba [33], [52], Isis [11], Transis [23], Totem [47], Horus [53], Relacs [5], and more recently Phoenix [40] and RAIN [12]. Researchers developing these systems are, of course, aware of the original impossibility result [28] and of its potential application to their membership protocols [17], but they also believe that their systems can work under assumptions that can easily be verified in practice. This motivates the study of formal specifications that are solvable in completely asynchronous settings. Yet, despite the wide interest it has attracted and the number of publications on this subject, it appears that currently there is no specification that is, at the same time, simple, practical, and does not rely on any extension to the asynchronous model of computation (e.g., the existence of global failure detectors).

Our work tries to fill this gap.

3 The Model

We consider an asynchronous distributed system, where processes communicate by exchanging messages. Processes are identified by unique id’s. The asynchronous model of execution of concurrent processes follows the one described in [35]. The communication model follows the one described in [32]. Every pair of processes is connected by a communication channel. That is, every process can send messages to and can receive messages from any other. We assume processes are able to probe a communication channel for incoming messages, using a Boolean primitive as defined in [42]. Communication channels are considered to be reliable, FIFO, and to have an infinite buffer capacity. Message transmission and node processing times are finite but unpredictable; that is, no upper or lower bounds are assumed on the execution speeds of the processes or on the delays experienced by messages in transit.

The failure model allows processes to crash, silently halting their execution. Because of the unpredictable delays experienced by the system, it is impossible to use time-outs to accurately detect a process crash. A process that has been infinitely slow for some time and has been unresponsive to other processes may become responsive again at any time. Therefore, processes can only suspect other processes to have crashed, using local failure detectors. Local failure detectors are assumed to be inaccurate and incomplete. That is, local failure detectors may erroneously suspect that other, operational processes have crashed or that crashed processes are operational. Since local failure detectors run independently at each process, one local failure detector may perceive a failure, but other detectors may perceive it at a different time or not at all.

We assume that a process communicates with its local failure detector through a special receive-only channel on which the local failure detector may place a new list of id’s of processes not suspected to have crashed. We call this list the local connectivity view of the process. Each process considers the last local connectivity view received from its local failure detector as the current one.

We can summarize our model as follows:

- Sequential processes exchange messages on FIFO reliable channels with unbounded buffering capability and unpredictable delay, following the CSP specification in [32].
Probes for incoming messages described in [42] extend the model in [32].

The failure model includes process crashes. Local failure detectors inform processes of suspected changes in connectivity through special unidirectional communication channels.

4 GROUP MEMBERSHIP SPECIFICATION

Each process $p$ maintains two fundamental data structures: a set $v_p$ containing the current local connectivity view and an infinite sequence $S_p = [V_{p1}, V_{p2}, \ldots]$ of global views. Initially, all but the first view in $S_p$ are empty, the first view being the initial state of the system. The property is to extend the sequence of nonempty views, based on changes in the local views, and to do so consistently at different processes. Local connectivity views may change independently and arbitrarily at different processes, according to messages from local failure detectors. Sequences $S_p$ must be extended at different processes maintaining a consistent global history in the sequence.

We define consistent history as follows:

**Definition 1 Consistent History.** A set of processes has a consistent history of views if sequences $S_p$ are the same at all processes, unless views $V_{pj}$ and $V_{qj}$ are disjoint. Namely,

$$\text{ConsistentHistory} \equiv \forall p, q (V_{pj} = V_{qj}) \lor (V_{pj} \cap V_{qj} = \emptyset)$$

where $p$ and $q$ are process id’s and $V_{pj}$ and $V_{qj}$ are the $j$th elements in $p$’s and $q$’s global sequences of views, respectively.

We define a quiescent state as follows:

**Definition 2 Quiescent State.** A process $p$ is in a quiescent state if it does not change its sequence of global views anymore. Namely,

$$\text{QuiescentState}_p \equiv \Box (S_p = \hat{S}_p)$$

where $\hat{S}_p$ is a constant sequence of views and $\Box$ is the always or henceforth operator [35], the notation $\Box A$ meaning that $A$ holds at all time points after the reference point.

We make some remarks regarding the definitions above.

A quiescent state is stable in the sense that, once a process reaches a quiescent state, it stays in that state forever.

Consistent history requires all processes to have the same sequence of views, except for processes that were part of disjoint memberships (i.e., memberships that excluded each other); such processes are allowed to maintain the disjoint parts of their histories when they are connected again. Moreover, consistent history does not require processes that were excluded from all memberships and then readmitted to some membership to have in their sequence views in which they did not participate.

We now define a specification, consisting of four properties, for a group membership algorithm. We assume the system to be initialized to a start state where the sequences $S_p$ are the same at all processes and the last nonempty views in their sequences are the ones reported by all failure detectors.

**Property 1 Agreement.** At any point in time, all processes have a consistent history.

$$\text{True} \Rightarrow \Box(\text{ConsistentHistory}).$$

**Property 2 Termination.** If there are no more changes in the local views of the processes, they eventually reach their quiescent states

$$\forall p (\Box (v_p = \hat{v}_p)) \Rightarrow \forall p (\Box (\text{QuiescentState}_p)),$$

where each $\hat{v}_p$ is a constant set and $\Box$ is theetime operator [35], the notation $\Box A$ meaning that there is a time point after the reference point at which $A$ holds.

**Property 3 Validity.** If all processes in a view $v'$ perceive view $v'$ as their local view and they have reached their quiescent states, then the last nonempty elements of their sequences of global views are all at position $j$ and must be equal to $v'$.

$$\forall p (v' \Box (\text{QuiescentState}_p \wedge v_p = v')) \Rightarrow (\exists j (\forall p (v' (j = \max _{v_p \neq \emptyset} V_{pj} = v')))).$$

**Property 4 Safety.** Once a view is “committed” in the sequence of global views, it cannot be changed.

$$\forall p, j ((V_{pj} = v' \neq \emptyset) \Rightarrow \Box (V_{pj} = v')).$$

The first property expresses agreement. Consistent history must be an invariant for any program that satisfies the specification.

The second property expresses termination. When the inputs of all processes are stable, the processes are eventually going to stop changing their output sequences.

The third property rules out trivial solutions where protocols never decide on any new view or always decide on the same view. It ensures that a protocol that satisfies the specification does something useful, by stating that when all processes in a set agree on such set, they must commit this common view at the same position $j$ in their sequences of global views. Note that this requirement is weak because a new membership is created only if the local views of the different processes in the membership reach agreement.

The fourth property also rules out trivial solutions, requiring processes not to change old views in their sequences.

5 CIRCUMVENTING THE IMPOSSIBILITY RESULT

In this section, we relate our specification to other ways to solve the group membership problem, summarized in Table 1.

- In an asynchronous model augmented by global failure detectors, processes have access to modules that (by definition) eventually reflect the state of the system. Therefore, progress can be guaranteed unconditionally.
- In a timed asynchronous model, processes must react to an input, producing the corresponding output or changing state, within a known timebound.
Under this model, progress can be guaranteed if no failures and recoveries occur for a known time needed to communicate in a timely manner.

- In a completely asynchronous model, progress cannot always be guaranteed and failure detectors in practice eventually reflect the system state, but they must be considered arbitrary. Correct processes react in practice within finite time, but this time cannot be quantified. Therefore, in order to guarantee a solution, we need a weaker specification of the problem.

Our approach falls into the last category that originated with Neiger’s work [48]. Our specification, however, differs from Neiger’s in several ways.

- Processes in Neiger’s model do not need to wait for convergence in their local views to change their membership. If one process suspects that another failed, it may attempt to remove the suspect process. Neiger’s specification says that, if one process attempts to remove another, they will eventually not be in the same membership. Our specification requires all processes in a set to agree on that set before changing their membership.

- Neiger’s specification allows a solution in which the attempted removal of a single process blocks all processes. Our specification does not allow such a single point of failure because it states that if all processes in a set agree on such set, they must eventually commit such set.

- Finally, Neiger’s specification does not consider processes joining the group. It states that the membership changes only by processes leaving the group. Our specification allows removal and rejoining of any number of processes.

We now relate our solution to the other methods quoted in Table 1.

In a timed asynchronous model, processes must react to an input, producing the corresponding output or changing state, within a known time period. Progress under this model is achieved if there are no failures or recoveries in the system for an a priori known duration. This implies a certain amount of synchronism, which is absent from our totally asynchronous model.

Global Failure Detectors strengthen the asynchronous time-free model, based on the observation that the system eventually stabilizes: This essentially implies that, after a “sufficiently long” time, failure detectors are accurate and complete, thus reflecting the actual system state. In our model, stability is also required for progress, but, at variance of the above case, it is not necessarily related to the state of the system. In other words, eventual progress is required when there is agreement among a subset of the local failure detectors, even if failures and recoveries continue to occur in the system.

If our local failure detectors do not necessarily reflect the system state because they are inaccurate and incomplete, how is it possible to require progress in a group that agrees on a wrong connectivity? For example, if processes $p_1$, $p_2$, and $p_3$ agree on the view $\{p_1, p_2, p_3\}$, but process $p_3$ has stopped and cannot communicate with $p_1$ and $p_2$, how can $p_3$ commit the view $\{p_1, p_2, p_3\}$? There are two answers to this question. First, $p_3$ may remain stopped for a while (or just be infinitely slow) and may become responsive again at any time. In this case, if the three processes do not change their local view and since channels are reliable, all three will be able to commit the view $\{p_1, p_2, p_3\}$ when communication with $p_3$ is restored. This is enough to guarantee eventual progress. Second, if $p_3$ has crashed permanently (consider for example, turning off a computer), it does not have a local view and does not agree on the view $\{p_1, p_2, p_3\}$. Therefore, the specification does not require the two remaining processes to commit such view.

In practice, local failure detectors are typically implemented using timeouts and “heartbeats” and are at least complete, if not accurate, since all crashed processes eventually time out. In the example above, the permanently crashed process $p_3$ would be timed out by the remaining two processes and those processes would be able to commit the view $\{p_1, p_2\}$. However, in our specification, we ignore global properties of failure detectors and consider their reports to be as random inputs.

### 6 GROUP MEMBERSHIP ALGORITHM

We provide an algorithm that solves the Group Membership Specification given in Section 4. The algorithm is based on the three asynchronous phases depicted in Fig. 1.

- a preparation phase, in which a process proposes a new view that matches the view of the other processes,
- a ready phase, in which all processes that agree on the new view acknowledge the reservation of a

![Fig. 1. Phases of the algorithm.](image-url)
position in their sequences of global views to commit such view,
- a commit phase, in which the new view is finally installed, and the sequences $S_p$ of global views are extended consistently at different processes.

### 6.1 Solution Sketch

The main idea for the algorithm is as follows: A process $p$ that is informed by its local failure detector of a change in its local connectivity view and that has the smallest id among processes in its new local connectivity view sends a message to all processes in its view proposing to update the current membership with the new view. Each process records this proposal until its local view is the same as the proposed view. At which point, it responds by sending back an Accept or Retry message to the process that proposed the membership update. The Accept message is sent if the process agrees on the proposed group index, namely, on the position in its sequence $S_p$, where to place the new view. Upon sending the Accept message, the process reserves the corresponding position in its sequence $S_p$, so that no other proposal is accepted for that group index. Upon receiving a Retry message, the proposing process restarts the first phase of the algorithm, sending a new Propose message to all processes in its view, this time with a new group index. When the proposing process has collected Accept messages from all processes in its view, it starts the commit phase by sending commit messages, ordering other processes in its view to commit the membership update. Upon receiving a commit message, processes extend their sequences $S_p$ accordingly.

### 6.2 Channels and Data Structures

Our communication model is depicted in Fig. 2. A process $p$ is connected to a process $q$ through a send channel $SN_q$ and a receive channel $R_q$. Process $p$ also has a receive channel, named local, coming from its local failure detector.

The local view of process $p$ is stored in the variable $v_p$. A global view that has been committed by process $p$ at position $i$ in its sequence $S_p$ of global views is represented by $V_{pi}$. The index $Next$ of process $p$ always points to the first position in the sequence $S_p$, where a new view can be committed. The remaining local variables of process $p$ are summarized in Table 2.

### 6.3 CSP Notation

We specify our algorithm using Hoare's CSP [32]. A full description of our notation and its semantics can be found in [43] and [44]. What follows is a short summary of the notation we use.

- **Statements:**
  - Assignment: $a := b$.
  - Send: $X!e$ means send the value of $e$ over channel $X$.
  - Receive: $Y?v$ means receive a value over channel $Y$ and store it in variable $v$.
  - Probe: The Boolean expression $X$ is true iff a Receive statement over channel $X$ can complete without suspending.

- **Control Structures:**
  - Selection: $\langle G_1 ! S_1 \ldots ! G_n ! S_n \rangle$, where $G_i$'s are Boolean expressions (guards) and $S_i$'s are program parts. The execution of this command corresponds to waiting until one of the guards is true and executing one of the statements with a true guard.
  - Repetition: The notation $* [S]$ means repeat $S$ forever.
  - Sequential execution: $S; T$.
  - Parallel execution: $S \parallel T$.

<table>
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<th><strong>Table 2</strong></th>
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<tr>
<td><strong>Variables of Process p</strong></td>
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<td>$p$</td>
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<tr>
<td>$v_p$</td>
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<tr>
<td>$V_{pi}$</td>
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<td>$Next$</td>
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<td>$v^*$</td>
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<tr>
<td>$old, new, view, i$</td>
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Parameterization: The notation $$h_{q}$$ means:

$$\forall q \in U \left( S_{q} \rightarrow S_{q+1} \rightarrow \ldots \right)$$

where $$S_{q}$$ is the program part $$S$$ with $$q$$ replaced by the $$i$$th member of the set $$U$$.

The notation $$q::$$ is a shorthand for $$q::\forall g \in \text{set of all processes}$$.

6.4 Code Description

The code is shown in Fig. 3. The first guarded command in Fig. 3 shows how a process $$p$$, when informed of a change in its local connectivity view, checks if it has the minimum $$id$$ among the processes in $$v_{p}$$. If $$p$$ has the minimum $$id$$, it proposes to extend the sequence of global views at position $$PropOut$$ with the view $$v_{p}$$ by broadcasting $$v_{p}$$ and $$PropOut$$ to all processes in $$v_{p}$$ and initializes its $$ack$$ array to zero.

The second guarded command in Fig. 3 checks for incoming messages from other processes. These may be proposals for a new membership ($$Propose$$), invitations to retry proposing a membership with a new group index ($$Retry$$), acceptances of a proposed membership ($$Acc$$), or orders to commit a new membership ($$Commit$$).

Upon receiving a proposal message from process $$q$$, process $$p$$ stores the view proposed by $$q$$ at position $$q$$ of the array $$v$$ and stores the proposed group index at position $$q$$ of the array $$PropIn$$, then sets position $$q$$ of the array $$prop$$ to true to record the receipt of the proposal from $$q$$.

If process $$p$$ later agrees on the proposed view, it sends a response to process $$q$$ (see last guarded command in Fig. 3). The response is either an acceptance of the view $$v_{q}$$ at position $$PropIn[q]$$ if the next available slot of process $$p$$ to commit a view is at a position that is less or equal than the proposed index $$PropIn[q]$$; or it is an invitation to retry with a different index if the next available slot of process $$p$$ is at a position that is greater than the proposed index.
invitation to retry consists of sending back to \( q \) the proposed index and the current next available index to be used for the retry. An acceptance consists of acknowledging the proposed view at position \( \text{PropIn}[q] \). Following an acceptance, process \( p \) also increments its index \( \text{Next} \) to the value \( \text{PropIn}[q] + 1 \) in order not to accept any other proposal for that position.

We now examine the guarded commands of the remaining message types.

A process \( p \) that receives an invitation to retry its proposal, receives two indices, \( \text{old} \) and \( \text{new} \). The former is the one \( p \) has sent in the proposal, the latter is the one \( p \) will use in its next proposal. If \( \text{old} \) is equal to the index of the most recent proposal sent by process \( p \) and process \( p \) still has the minimum \( \text{id} \) among the processes in its local view, it proposes to extend the sequence of global views at the new position \( \text{new} \) with the view \( v_p \) and reinitializes the \( \text{ack} \) array to zero.

A process \( p \) that receives an acceptance regarding its proposed view receives an index \( i \). If \( i \) is equal to the index of the most recent proposal sent, process \( p \) sets the element at position \( q \) in the array \( \text{ack} \) to 1 to record the acceptance. Then, it inspects the \( \text{ack} \) array to check if all entries are 1. If so, \( p \) starts the commit phase by broadcasting its previously proposed view \( v^* \) and the corresponding proposed index \( \text{PropOut} \) to all processes in \( v^* \) and reinitializes the \( \text{ack} \) array to zero.

A process \( p \) that receives an order to commit a view at position \( i \) from process \( q \), simply sets the view at position \( i \) of its sequence of global views to the received view.

### 6.5 Local Failure Detectors Implementation

Our algorithm relies on an implementation of the local failure detectors that satisfies the following property:

- Given any pair of processes \( p \) and \( q \), every time \( p \) suspects \( q \) failed, it cannot unsuspect \( q \) until \( q \) has also suspected \( p \) failed.

Note that such an implementation does not introduce any global property in the failure detection mechanism. In fact, it does not relate suspected failures to actual failures in the system in any way.

It is easy to implement local failure detectors that fulfill this requirement. In particular, an implementation of a very simple protocol for this kind of local failure detectors is reported in [12] and [36] and its state machine description is depicted in Fig. 4. Instead of replicating this state machine into the CSP description of our algorithm, we assume to have incorporated it within the local failure detector implementation.

The state machine depicted in Fig. 4 shows process \( p \)'s local view of process \( q \). It shows the reaction to \( t_{\text{out}} \) events and \( T \) (token-receipt) events by process \( p \) that is at one end of the communication channel to process \( q \). The protocol consists of two parts:

- First, we have the sending and receiving of tokens, using reliable messaging. Tokens are sent whenever process \( p \) sees a state transition over the channel to process \( q \), i.e., it suspects or unsuspects process \( q \).

- Second, we have an (unreliable) hint from the underlying system, such as a time-out, that indicates that communication to process \( q \) has (perhaps) been lost.

In Fig. 4, each state is characterized by whether process \( p \) considers process \( q \) to be \( \text{Up} \) or \( \text{Down} \) and by how many tokens are held by process \( p \). The state transitions are labeled by the action triggering the transition and by the action taken upon transition. A trigger event is either a time-out \( t_{\text{out}} \) or a receipt of a token \( T \). The action taken is always whether the token is sent (1) or not (0). Note that a token \( T \) is sent whenever a transition from an \( \text{Up} \) state to a \( \text{Down} \) state or from a \( \text{Down} \) state to an \( \text{Up} \) state is made.

### 6.6 Example

In order to clarify the behavior of the overall algorithm, we show the following example: Let us consider the four processes system depicted in Fig. 5 as a complete graph with four nodes. Initially (Step (a) of Fig. 5), all four nodes have the same view sequence \( V = \{V_1, \{\}, \{\}, \{\}, ...\} \) with \( V_1 = \{1,2,3,4\} \). Suppose node 1 is disconnected from the network and suppose that local failure detectors of different nodes discover the failure at different times, reporting the connectivity views represented in Steps (b) through (d).

At Step (b), node 1 suspects 2 has failed and node 2 suspects 1 has failed. Accordingly, at Step (b), both nodes 1 and 2 have the minimum \( \text{id} \) among their local views. Therefore, node 2 proposes the new membership \( v = \{2,3,4\} \) tagged with group index \( i = 2 \), while node 1 proposes \( v = \{1,3,4\} \), which is also tagged with group index \( i = 2 \). At Step (b) nodes 3 and 4 do not send any response because their local views are still \( v = \{1,2,3,4\} \).

At Step (c), node 1 suspects 2 and 3 have failed and nodes 2 and 3 suspect 1 has failed. Accordingly, at Step (c), node 1 proposes \( v = \{1,4\} \), tagged with group index \( i = 3 \), while node 3 accepts the view \( v = \{2,3,4\} \) proposed by node 2 at Step (b). Finally, at Step (d), node 4 also accepts the view \( v = \{2,3,4\} \) proposed by node 2 at Step (b) while node 1 proposes and commits the singleton view \( v = \{1\} \) at position \( i = 4 \).
By Step (d), node 2 has collected *Accept* messages from all nodes to which it sent its proposal; therefore, it is able to start the commit phase. Suppose that the *Commit* message sent to node 4 is delayed (Step (e)) so that nodes 2 and 4 suspect each other to have failed. By Step (e), the sequences of views at the different nodes are:

- **Node 1.**
  \[ S = \{\{1, 2, 3, 4\}, \{\emptyset\}, \{\emptyset\}, \{\emptyset\}, \{\emptyset\}, \{\emptyset\}, \ldots\}. \]

- **Nodes 2 and 3.**
  \[ S = \{\{1, 2, 3, 4\}, \{2, 3, 4\}, \{\emptyset\}, \{\emptyset\}, \{\emptyset\}, \ldots\}. \]

- **Node 4.**
  \[ S = \{\{1, 2, 3, 4\}, \{\emptyset\}, \{\emptyset\}, \{\emptyset\}, \ldots\}. \]

Now, suppose that node 1 is reconnected to the system (Steps (f) and (g)). Accordingly, node 1 proposes \( v = \{1, 3, 4\} \) with group index 5 at Step (f), while node 2 proposes \( v = \{2, 3\} \) with group index 3. Node 3 does not respond to any of these proposals because its local view is \( v = \{1, 2, 3, 4\} \). Node 4, sends an *Accept* message to node 1, regarding the new proposed view \( v = \{1, 3, 4\} \) at position 5, but the commit phase does not start for this view. Node 1 then proposes \( v = \{1, 2, 3, 4\} \) with group index 6 at Step (g). Node 3 accepts this proposal at Step (g); nodes 2 and 4 accept it at Step (h). Node 4 also receives the delayed commit message from node 2 at Step (h). By Step (h), node 1 has collected *Accept* messages from all the nodes to which it sent its proposal; therefore, it starts the commit phase. Supposing node 1 completes the commit phase at Step (h), the final sequences of views at the different nodes become:

- **Nodes 2, 3, and 4.**
  \[ S = \{\{1, 2, 3, 4\}, \{2, 3, 4\}, \{\emptyset\}, \{\emptyset\}, \{\emptyset\}, \{1, 2, 3, 4\}, \{\emptyset\}, \{\emptyset\}, \{\emptyset\}, \ldots\} \]

## 7 Correctness

Correctness of the algorithm is ensured by proving that it satisfies the four properties of the specification given in Section 4.

### 7.1 Agreement

**Theorem 1.** The algorithm described in Section 7 satisfies the agreement condition of the specification (Property 1, Section 4): At any point in time, all processes have a consistent history.

**Proof.** Either all processes remain in the start state or some process \( p \) extends its sequence \( S_p \) of global views. In the start state, the consistent history property holds. If some process \( p \) extends its sequence \( S_p \) by committing a new view \( v^* \) at a given position \( i \), it must have received a *Commit* message from some process \( q \); therefore, \( q \) must have received *Accept* messages regarding its proposal of \( v^* \) at position \( i \) from all processes in \( v^* \), including \( p \). It follows from the last guarded command in Fig. 3 that, if process \( p \) has accepted the proposal of process \( q \), it has also increased its \( Next \) variable to the value of \( i + 1 \) and will not accept any other proposal for that position. Therefore, process \( p \) either commits view \( v^* \) at position \( i \) or ends up with position \( i \) of its sequence of global views empty. The consistent history property follows. \( \square \)

### 7.2 Termination

**Theorem 2.** The algorithm described in Section 7 satisfies the termination condition of the specification (Property 2, Section 4): All processes eventually reach their quiescent states if there are no more changes in their local views.

**Proof.** By contradiction, a nonquiescent state means that the sequence of global views is extended infinitely often at some process; therefore, an infinite number of *Commit* messages must be sent. Since the number of processes is
finite, there must be at least one process that sends infinitely many \textit{Commit} messages. Call this process $p$. By the code in Fig. 3, we see that, each time $p$ sends a commit message, it reinitializes its \textit{ack} array to 0. It follows that, in order to send infinitely many \textit{Commit} messages, process $p$ must refill the array with 1’s infinitely often. Since each process sends at most one \textit{Ack} message for each proposal, process $p$ must send infinitely many proposals. Proposals are sent either when there is a change in the local view or because of an invitation to retry has been received (see Fig. 3). The first case is ruled out because it implies that process $p$ changes its local view infinitely often and by hypothesis there are no more changes in the local views of the processes. The second case is also ruled out, because it implies that process $p$ sends infinitely many proposals of the same view, each time increasing the proposed index. Since the number of processes is finite, and there are no more changes in their local views, process $p$ must eventually propose an index large enough, that does not generate any \textit{Retry} messages, leading to the desired contradiction. \qedsymbol

7.3 Validity

\textbf{Theorem 3.} The algorithm described in Section 7 satisfies the validity condition of the specification (Property 3, Section 4): if all processes in a view $v^*$ perceive view $v^*$ permanently as their local view and, if they have reached their quiescent states, then they must have installed view $v^*$ at position $j$ as the last nonempty element of their sequences of global views.

\textbf{Proof.} Let all processes in $v^*$ perceive view $v^*$ permanently as their local view. We first show that, when the processes change their local views to $v^*$, process $p = \min(v^*)$ proposes view $v^*$. We distinguish two cases:

- \textbf{Case 1.} All processes in $v^*$ always perceive $p$ as not failed. In this case, since $p = \min(v^*)$, the only process that can ever send a proposal is process $p$. Moreover, by the assumption on the behavior of the local failure detectors (see Section 6.5), $p$ can never suspect any $q \in v^*$ and perceive $v^*$ again. It follows that either $p$ proposes $v^*$ by excluding some process that is not in $v^*$ or $p$ never proposes $v^*$; in which case, $v^*$ must be the initial view of all processes in the system.

- \textbf{Case 2.} There is at least a process $q \in v^*$ that perceives a transition from considering $p$ failed to considering $p$ not failed. In this case, by the assumption on the local failure detectors (see Section 6.5), $p$ must suspect $q$ before $q$ unsuspects $p$. It follows that, to perceive $v^*$, $p$ must at some time perceive a transition to unsuspect $q$ and propose a view letting $q$ join the group. The last process $q$ for which process $p$ perceives such a transition makes $p$ send a proposal for view $v^*$. When process $p$ sends a proposal for view $v^*$ and all processes in $v^*$ agree on such view, the proposal is either accepted or an invitation to retry is sent by some process back to process $p$. Since for every \textit{Retry} message received, process $p$ sends a new proposal for $v^*$ with a larger proposed index and, since the local views of all processes are stable, process $p$ must eventually propose an index that is large enough to be accepted by all processes in $v^*$ and, therefore, must eventually send \textit{Commit} messages to all processes in $v^*$. It follows, that by the time they reach a quiescent state, all processes in $v^*$ have installed view $v^*$ at the same position in their sequences of global views. \qedsymbol

7.4 Safety

\textbf{Theorem 4.} The algorithm described in Section 7 satisfies the safety condition of the specification (Property 4, Section 4): Once a view is “committed” in the sequence of global views, it cannot be changed.

\textbf{Proof.} Every time a process sends an \textit{Ack} message, regarding a view at a given position $i$, it increases the value of its \textit{Next} variable to $i + 1$; therefore, it will not accept any other proposal for position $i$, making it possible to commit at most a single view at position $i$. \qedsymbol

8 Conclusion

We have presented a specification for the Group Membership Problem in completely asynchronous systems and a corresponding algorithm that solves it.

Our specification requires processes to maintain a consistent history in their sequence of views. This allows processes to order failures and recoveries in time and simplifies the programming of many high level applications (see for example the discussion in [22]).

We have assumed our local failure detectors to be inaccurate and incomplete. With this approach, the specification states explicitly that progress cannot always be guaranteed.

In practice, our requirement for progress is weaker than that stated in the specification of having a set of processes sharing the same connectivity view indefinitely. In fact, if the rate of perceived failures in the system is lower than the time it takes the protocol to make progress and commit a new membership, then it is possible for the algorithm to make progress every time there is a failure in the system. This depends on the actual rate of failures and on the capacity of the failure detectors to track such failures.

In [15], Chandra and Toueg note that failure detectors defined in terms of global system properties cannot be implemented. This result gives strength to the approach of relaxing the specification and of having a protocol in continuous search for convergence. In real world systems, where process crashes actually lead a connected cluster of processes to share the same connectivity view of the network, convergence on a new membership can be easily reached in practice.

In the presence of partitions, our protocol does not attempt to solve inconsistencies in the history of views of processes that commit disjoint memberships. Therefore, two processes that partition and evolve separately as part of different components maintain separate histories in their view sequences. This information may be used later by
other algorithms to place a total order on the memberships of the whole network.

We believe that the weak progress approach, introduced by Neiger [48] and expanded here, can be generalized to other agreement problems in asynchronous systems.

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