Time-variable emission from transiently accreting neutron stars in quiescence due to deep crustal heating

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ABSTRACT

Transiently accreting neutron stars in quiescence ($L_X \leq 10^{34}$ erg s\(^{-1}\)) have been observed to vary in intensity by factors of few, over time-scales of days to years. If the quiescent luminosity is powered by a hot neutron star core, the core cooling time-scale is much longer than the recurrence time, and cannot explain the observed, more rapid variability. However, the non-equilibrium reactions which occur in the crust during outbursts deposit energy in isodensity shells, from which the thermal diffusion time-scale to the photosphere is days to years. The predicted magnitude of variability is too low to explain the observed variability unless – as is widely believed – the neutrons beyond the neutron-drip density are superfluid. Even then, the variability due to this mechanism in models with standard core neutrino cooling processes is less than 50 per cent – still too low to explain the reported variability. However, models with rapid core neutrino cooling can produce a variability by a factor as great as 20, on time-scales of days to years following an outburst. Thus, the factors of \textasciitilde few intensity variability observed from transiently accreting neutron stars can be accounted for by this mechanism only if rapid core cooling processes are active.

Key words: nuclear reactions, nucleosynthesis, abundances – stars: neutron – X-rays: binaries.

1 INTRODUCTION

Many neutron star (NS) X-ray binaries go through accretion outbursts ($L_X \sim 10^{37}$ erg s\(^{-1}\)) followed by long periods (months–decades) of relative quiescence ($L_X \leq 10^{34}$ erg s\(^{-1}\)). The origin of these outbursts remains under debate, although many agree that in the wider binaries an accretion disc instability is the cause (Van Paradijs 1996). During these outbursts, of the order of \textasciitilde 10\textsuperscript{23} g may be accreted, over a period of \textasciitilde few–30 d (for a review of these transients see Tanaka & Shibazaki 1996; Campana et al. 1997; Chen, Shrader & Livio 1997).

Following an outburst, the NSs return to quiescence, and have typically been detected with luminosities in the $10^{32}–10^{33}$ erg s\(^{-1}\) range (Van Paradijs et al. 1987; Verbunt et al. 1994; Asai et al. 1996a,b; Campana et al. 1998a; Garcia & Callanan 1999; Rutledge et al. 1999; Campana et al. 2000; Rutledge et al. 2000). The quiescent X-ray spectrum has been described as being comprised of two components, a soft (blackbody $kT \sim 0.2$ KeV) thermal component, and a power-law component dominating the emission above 2 keV (Asai et al. 1996b; Campana et al. 1998b; Campana et al. 2000).

The basal luminosity has been attributed to the continued accretion in quiescence (Van Paradijs et al. 1987), possibly through an advection-dominated accretion flow (Narayan et al. 1997; Menou et al. 1999); accretion on to the NS magnetosphere (Campana et al. 1997) following a ‘propeller phase’, in which the NS magnetosphere is larger than the Keplerian orbit with an orbital period equal to the spin period of the NS (Illarionov & Sunyaev 1975; Stella, White & Rosner 1986); and to thermal emission from a hot NS core, heated by non-equilibrium reactions deep in the NS crust (Brown, Bildsten & Rutledge 1998, BBR98 hereafter). Of these possibilities, only deep-crustal heating predicts the similar luminosities observed in the thermal spectral component of these systems. The observed temperatures of the thermal component ($kT = 0.08–0.20$ keV) are as expected in this scenario. In addition, the emission area radii of the thermal component are consistent with the theoretically predicted NS radii (~10 km; Rutledge et al. 1999, 2000); this supports the interpretation of this spectral component as a thermal NS photosphere. It is possible that the quiescent luminosity is due to a combination of these three mechanisms, in which case they are only separable by their spectral and intensity variability properties.

The detected flux during quiescence from some of these transients has been observed to vary, by a factor of as much as \textasciitilde 3 or more, over time-scales of days to years. No clear cause for this
variability has yet emerged. There are three means discussed in the literature:

(i) Variable accretion on to the NS surface. If accretion on to the NS surface dominates the luminosity in quiescence, changes in the accretion rate could account for variations in the thermal ($kT_{\text{eff}} \sim 0.2\,\text{KeV}$) part of the spectrum, since the photospheric spectrum at the low implied accretion rate is thermal in the absence of a shock (Zampieri et al. 1995), assuming quasi-steady-state accretion at the appropriate accretion rates.

(ii) Variable accretion on to the NS magnetosphere. The power-law spectral ‘tail’ which can dominate emission above 2 keV may be due to accretion on to a NS magnetosphere (Campana et al. 1998a). Variations in the accretion rate on to the magnetosphere would cause intensity variability in this power-law component.

(iii) Variable absorption column density ($N_H$). Outbursts may be accompanied by outflows, increasing locally the column density of absorbing material. This would cause variations in the absorption column density ($N_H$), which may account for the factors of 2–3 variation in the observed intensity (Rutledge et al. 1999).

Moreover, intensity variability due to accretion – either photospheric or magnetospheric – is likely to be stochastic; there have been no theoretical estimations of its magnitude or variability time-scale. In the variable $N_H$ scenario, the amount of absorption should decrease (and intensity, increase) with time after the outburst.

In this paper, we investigate the time dependence of the thermal emission mechanism proposed by BBR98. Compression of the NS crust by accretion during outbursts induces electron captures, neutron emissions, and pycnonuclear reactions deep in the crust (at densities $> 10^9\,\text{g}\,\text{cm}^{-3}$), and these reactions deposit $Q_{\text{nuc}} \approx 1\,\text{MeV}$ per accreted baryon of heat into the crust (Haensel & Zdunik 1990b). BBR98 argued that these reactions heat the NS core to an equilibrium temperature of $\approx 10^8\,\text{K}$, and, during quiescence, the hot core shines with a typical luminosity of $L_q = \langle M \rangle Q_{\text{nuc}} / m_0$, where $\langle M \rangle$ is the mean accretion rate, averaged over the thermal time of the core, i.e. over many recurrence intervals $\tau_{\text{rec}}$. Consequently, the core luminosity is not expected to change on time-scales shorter than its thermal time, i.e. $\approx 10^3\,\text{yr}$.

Transient X-ray emission from the non-equilibrium reactions had not been previously considered. Similar work regarding transient energy deposition in NSs has either focused on the atmosphere, well above the crust (Eichler & Cheng 1989), or investigated the thermal response to pulsar glitches (van Riper 1991; Chong & Cheng 1994; Hirano et al. 1997; Cheng, Li & Suen 1998). However, the crucial difference is that the depths and amounts of energy deposited by non-equilibrium reactions can, at least in principle, be obtained from an $ab\ initio$ calculation (and have been, for the case of iron, by Haensel & Zdunik 1990b), while, on the other hand, the total amount of energy deposited by the crust-breaking glitches is not precisely known (it is bounded by $2I/\Omega \delta I$), and neither the depth nor the distribution of energy deposition is well constrained.

Colpi et al. (2001) examined the effect of non-equilibrium reactions on the temperature of the core. They found that in transient NSs with recurrence time-scales of $\sim 1\,\text{yr}$, the core heats to an equilibrium temperature in $\sim 10^3\,\text{yr}$, after which the core temperature varies by $\sim 0.5$ per cent in response to the periodic input of energy from the non-equilibrium reactions during individual accretion outbursts. They confirmed the steady-state assumption made by BBR98. Our study is complimentary to that of Colpi et al. (2001), as we examine the time dependence of the temperature in the crust and the thermal luminosity from the NS surface between outbursts. We focus in detail the thermal relaxation of the crust and find that, depending on the microphysics and the accretion history, the magnitude of the variability can be as small as $< 1$ per cent, or as large as a factor of 20. Only the reactions that deposit energy at depths where $\tau_{\text{nuc}} \approx \tau_{\text{rec}}$ lead to variable thermal emission. The luminosity as a result of the individual reactions is largely blended together, simply because the difference between the $\tau_{\text{nuc}}$ of adjacent reactions is typically much shorter than $\tau_{\text{rec}}$. However, the emission from the reactions in the outer crust as a whole is typically well separated from that due to the reactions in the inner crust, resulting in a characteristic ‘double-hump’ luminosity evolution. In addition, the amplitude of the variability is not simply related to the energy deposited in the individual reactions. We quantify the dependence of this variability on the conductivity of the crust and the physics of neutrino emission in the core.

In Section 2 we briefly recount the observational evidence for quiescent intensity variability. In Section 3, we describe detailed calculations of time evolution in the thermal NS flux, in response to a ‘delta-function’ (1-day long) accretion event, for different time-averaged accretion rates (and thus core temperatures), different dominant conductivities and core neutrino cooling prescriptions. We also present the resulting light curves in this section. Finally, in Section 4, we compare these results with observations, and in Section 5, we present the conclusions.

2 OBSERVATIONS OF VARIABILITY IN QUIESCENCE

Intensity variability in quiescent transients is, at present, not well studied observationally, largely due to the low signal-to-noise of the data, from a small number of observations (only a few of which are obtained with the same instrumentation), and different assumed intrinsic energy spectra. Multiple observations in quiescence ($L_q < 10^{34}\,\text{erg}\,\text{s}^{-1}$) are found in the literature only for three NSs, Cen X-4, 4U 2129+37 and Aql X-1.

For Cen X-4, Van Paradijs et al. (1987) found that the luminosity increased by a factor of $\sim 2–5$ over $\sim 5.5\,\text{yr}$ (1980 July–1986 February), for the same assumed thermal bremsstrahlung spectrum. There was no intervening outburst observed, and the observations were made using two different instruments [EXOSAT/ Low Energy (LE) and Einstein/Imaging Proportional Counter (IPC)]. Campana et al. (1997) reanalysed this data, and contrarily concluded that they are consistent with the same luminosity. Also, Campana et al. (1997) found that in observations with ROSAT/High Resolution Imager (HRI) of Cen X-4 over a 4–8 d period, the source count rate varied by $\sim 3$, with an average luminosity of $\sim 7 \times 10^{33}\,\text{erg}\,\text{s}^{-1}$; Campana did not discuss as to whether the 1995 ROSAT/HRI observations were consistent with the luminosities of the EXOSAT and Einstein observations; a comparison between their values and passbands indicates that the luminosities of Van Paradijs et al. (1987) were greater by more than a factor of 3, although part of this may be due to a different assumed spectrum. Finally, Rutledge et al. (2001) found that Cen X-4 varied by $< 18$ per cent ($0.2–2\,\text{keV}$) during a 10 k observation, while the 0.5–10.0 keV luminosity had decreased by $40 \pm 8$ per cent in comparison with an ASCA observation taken 5 yr previously.

Aql X-1 was also found to have a variable flux in quiescence (Rutledge et al. 1999) between three observations with ASCA (1992 Oct, 1993 Mar and 1996 Oct). If the same intrinsic spectrum
is assumed, the three observations are not consistent with having the same intensity with high confidence (probability $= 10^{-5}$). The difference in observed flux is a factor of $\sim 2 - 3$.

4U 2129+37 decreased in flux by a factor of $3.4 \pm 0.6$ between 1992 November–December (ROSAT/HRI) and 1994 March (ROSAT/Position Sensitive Proportional Counter (PSPC)) (Garcia & Callanan 1999; Rutledge et al. 2000) in the unabsorbed luminosity, for the same assumed spectrum.

The observations of Cen X-4 with ROSAT/HRI, and of Aql X-1 with ASCA are the only two instances of repeated observations with the same instrumentation. Cen X-4 has not been observed in any outburst since 1979 May (Chen, Shrader & Livio 1997), while Aql X-1 goes into outburst every $\sim 220$ d; and it therefore seems likely that there were intervening outbursts between the three ASCA observations.

Thus, observations of transiently accreting NSs in quiescence, taken at face value, indicate that their quiescent luminosities vary by a factor of up to 3–5 on time-scales of days to years. However, it is not clear as to what fraction of this variability is intrinsic, and what can be attributed to the systematic differences in instrumentation.

3 THERMAL EMISSION FROM THE CRUST

We now describe our simulations of the thermal relaxation of the crust following an accretion outburst, and the resulting time dependence of the quiescent thermal emission. As argued by BBR98 and confirmed by Colpi et al. (2001), the nuclear energy release in the crust heats the NS core to a temperature corresponding to steady accretion at the corresponding time-averaged accretion rate. We performed several simulations starting with a very cold NS and subjecting it to a series of accretion outbursts and found that indeed, the core is heated to the appropriate temperature on a $10^{5}$ yr time-scale, in agreement with Colpi et al. (2001). For the remainder of our analysis we first construct a steady-state thermal model corresponding to the time-averaged ($\dot{M}$), and then subject it to outbursts. After several outbursts, the model reaches a limit cycle. In this paper, we report the results of only these latter simulations. Our accretion events (‘outbursts’) are 1-day long ‘delta-functions’, with no accretion outside these events. More realistic time-evolved light curves may in principle be found by convolving our ‘delta-function’ thermal response light curves with a more realistic accretion outburst time-dependent profile for the small perturbation light curves ($\delta L/L \ll 1$); light curves in which the variations are larger would require a more detailed calculation. We simulate two different outburst recurrence time-scales (1 and 30 yr) and three time-averaged accretion rates ($10^{-10}, 10^{-11}$ and $10^{-12} M_\odot$ yr$^{-1}$). As we are primarily interested in illustrating the response of the NS surface flux from the crust and core to a ‘delta-function’ accretion profile, it is unimportant that the implied 1-day accretion rates may be super-Eddington, and we do not include accretion luminosity in our results.

3.1 Microphysics of the crust and core

The hydrostatic and steady-state thermal models of the crust used in this paper are substantially identical to the ones used by Ushomirsky, Cutler & Bildsten (2000). We briefly recount the major ingredients of these models, and then describe our time-evolution code.

Since the thermal time-scale for the crust (days to years, see below) is so much longer than the sound crossing time (milliseconds), and the equation of state is nearly independent of temperature, the hydrostatic structure of the crust is effectively decoupled from its thermal evolution. Therefore, one can solve the thermal evolution equations assuming that the density and pressure as a function of the position in the star are fixed. This approach to model the thermal state of a NS is fully described by Brown (2000) and is followed here.

The composition of the crust (i.e. the mass A and charge Z of the nuclei, as well as the neutron fraction $X_n$) is taken from the tabulation of Haensel & Zdunik (1990a,b). The pressure is the sum of the contributions from degenerate, relativistic electrons and free neutrons at densities exceeding neutron drip ($\rho_{nd} = 6.1 \times 10^{11}$ g cm$^{-3}$; Haensel & Zdunik 1990a). With the equation of state as described above, we take $M = 1.4 M_\odot$, $R = 10$ km, and solve the Newtonian equations of mass continuity and hydrostatic balance. Our (purely Newtonian) crust has a thickness of 1.1 km and a mass of 0.06 $M_\odot$.

With the hydrostatic structure specified, we solve the heat equation,

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 k \frac{\partial T}{\partial r} \right) + \rho (\epsilon_{\text{nuc}} - \epsilon_{\nu}),$$

(1)

where $c_p$ is the heat capacity per gram (which has contributions from the ionic lattice, degenerate electrons, and free neutrons in the inner crust), $\epsilon_{\text{nuc}}$ is the nuclear energy release, $\epsilon_{\nu}$ is the neutrino emissivity, and $K$ is the thermal conductivity. We neglect the overall downward motion of the material due to accretion, as well as the heat release due to compression. (These effects are not important in the crust.) Therefore, the only energy source in the crust is the heat release as a result of non-equilibrium electron captures, neutron emissions, and pycnuclear reactions. A detailed treatment of the energy release is described in Ushomirsky et al. (2000); we note here that $\epsilon_{\text{nuc}}$ is proportional to the instantaneous mass accretion rate $\dot{M}$. The total energy released in the crust is taken to be $Q_{\text{nuc}} = 4 \pi r^2 \rho_{\text{nuc}} \nu dr = 1.45$ MeV per accreted baryon (Haensel & Zdunik 1990b). At the temperatures of interest, neutrino losses in the crust are negligible.

Instead of fully modelling the thermal evolution of the NS core, we presume that it is isothermal, and is characterized by a single temperature $T_{\text{core}}$ which is equal to the temperature of the crust at the interface with the core, $T_{(\text{core})}$. The core temperature obeys

$$C_{\text{core}} \frac{dT_{(\text{core})}}{dr} = 4\pi r^2 \left[ \frac{\partial T}{\partial r} - L_{\nu} \right],$$

(2)

where $C_{\text{core}}$ is the total heat capacity, and $L_{\nu}$ is the total neutrino emissivity of the core (see below). This approximate treatment is justified because of the large thermal conductivity and heat content of the core. In effect, the temperature of the core changes by a negligible amount compared to the temperature variations in the crust, and the core acts as an energy sink at the bottom edge of the crust. With this approximation, we do not need to integrate the heat equation in the core, and instead use equation (2) as a boundary condition for equation (1) at the bottom edge of the crust.

To survey the parameter space, we use two different prescriptions for core cooling. ‘Standard’ cooling models use modified Urca (Friman & Maxwell 1979; Yakovlev & Levenfish 1995) and $e-e$ neutrino bremsstrahlung (Kaminker & Haensel 1999), while ‘rapid’ cooling models assume the presence of a pion...
condensate (Shapiro & Teukolsky 1983).¹ Nucleon superfluidity significantly reduces neutrino emissivity in the core (see Yakovlev et al. 1999 for an in-depth review). In our calculation we use the superfluid parameters collated by Brown (2000). Maxwell (1979) argued that at temperatures well below the superfluid transition temperature $T_s$, this suppression is exponential. However, detailed calculations summarized by Yakovlev, Levenfish & Shibanov (1999) show that this suppression is weaker by a power-law factor in $T/T_s$, leading to lower core temperatures than one would expect based on the simple exponential reduction in emissivity.

Core heat capacity is the sum of contributions from electrons, neutrons and protons. However, superfluidity alters the heat capacity of the nucleons (see Yakovlev et al. 1999 for an in-depth review). At $T < T_s$, the reduction of the heat capacity is similar to the suppression of the neutrino emissivity. Maxwell (1979) gives a fitting formula for $^3$S$_1$ superfluid (applicable to neutrons in the NS crust and protons in the core), while Levenfish & Yakovlev (1994) give fitting formulae for the $^3$P$_2$ superfluid as well (applicable to neutrons in the core). At temperatures of interest ($\sim 10^7$–$10^8$K) electrons dominate the heat capacity.

Throughout the crust, heat is transported by degenerate, relativistic electrons, and the thermal conductivity $K$ is very sensitive to the purity of the crust. If the crust is a pure crystal, electron–phonon scattering (Baiko & Yakovlev 1995) sets the conductivity. However, crusts of accreting NSs are unlikely to be pure crystals, since they are composed of (likely impure) products of nuclear burning in the upper atmosphere. In fact, calculations of Schatz et al. (1999) suggest that the typical values of the impurity parameter $Q_{\text{imp}} \equiv T_{\text{imp}}/(Z_{\text{imp}} - Z)^2$, where $Z_{\text{imp}}$ and $T_{\text{imp}}$ are the charge and fraction of the impurities, could be comparable to the average charge $(Z)^2$. Following Brown (2000), we set the absolute lower bound on the conductivity by using electron–ion scattering (Yakovlev & Urpin 1980), which has the same form as the electron–impurity scattering with $Q_{\text{imp}} = Z^2$. The much smaller conductivity in the case of electron–impurity scattering with large $Q_{\text{imp}}$ leads to much longer crustal thermal time, and much smaller variability of quiescent thermal emission. This approach allows us to survey the entire range of possibilities for the crustal conductivity.²

At the top of the crust ($\rho = 10^8$ g cm$^{-3}$) we utilize the results of Potekhin, Chabrier & Yakovlev (1997) for the relation between the core temperature and the surface temperature.³ They extended the original calculations of Gudmundsson, Pethick & Epstein (1983) by considering the opacities of accreted envelopes, rather than iron envelopes. Because of the smaller opacity of a light-element envelope, a $\sim 50$ per cent smaller core temperature is necessary to carry a given flux (see also Blandford, Applegate & Hernquist 1983). The thermal time at the top of the crust, where the outer boundary condition is applied, is $\approx 1$ d in our models, so our simulations cannot follow variability on shorter time-scales.

³Instead, one could assume that the NS mass is high enough to allow direct Urca neutrino emission [see e.g. Colpi et al. (2001)], as the emissivities are similar.

²The presence of impurities does not dramatically affect the heat capacity of the ionic lattice. While the phonon spectrum will be affected by the presence of impurities, the heat capacity should still approach the Debye limit.

³Potekhin et al. (1997) express the surface temperature in terms of the temperature in the isothermal region of the crust, which they assume to be at $\rho > 10^{10}$ g cm$^{-3}$. However, since the authors assume that the crust has high thermal conductivity, the temperature difference between $10^{10}$ and $10^{8}$ g cm$^{-3}$ is negligible in their models.

3.2 Time-averaged quiescent luminosity

Before describing the results of our time-evolution simulations, we discuss the thermal luminosity of NSs assuming steady accretion at the rate $\dot{M}$ corresponding to the time average over the outburst recurrence interval $t_{\text{rec}}$. This provides a reasonable estimate of the average luminosity level of NS transients. The variations of the luminosity discussed in Section 3.3 are excursions about this average level. As outlined in Section 3.1, we survey the parameter space by considering two different strengths of core–neutrino emission (standard and enhanced), and two different assumptions regarding the conductivity of the crust.

The average core temperature and luminosity of the NS are set by the balance between the heat input because of the non-equilibrium reactions in the crust (at the rate $Q_{\text{nuc}}(\dot{M})/m_b$) and the heat loss because of the neutrino emission from the core ($L_\nu$) and photon luminosity from the surface ($L_\gamma$). At the low core temperatures characteristic of NS transients ($T_{\text{core}} \lesssim 10^8$ K) modified Urca neutrino emission (standard cooling case) is substantially suppressed by nucleon superfluidity and cannot compete with photon losses from the surface of the star, $L_\gamma \ll L_\nu$. Therefore, most of the heat deposited by the non-equilibrium reactions is radiated from the surface, i.e. the quiescent luminosity is just (BBR98)

$$L_q \approx \frac{Q_{\text{nuc}}(\dot{M})}{m_b} = 8.7 \times 10^{33} \left(\frac{\dot{M}}{10^{-10} M_\odot \text{ yr}^{-1}}\right) \left(\frac{Q_{\text{nuc}}/m_b}{1.45 \text{ MeV}}\right) \text{ erg s}^{-1}. \quad (3)$$

This estimate is independent of the conductivity of the crust. The core temperature, however, depends on the crustal conductivity. If the crustal conductivity is large (i.e. set by electron–phonon scattering), the crust is nearly isothermal, and the core temperature, $T_{\text{core}} \approx 1.2 \times 10^8 \text{ K}(\dot{M}/10^{-10} M_\odot \text{ yr}^{-1})^{0.41}$, can be obtained by simply using the relation of Potekhin et al. (1997). If the crust has low thermal conductivity, then a substantial temperature gradient is needed to carry the heat from the inner crust to the surface, and the core temperature is $T_{\text{core}} \approx 2.2 \times 10^8 \text{ K}(\dot{M}/10^{-10} M_\odot \text{ yr}^{-1})^{0.45}$ (obtained by fitting our detailed calculations).

On the other hand, if rapid cooling processes are allowed in the core, then, despite the suppression of neutrino emission by superfluidity, most of the heat deposited by non-equilibrium reactions is radiated away by neutrinos, $L_q \gg L_\gamma$. The core temperature is then set by balancing the energy input from crustal reactions with the neutrino luminosity. Except at very low-accretion rates ($\lesssim 5 \times 10^{-12} M_\odot \text{ yr}^{-1}$), where superfluid suppression of neutrino emission makes $L_q$ comparable to $L_\nu$, a good fit to the results of our calculations for the particular case of enhanced neutrino emission from a pion condensate is $T_{\text{core}} \approx 2.7 \times 10^7 \text{ K}(\dot{M}/10^{-10} M_\odot \text{ yr}^{-1})^{0.09}$, regardless of the conductivity of the crust. The thermal luminosity, however, depends on the relation between the core temperature and that at the top of the crust. When the conductivity is determined by the electron–phonon scattering (i.e. $K$ is large), the crust is nearly isothermal, and, using the relation of Potekhin et al. (1997), we find

$$L_q \approx 2.2 \times 10^{32} \left(\frac{\dot{M}}{10^{-10} M_\odot \text{ yr}^{-1}}\right)^{0.22} \left(\frac{Q_{\text{nuc}}/m_b}{1.45 \text{ MeV}}\right)^{0.22} \text{ erg s}^{-1}, \quad (4)$$

which agrees well with our detailed calculations. On the other
hand, if the crustal conductivity is low, the relation between the core and the surface temperature is non-trivial, since an appreciable temperature gradient between the neutron drip and the core is needed to carry the flux from the non-equilibrium reactions into the core. In this case, the crust is warmer than in the electron–phonon scattering case, and the thermal luminosity is

$$L_q \approx 7 \times 10^{32} \left( \frac{\langle M \rangle}{10^{-10} \, M_\odot \, \text{yr}^{-1}} \right)^{0.8} \left( \frac{Q_{\text{max}}/m_\nu}{1.45 \, \text{MeV}} \right)^{0.8} \, \text{erg s}^{-1}$$  \hspace{1cm} (5)

(which we obtained by fitting the results of our detailed calculations). Comparing equation (3) with equations (4) and (5), we see that when enhanced neutrino cooling processes are allowed in the NS core only < 10 per cent of the non-equilibrium energy release is radiated from the surface, while the remaining > 90 per cent of the heat is emitted in the neutrinos. Regardless of the crustal conductivity, the core temperature in the rapid cooling case is set entirely by the core–neutrino emission. However, depending on the conductivity of the crust, a temperature gradient between the core and the surface may or may not be present, resulting in different surface temperatures and photon luminosities [cf. equations (4) and (5)]. Thus, NSs with rapid cooling processes active in the core appear dimmer than those with just standard cooling, and, when their cores are very cold as a result of rapid neutrino cooling, NSs with high conductivity crusts will appear dimmer than those with lower conductivity crusts.

### 3.3 Time-variable emission

As we now describe, the approximate steady-state luminosity is only a part of the story. Depending on the microphysics of the crust and core, the time-dependent thermal luminosity of the NS may either always be very close to the steady-state estimate ($\delta L/L < 1$ per cent), or vary wildly around it ($\delta L/L \gtrsim 1$).

In Fig. 1 we display the time evolution of the temperature in the NS crust for a model with low thermal conductivity, standard neutrino emission, $\langle M \rangle = 10^{-10} \, M_\odot \, \text{yr}^{-1}$, and $\tau_{\text{rec}} = 30 \, \text{yr}$. Other models are similar qualitatively, but, of course, differ in the magnitude of variability. The slices of the figure in the $x$–$z$ plane represent the temperature in the crust as a function of density, and the $y$ axis is the time (in days) since the beginning of a 1-day outburst. At early times, the temperature rises locally at densities where the energy is deposited. It reaches the maximum after 1 d (i.e. at the end of the outburst). In the ensuing cooling period, the heights of the temperature peaks decrease, and their widths increase due to heat diffusion. After ~6 d the ‘heat wave’ due to the very first reaction, at $\rho = 1.5 \times 10^9 \, \text{g cm}^{-3}$, reaches the top of the crust. This temperature increase at the top of the crust is then directly translated into the increase in the thermal luminosity of the NS (Potekhin et al. 1997). At a much later time (~1000 d after the outburst) the heat wave due to the reactions in the inner crust, at $\rho \approx 10^{12} \, \text{g cm}^{-3}$, reaches the surface. Even though the energy release in these reactions is much larger, the change in the outer boundary temperature and, hence, the variation of the thermal luminosity is much smaller, because the temperature peak spreads out to a width corresponding to roughly the thermal time at the deposition depth as it travels towards the surface of the star, and because some of the heat is conducted into the core of the star. Moreover, while the temperature peaks in the crust because of individual reactions are well-separated at the early times, they become blended together by the time they reach the surface.

Fig. 1 clearly suggests that, in order to produce appreciable variability in the thermal emission from the surface, the energy deposition by the non-equilibrium reactions should be sufficient to
heat the crust locally by $\delta T \sim T$. The typical outburst fluences indicate that only $\Delta M \lesssim 10^{23} \, \text{g}$ of material is accreted during an outburst, and hence $\lesssim 10^{43} \, \text{erg}$ of energy is deposited in the crust. This amount is small, and allows us to place some interesting constraints on the heat capacity of the NS crust. The thermal time at the depths where this energy is deposited is at least comparable to and usually much greater than a typical outburst duration. Thus, during the outburst, the heat is not conducted away, but primarily heats the crust locally. Neglecting heat diffusion, the nuclear energy deposited in the region near neutron drip will heat this region by an amount

$$\delta T \sim 10^4 \, \text{K} \left( \frac{C}{k_B/\text{baryon}} \right)^{-1} \left( \frac{\rho_B}{10^{30} \, \text{erg cm}^{-3}} \right)^{-1} \left( \frac{Q_{\text{acc}}}{1 \, \text{MeV}} \right) \times \left( \frac{\Delta M}{10^{23} \, \text{g}} \right)$$

(6)

where $C$ is the heat capacity in units of $k_B$ (Boltzmann constant) per baryon and $\rho_B$ is the pressure at which the energy is deposited. As outlined in Section 3.1, the typical crustal temperature is $\sim 10^{7}$–$10^8$ K. Therefore, if the heat capacity of the NS crust is $\sim k_B$ per baryon, the crust relaxation luminosity is only a small perturbation on the overall cooling of the core. In order to produce observable variability of quiescent thermal emission, the NS crust must have $C \ll k_B$ per baryon.

Fig. 2 shows contributions to the heat capacity in the crust, for a representative case of $\langle M \rangle = 10^{-11} \, \text{M}_\odot \, \text{yr}^{-1}$, in the outer crust. The heat capacity of the ionic lattice, $C \approx 10^{-2} k_B$ per baryon (dotted line, Shapiro & Teukolsky 1983; van Riper 1991; Chong & Cheng 1994), dominates. Strongly degenerate, relativistic electrons (dashed line) contribute a negligible amount to the heat capacity everywhere except in the deep inner crust ($\rho \gtrsim 2 \times 10^{13} \, \text{g cm}^{-3}$). If densities above neutron drip, free neutrons, if not superfluid, would have appreciable heat capacity, approaching $k_B$ per baryon (dotted-dashed line). Thus, if the neutrons in the inner crust are not superfluid, their heat capacity is so large that the energy release of $\lesssim 10^{44} \, \text{erg}$, characteristic of the heating during an outburst, is not sufficient to heat the crust substantially. However, it is commonly believed that the electrons in the inner crust form a $\nu_\mathrm{e}$–$\bar{\nu}_\mathrm{e}$ superfluid, and that their heat capacity is greatly reduced (Maxwell 1979; Yakovlev et al. 1999). Superfluidity completely suppresses free neutron heat capacity at densities $> 8 \times 10^{11} \, \text{g cm}^{-3}$ (solid line in Fig. 2). In this case, the ions dominate the heat capacity in the inner crust, and $C \approx 10^{-2} k_B$ per baryon. Therefore, observing the evolution of quiescent luminosity provides a test for the superfluidity of neutrons: if late-time evolution is observed ($\delta T/T > 10^{-3}$), then the neutrons must be superfluid. While this may not be a controversial conclusion, it is worth noting that the only existing observational evidence for superfluidity of crustal neutron gas is the interpretation of pulsar glitches as being initiated in the crust. Variability of quiescent thermal emission from NS transients, if disentangled from accretion in quiescence or from other sources of variability outlined in Section 1, would be a completely separate direct observational confirmation of this view.

The spreading and blending of the temperature peaks due to different reactions, clearly evident in Fig. 1, as well as the relative magnitude of heat diffusion towards the surface and the core can be easily understood in terms of the thermal time at the place where the heat is deposited. To make the discussion more precise, we define the thermal diffusion time-scale (Heney & L’Ecuyer 1969; Maxwell & Maclay 1979)

$$\tau_{\text{th}} = \left[ \int_{r_1}^{r_2} \left( \frac{\rho_B}{K} \right)^{1/2} \, dr \right]^2$$

(7)

BBR98) as

Roughly speaking, $\tau_{\text{th}}$ is the time it takes for a heat impulse to diffuse from $r_1$ to $r_2$. In Fig. 3, we plot the thermal diffusion time-scales as a function of density in the crust to the NS surface (solid line) and the core (dotted lines). The top panel is for the model with electron–impurity scattering conductivity in the limit of very impure crust, while the bottom panel is for the model with electron–impurity scattering conductivity. Black vertical bars in the top panel show the locations of energy deposition owing to non-equilibrium reactions, with the height of the bars proportional to the energy deposited. Clearly, the energy depositions in the inner crust happen at very similar depths, such that the differences between the thermal times of the reactions are much smaller than the thermal times themselves. By the time the heat from these reactions diffuses to the surface, the differences between them are blended. It is also clear from this figure that, if the thermal diffusion time to the core is smaller than that to the surface, then most of the heat will flow to the core, rather than to the surface.

In Figs 4 and 5 we survey the dependence of the transient emission on various model parameters following an accretion
high thermal conductivity have systematically lower crust temperatures (see Section 3.2), and hence have a larger amplitude of variability.

As discussed above, the first hump in the light curve is because of the reactions in the outer crust, while the second hump is because of those in the inner crust. This is a generic feature of all models except for the low-\(K\), short-recurrence ones (left column of Fig. 4). As is evident from the top panel of Fig. 3, the thermal diffusion time from the inner crust to the surface is \(\approx 10^3\) d, i.e. longer than the recurrence time. Thus, the second peak in the light curve does not make it to the NS surface before the next outburst, and only one peak is observed in the quiescence light curve. This mismatch between the thermal diffusion and the outburst recurrence time-scales explains why only one light-curve peak is observed in the \(\tau_{\text{rec}} = 1\) yr, low thermal conductivity simulations (Figs 4a,b) while two peaks are observed in all other simulations. For long \(\tau_{\text{rec}}\) sources, the time between the outburst and the second peak in the light curve is an excellent observational indicator of the thermal time in the inner crust. As we see in Fig. 3, the thermal time of the inner crust in low-\(K\) models is \(\approx 10^3 - 10^4\) d, while it is only \(\approx 10^2\) d for the high-\(K\) models. As a result, for low-\(K\) models with \(\tau_{\text{rec}} = 1\) yr, the crust does not completely cool down, and therefore remains at a roughly constant temperature. On the other hand, for \(\tau_{\text{rec}} = 30\) yr, the crust does have enough time to cool down, and hence the variability is much greater (tens of per cent to factors of few). In addition, it is clear from equation (6) that it is \(\Delta M = \langle M \rangle \tau_{\text{rec}}\), rather than \(\langle M \rangle\) or \(\tau_{\text{rec}}\) individually that is important for the overall magnitude of variability. For a given \(\langle M \rangle\), models with larger \(\tau_{\text{rec}}\) exhibit larger variability, simply because the amount of energy deposited into the crust is larger. This explains why the \(\tau_{\text{rec}} = 1\) yr light curves are (relatively) less variable than the \(\tau_{\text{rec}} = 30\) yr light curves. In general, one therefore expects a greater magnitude of (relative) intensity variability in quiescence from systems with longer recurrence time-scales (\(\approx 10\) yr).

4 COMPARISON WITH OBSERVATIONS

Is the phenomenon we describe here observed in transient NSs in quiescence? If so, both the time-averaged luminosity level and the variability around it need to be self-consistently accounted for. This constraint provides an observational discriminant for the various core cooling scenarios in NS transients.

The observed magnitude of variability (factors of \(\approx 3 - 5\)) cannot be achieved in models with standard neutrino cooling, which vary by less than 50 per cent in all cases that are investigate here. On the other hand, rapid neutrino cooling models are capable of producing intensity variability by factors of up to 20 on time-scales of months–years; this may be invoked for the observed variability of 4U 2129+37 (a factor of 3.4 over months), and for the longer time-scale variability of Cen X-4 (factor of 2–5 over 6 yr, an additional factor of 3 over 10 yr). The factor of 2–3 variability in Aql X-1 over months–years can be similarly explained, again requiring rapid cooling to be operative in its core. However, to explain the factor of \(\approx 3\) variation over a period of 4–8 d in the quiescent X-ray flux of Cen X-4 observed by Campana et al. (1997), one must invoke an unobserved, short (\(\approx 1\) d) outburst, preceding those observations by only \(\approx\) few days, which seems unlikely although it cannot be excluded due to the lack of X-ray coverage.

However, as argued by BBR98, the inferred quiescent luminosity of Aql X-1 can be naturally understood if the neutrino losses from its core are negligible, i.e. if only standard cooling.

**Figure 3.** Thermal diffusion time to the surface (solid lines) and to the core (dotted lines) as a function of density for (reading lines top to bottom) \(\langle M \rangle = 10^{-10}, 10^{-11}, \text{ and } 10^{-12} M_\odot \text{yr}^{-1}\). The top panel corresponds to models with standard core cooling and low-crustal thermal conductivity, while the bottom panel is for models with standard core cooling and high-crustal thermal conductivity. The filled bars in the top panel schematically regions of nuclear energy deposition indicate. The height of the bars are proportional to the deposited energy, with the highest bar at \(\rho = 1.5 \times 10^{12} \text{g cm}^{-3}\) corresponding to 0.47 MeV per accreted baryon deposited.

outburst. Fig. 4 shows the results for a crust with low thermal conductivity (electron–impurity scattering with \(Q_{\text{imp}} = Z^2\)), while Fig. 5 shows the results for high thermal conductivity (electron–phonon scattering). These figures are plotted with the luminosity normalized by the thermal luminosity just prior to the outburst, which is the lowest luminosity of the outburst cycle. This tends to exaggerate visually the range of luminosities. It also does not display the absolute differences in the luminosities due to different \(\langle M \rangle\), which are discussed in Section 3.2. However, it does make the relevant factors of variability, which can be produced over a recurrence cycle due to this mechanism, clear and that is our goal here.

As is clear from the figures, the variability is the smallest (\(\approx 1\) per cent) for models with standard cooling and short- (1 yr, in our case) outburst recurrence times. The variability is the largest for models with rapid core cooling and long- (30 yr) outburst recurrence time. Under the same conditions, the variability of the models with high-crustal thermal conductivity (Fig. 5) is always larger than that of the low-conductivity models (Fig. 4). All of these features are straightforward to understand in the light of the above discussion. Since the energy deposited into the crust is not large (\(\approx 10^{41}\) erg), the colder the crust, the larger the transient thermal response. Models with rapid core cooling and

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processes operate [cf. our equation 3]). Time-averaged $L_q$ of Aql X-1 is inconsistent with rapid cooling, regardless of the assumed crustal conductivity [cf. equations (4) and (5)]. This is also the case for 4U 1608 − 52. For these sources, variability in quiescence must be due to processes other than the thermal relaxation of the crust (outlined in Section 1).

Of the observed recurrent NS transients, only Cen X-4 has time-averaged $L_q$ much lower than that given by equation (3). BBR98 conjectured that this discrepancy is because of the fact that only $Q_{\text{nuc}} \sim 0.1 \text{MeV}$, rather than $\sim 1 \text{MeV}$ is deposited in its crust.\(^5\) While Colpi et al. (2001) argued that $Q_{\text{nuc}} \sim 1 \text{MeV}$, and rapid neutrino cooling is operating in Cen X-4. Clearly, an analysis of the variability of Cen X-4 in quiescence (or other transients with long-recurrence times) provides a discriminant between the various core cooling mechanisms. If enhanced neutrino cooling is responsible for the relatively low $L_q$ of Cen X-4, then its quiescent luminosity must vary in a well-defined way (see panels d of Figs 4 and 5). In the contrary case of standard cooling with low $Q_{\text{nuc}}$, the baseline thermal luminosity of Cen X-4 should be constant to be better than 50 per cent, and any variability must be accounted for by the processes outside the NS crust. Finally, it is difficult to reliably predict the expected quiescent luminosity of 4U 2129+47, due to its unusual accretion history (Pietsch et al. 1986), and so we draw no additional conclusions for this source.

If transiently accreting pulsars in quiescence fully exclude accretion on to the NS surface due to the propeller effect, then observations of the thermal pulse during quiescence will be due only to crustal and core emission. Moreover, optical observations can indicate when the accretion disc is absent in these sources (Negueruela et al. 2000), and observations which follow the pulsed intensity profile of such an object over several years following an outburst can readily constrain the NS atmospheric luminosity. The quiescent emission of transient pulsars, such as the pulsed emission recently detected from A0535+35 (Negueruela et al. 2000) and the persistent emission from SAX J1808.4−3658 (Stella et al. 2000; Dotani, Asai & Wijnands 2000), can be used potentially for measuring the late-time emission predicted here. However, theoretical modelling of the quiescent emission in this case is complicated by the need to understand the effects of the magnetic field, transverse heat flow in the crust, and whether or not the crust is replaced by accretion in these systems.

\(^5\)The low-quiescent luminosity could also be potentially explained by an overestimation of the time-average accretion rate of Cen X-4, as a result of the small number of observed outbursts.

Figure 4. Thermal emission luminosity from the NS photosphere, due to crustal heating and NS core temperature (neglecting accretion luminosity), as a function of time since a 1-day long ‘outburst’ – for a crustal conductivity as a result of the electron–impurity scattering. The luminosity is normalized by its value just prior to the outburst (i.e. by the lowest value over the recurrence time). Panels a and c are for models with modified Urca (‘standard’) cooling, while panels b and d are for models with accelerated cooling (in this case, pion condensate). The assumed outburst recurrence time is indicated in each panel. The time average accretion rates used are: $\dot{M} = 10^{-10}$ (solid line), $10^{-11}$ (dotted line), and $10^{-12}$ (broken line) $M_\odot\text{yr}^{-1}$.
5 CONCLUSIONS

We have investigated the time-variable luminosity of a transiently accreting NS, following a ‘delta-function’-like accretion outburst (one day in duration), for three different values of $\dot{M}$, for NS cores with both standard and rapid core cooling (specifically, for $\nu$ emissivity from modified Urca and from pion condensate), and for high-crustal thermal conductivity dominated by electron–phonon scattering (for the case of a pure crystal crust) or for low conductivity owing to the electron–impurity scattering with large impurity fraction (for the more likely case of very impure crust expected in an accreting NS).

The magnitude and time dependence of late-time emission due to heat deposited by non-equilibrium reactions in the crust depends intimately on the microphysics active in the crust and the core, the time-average accretion rate (and thus the time-average quiescent luminosity itself) and – most importantly – on the recurrence time-scale of the transient system. In the case of the standard core cooling, the magnitude of the luminosity swings (peak-to-peak) is $\lesssim 1$ per cent for $t_{\text{rec}} = 1$ yr, and up to 15–40 per cent for $t_{\text{rec}} = 30$ yr, independent of the crust conductivity and $\dot{M}$. For rapid core cooling, the magnitude of the luminosity swings varies between $\lesssim$ few per cent, to as much as a factor of 20, and is largest for long $t_{\text{rec}}$ and high-crustal thermal conductivity.

It is possible to observe either one peak in the light curve (as a result of the blended emission from the several reactions in the outer crust) or two peaks (the second is because of the blended emission from reactions in the inner crust). Observing the second peak requires a recurrence time long enough to permit the inner-crust emission to reach the surface. For low-crustal thermal conductivity, this thermal time is $\sim 10$ yr, so the second peak is only observed in transients with longer recurrence times. In the electron–phonon scattering dominated crust, with thermal time of $\lesssim 1$ yr, the second peak may be observed in short-$t_{\text{rec}}$ sources. In general, only the energy deposited, at depths where the thermal time is smaller than the outburst recurrence interval, will produce observable variability in the quiescent thermal emission.

The changes in luminosity which would permit one to follow the time-variable emission due to the deep-crustal heating at the $\sim$ few per cent level are $\delta L_X \sim 10^{38}$ erg s$^{-1}$ ($= 5 \times 10^{-16}$ (d/4.0 kpc)$^2$ erg cm$^{-2}$ s$^{-1}$), which is detectable with Chandra, XMM and Con-X in $\sim 10^3$, $5 \times 10^4$, and $10^4$ s, respectively. Complicating such observations is the contribution of the accretion on to the NS, which could add a stochastic variability component to the thermal spectrum which may not easily be distinguished. If accretion only occurs at the magnetosphere, this contributes to the power-law component, which can be separated spectrally.

Our comparisons, between these results and the previously reported observations of transiently accreting NSs in quiescence, indicate that the observed magnitude of intensity variability is much greater than what we predict for standard NS core cooling and crust conductivity (factors of $\sim$ few for the former versus $< 50$).
per cent for the latter). Hence, for the recurrent transients of Aql X-1, 4U 1608-52, and SAX J1808.3–36, whose time-averaged luminosity agrees well with the predictions of standard neutrino cooling (BBR98; cf. our Section 3.2), it seems likely that some other process – such as continued, variable accretion in quiescence – is required to explain the majority of the observed variability.

In the case of Cen X-4, however, the observed quiescent luminosity is consistent with the rapid core cooling (Colpi et al. 2001, cf. our Section 3.2), and hence its long-time-scale (months to years) variability could be accounted for by crustal relaxation. On the other hand, if Cen X-4 appears too dim because its time-averaged accretion rate has been overestimated or because non-equilibrium reactions deposit an unusually small amount of energy into its crust (BBR98), then the majority of its quiescent variability must be because of the stochastic sources external to the NS. Disentangling the true thermal emission of Cen X-4 from the contamination due to continued accretion in quiescence (or other sources) would allow one to discriminate between these two hypotheses.

Finally, we note that the observations are of low quality ($S/N$~few); there are only two instances of repeated observation with the same instrument, and in those cases the number of observations were only 2–3 (not densely sampled light curves); and the inferred time-average accretion rate (and, hence, the predicted quiescent luminosity) of Cen X-4 is highly uncertain due to the small number of outbursts observed. Therefore, a definitive statement on this subject requires greater observational scrutiny, with higher $S/N$ data than that presently available, and with densely sampled light curves following an outburst, which would be capable of distinguishing between the temporal variability predicted here and the more stochastic variability which is likely to accompany a variable accretion rate.

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However, the short time-scale of variability (~few days) requires that a short outburst (~days) would have had to occur only a few days prior to the observation; this seems unlikely, but cannot be excluded.