A COMPOUND INTERFEROMETER FOR FINE STRUCTURE WORK

By William V. Houston*

Abstract

The overlapping of orders in a Fabry-Perot interferometer can be avoided by using two interferometers in series. The fundamental equation shows that the dispersion is independent of the plate separation, while the distance between orders is inversely proportional to it. Thus an instrument with a small separation may be used as a preliminary filter to eliminate some of the orders in one of larger separation. This will not affect the fine structure pattern, and the resolution will be even greater than that due to the larger separation. Such an instrument has been built and satisfies the predictions of the theory. A plate, taken with the green line of mercury, is shown to illustrate the effect of the preliminary interferometer.

As an instrument for studying spectral fine structure the Fabry-Perot interferometer has several advantages over other high resolution instruments. It has a uniform intensity distribution over the whole image;[1] it is free, if properly adjusted,[2] from the errors due to the overlapping of two members of a close doublet; and it can be used to measure absolute wave-lengths. Furthermore, the distance between orders, measured in frequency units, is independent of the wave-length, so that it is useful in the identification of spectral series. The outstanding difficulty, however, is that when the plates are separated far enough to give high resolution, the successive orders are so close together that the patterns overlap. It is to overcome this difficulty, without sacrificing the advantages, that this combination of two interferometers has been devised.

Light is transmitted through a Fabry-Perot interferometer with maximum intensity if it is incident at an angle θ such that $2d\cos\theta = n\lambda$, where d is the distance between the plates, λ is the wave-length, and n is an integer. Thus the distance between orders is given by

$$d\,(\cos\theta)/dn = \lambda/2d \tag{1}$$

and the dispersion by

$$d(\cos\theta)/d\lambda = n/2d = \cos\theta/\lambda \tag{2}$$

which is independent of d. The resolving power is proportional to the order of interference n, and hence to the separation d. Thus, to get a high resolving power it is necessary to use a large d. This brings the

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[1] This is particularly true as contrasted with the echelon.

[2] W. V. Houston, Astrophys. J., **64**, 81 (1926).

successive orders close together, and sometimes causes overlapping. But, if light is incident on the interferometer at an angle θ_0 corresponding to $n=n_0$, and no light is incident at the angle θ_1 corresponding to $n=n_0+1$, it is evident that no light can be transmitted at this angle. Thus this order will be missing from the transmitted pattern.

If an interferometer of small separation is used as a preliminary filter, it will transmit light to the second in certain directions only; but since the dispersion is independent of the separation, the fine structure pattern will be transmitted or destroyed as a whole. Thus if the separation of the second is twice that of the first, every other order will be transmitted, and the fine structure pattern will not be disturbed. Furthermore, the resolution will be even greater than with the second interferometer alone.

THE COMPLETE THEORY

Consider the four lightly silvered surfaces 1, 2, 3, and 4 in Fig. 1. Surfaces 1 and 2, and 3 and 4 are accurately parallel to each other,

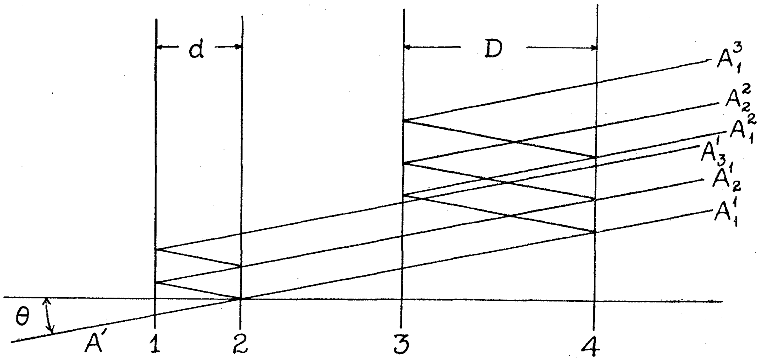

Fig. 1.

while 2 and 3 are inclined at just enough of an angle to prevent interference between them. This angle must be so small that the axes of the two instruments are practically together. Let A' be the incident amplitude, and A the resultant emerging amplitude. Let the amplitudes due to successive reflections be indicated by A_m^n where the subscript indicates reflections between the first surfaces, and the superscript indicates reflections between the second pair. Let R be the intensity coefficient of reflection and let $R=\rho^2$. We may neglect the absorption since it can be applied as a constant factor to the resultant intensity. From Fig. 1 it is evident that we have

$$A_1^1=A'(1-\rho^2)^2 \qquad A_2^1=A_1^1\rho^2 \qquad A_3^1=A_1^1\rho^4$$
$$A_1^2=A_1^1\rho^2 \qquad A_2^2=A_1^1\rho^4 \qquad A_{m+1}^{n+1}=A_1^1\rho^{2\,(m+n)}$$

Then the resultant amplitude is given by

$$A = A_1{}^1 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \rho^{2(m+n)} \cos(wt - mb - nB) \tag{3}$$

where $B = 4\pi D \cos\theta/\lambda$ and $b = 4\pi d \cos\theta/\lambda$. Let $wt - mb = M$ and the summation becomes

$$\begin{aligned} A &= A_1{}^1 \sum_{m=0}^{\infty} R^m \sum_{n=0}^{\infty} R^n \cos(M - nB) \\ &= A_1{}^1(1 - 2R\cos B + R^2)^{-1} \sum_{m=0}^{\infty} R^m [\cos(wt - mb) \\ &\qquad - R\cos(wt - B - mb)] \end{aligned} \tag{4}$$

Carrying out the second summation and introducing a phase constant we have

$$A = A_1{}^1(1 - 2R\cos B + R^2)^{-1/2}(1 - 2R\cos b + R^2)^{-1/2}\cos(wt + \psi). \tag{5}$$

Squaring this, the intensity is given by

$$I = (A_1{}^1)^2/(1 - 2R\cos B + R^2)(1 - 2R\cos b + R^2) \tag{6}$$

Equation (6) is just the product of the corresponding equations for the two separate interferometers. This justifies the process, used in the qualitative theory, of considering the action of the second instrument on the light transmitted by the first, without reference to the interference processes in the first.

If the values of d and D are such that $2d\cos\theta = n\lambda$, and $2D\cos\theta = m\lambda$, both instruments will transmit a maximum at the angle θ. However, the next maxima of the two instruments will not coincide, in general, and so both will be greatly decreased in intensity. The next maxima that do coincide will be determined by the ratio of D to d. For example, if $d = D$ the pattern transmitted by the two will be the same as for one interferometer, but, as will be shown later, the maxima will be sharper. If $D = 2d$, the separation of orders is that due to the separation d, while the resolving power is that due to the separation D, i.e., only every other order of the second instrument is transmitted. If $D = 3d$, only every third order is transmitted. In all these cases the dispersion is the same so that computations of absolute wave-length and frequency difference can be carried out as with the simple instrument. This constitutes the principal advantage of the air interferometer over instruments in which the path difference is wholly or partly in glass.

To determine the resolving power we may assume that two fringes can be recognized as distinct if their maxima are separated by a distance equal to the width of one image where its intensity has dropped to one half its maximum value. The computations, of course, assume two purely monochromatic lines of equal intensity. We then wish to find the value of B for which the intensity drops to 1/2. In the case of a single interferometer we have

$$1-2R\cos B+R^2=2(1-R)^2 \tag{7}$$

and hence

$$B=\cos^{-1}\{1-(1-R)^2/2R\} \tag{8}$$

The spectral range, or the difference in wave-length corresponding to the distance between orders, is given by the fundamental equation to be $\lambda^2/2D$. Hence

$$\lambda/\Delta\lambda=2\pi D/\lambda \cos^{-1}\{1-(1-R)^2/2R\} \tag{9}$$

$2D/\lambda$ is the order of interference n so we have

$$\lambda/\Delta\lambda=\pi n/\cos^{-1}\{1-(1-R)^2/2R\} \tag{10}$$

For $R=0.75$ this gives the resolving power as about $11n$.

For the two interferometers we have

$$(1-2R\cos B+R^2)(1-2R\cos b+R^2)=2(1-R)^4$$

or,

$$\cos B+\cos b-2R(1-\cos B)(1-\cos b)/(1-R)^2=[4R-(1-R)^2]/2R \tag{11}$$

B and b will both be small enough so that the cosines may be represented by the first two terms of their series so that we have

$$(1+r^2)b^2+Rr^2b^4/(1-R)^2=(1-R)^2/R \tag{12}$$

where $r=D/d=B/b$. Here again we may neglect the fourth power of b, unless R is very large, so that

$$b=(1-R)/(1+r^2)^{1/2}R^{1/2} \tag{13}$$

Then we have

$$\Delta\lambda=\lambda^2(1-R)/2\pi d(1+r^2)^{1/2}R^{1/2}$$

or

$$\lambda/\Delta\lambda=n\pi R^{1/2}(1+r^2)^{1/2}/(1-R) \tag{14}$$

Thus the resolving power of the combination is equal to that of the smaller interferometer multiplied by $(1+r^2)^{1/2}$. When $R=0.75$ and $r=1$ the resolving power is about $15n$.

Fig. 2 shows the shape and the separation of successive maxima when $D=3d$ and $R=0.75$. The two small maxima represent the remnants of the maxima cut out by the smaller instrument. These will be smaller as R increases, and also as they are farther from the principal maxima. This puts a limit to the value of r that can be used, since when r is large these spurious maxima will be closer to the principal ones and hence they will be stronger. When dealing with a fine structure in which the satellites are faint compared with the principal line, care must be taken not to mistake these maxima for satellites. If plates are taken with two different values of r, the true satellites can be distinguished easily by the fact that their positions will be the same on both plates.

Description of the Instrument

An instrument of this kind has been built in the machine shop of this laboratory under the direction of Mr. Julius Pearson. An attachment was made for the interferometer previously built, so that the preliminary interferometer can be clamped to the frame of the other.

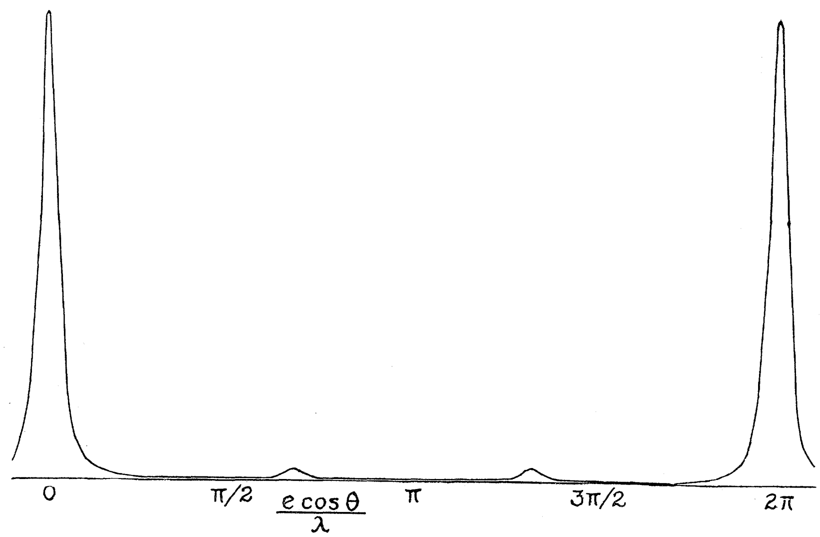

Fig. 2. Shape and separation of successive maxima when $D=3d$ and $R=0.75$.

This clamp carries a steel plate which can be rotated about two axes to set the optical axes of the two instruments together. This plate then carries one mirror which slides back and forth and can be fastened by set screws, and another which can be made parallel to the first. All the adjustments are made by screws working at the ends of levers which are held tightly against the screws by springs.

It has been found possible to make the necessary adjustments as follows. With the preliminary interferometer removed and the other set at about the separation to be used, the latter is adjusted until the plates are parallel. The other interferometer is then attached and its fringes are viewed from the side by means of a totally reflecting prism. In this way its plates can be made parallel. When the prism is removed the transmitted light shows the ring system of each interferometer as well as regions of brightness where the two systems coincide. If the axis of the preliminary interferometer is then adjusted until these regions of brightness are circles concentric with the other ring systems, the instrument is in adjustment. To make one separation an integral multiple of the other the movable interferometer is opened or closed to make the circles of bright rings move toward the center. As the desired separation as approached these regions become wider until they cover the whole field. A white light source is then put in and the adjustment continued until the colored fringes appear. When the white central fringe appears the desired point has been reached. This phenomenon of white light fringes was first used by Perot and Fabry[3] in the measurement of length.

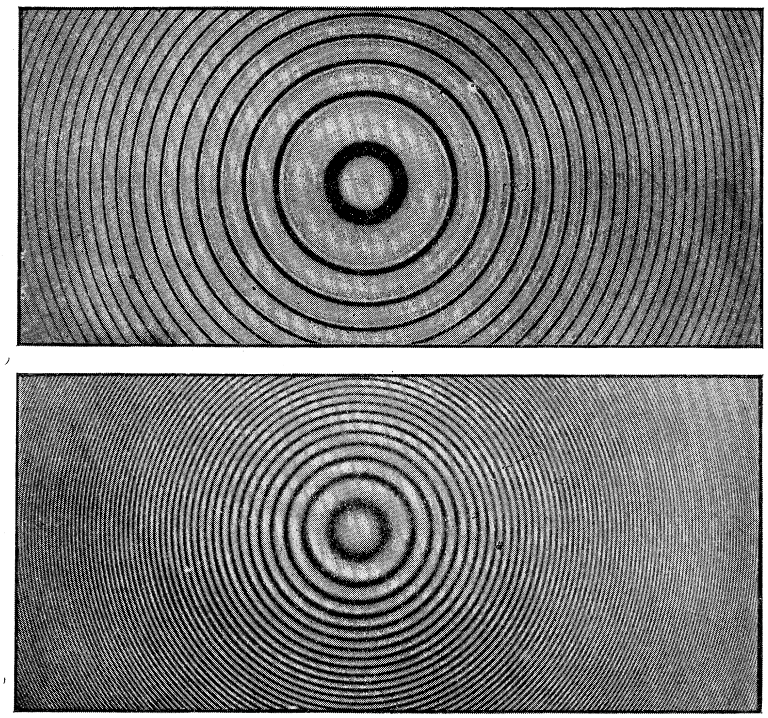

Fig. 3. Fringe system of the mercury line: above, through double interferometer; below, through the single interferometer.

A preliminary trial with rather inferior plane surfaces has given the plates reproduced in Fig. 3. The upper photograph shows the fringe system given by the green mercury line when one separation was about 3 mm and the other was about 9 mm. The lower photograph shows the same line through the 9 mm interferometer only. The overlapping of the fine structure patterns in this case causes the main line to appear fuzzy at the edges.

[3] Perot and Fabry, Ann. chim. et phys. **7**, 16 (1899).

Preparations are under way to apply this instrument to the study of the hydrogen doublets and to other fine structure problems. It should be valuable in the study of hyper-fine structure, such as that of mercury. The principal disadvantage is that a good deal of light is absorbed in passing through the four silver films, but this is characteristic of all high resolution instruments. On the other hand, this arrangement offers all the advantages of a fine grating in the direct measurement of fine structure, and is superior to the grating in resolving power and in the determination of absolute wave-lengths.

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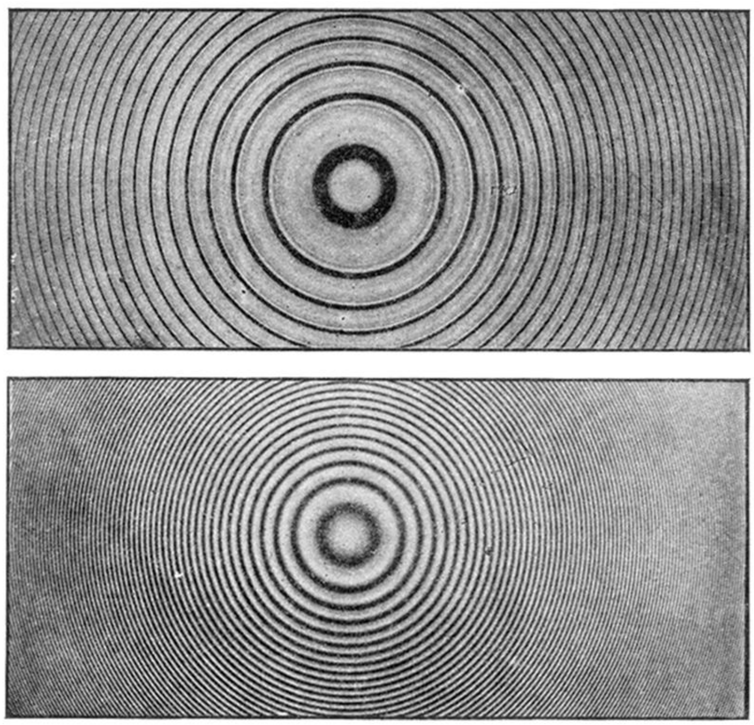

Fig. 3. Fringe system of the mercury line: above, through double interferometer; below, through the single interferometer.