A SPECTROSCOPIC DETERMINATION OF e/m

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Abstract

Rydberg constants for hydrogen and helium.—By an interferometer method the wave-lengths of the hydrogen lines at 6563A and 4861A were measured with reference to λ5015.6750 of He as standard. The values so obtained were, for the doublet at 6563A, 6562.7110 ± .0018 and 6562.8473 ± .0009; and for the doublet at 4861, 4861.2800 ± .0013 and 4861.3578 ± 0022. Similarly the wave-lengths of the 4686 lines of ionized helium were found to be 4685.7030 ± .0012 and 4685.8030 ± .0026. From these values of the wave-lengths the Rydberg constant for hydrogen, R_H is calculated to be 109,677.759 ± .008, and the Rydberg constant for helium, R_{He}, to be 109,722.403 ± .004.

Calculation of e/m.—The ratio of the mass of the electron to the mass of the hydrogen nucleus, m/m_H, is given by

$$m/m_H = (R_{He} - R_H)(m_{He} - m)/R_H(m_{He} - m_H) = 1.33648(R_{He} - R_H)/R_H.$$

The value of e/m is given in terms of the Faraday constant as $e/m = (m_H/m)(F/m_H)$. If the values of R_H and R_{He} calculated above be substituted in these expressions and if it be assumed that $F = 96470$ abamp. per gr. mol. and $m_H = 1.0072$, it is found that e/m has the value $(1.7606 \pm .0010) \times 10^7$ e.m.u. per gr.

Because of the accuracy attainable in spectroscopic measurements, a spectroscopic determination of a physical constant is usually preferable to any other kind. It has been known for some time that the value of e/m can be determined from purely spectroscopic measurements, but no single study has included all the measurements necessary in this determination. It is the purpose of this paper to describe a set of precision measurements made for this purpose. The work was done with a Fabry-Perot interferometer and prism, combined in the usual manner.

Standards of Wave-length

All measurements were referred to the helium line 5015 as a standard. Its wave-length was assumed to be 5015.6750 I.A., and the lines 6678 and 4387 were measured with reference to it. It was necessary to make these measurements in order to ascertain the dispersion of the phase change which takes place upon reflection at the silvered surfaces. It was impossible to use the method of differences on the lines of hydrogen and ionized helium because the fine structure is such that only a short range of orders of interference give the correct spacing of the components. And because the required order of interference is not the same for the two elements, it is impossible to determine all the lines from the same plate. However, since the hydrogen easily appears as an impurity in the helium, it is possible to measure all the lines with reference to the same ultimate standard, 5015.6750.

The lines 5015, 6678, and 4387 were selected because they are the only strong lines in this neighborhood which are strictly single and therefore

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presumably symmetrical. These three wave-lengths can be determined by the method of differences.

In using this method, it is necessary to take account of the temperature and barometric pressure under which the different exposures are made. This may be done as follows. Let λ be the standard wave-length and λ' the unknown wave-length, and let n be the order of interference of the standard and n' that of the unknown. Let $\delta n'$ represent the amount that must be added to the measured n' to take account of the dispersion of the phase change. Let the subscript 1 refer to the orders of interference determined with a different separation of the interferometer plates under different conditions of temperature, and let $d\lambda$ and $d\lambda'$ be the changes in the two wave-lengths due to the changes in temperature and pressure. Then we may write the fundamental equation of the interferometer for these two cases:

$$\lambda n = (n' + \delta n')\lambda'$$

$$n_1(\lambda + d\lambda) = (n_1' + \delta n')(\lambda' + d\lambda')$$

Taking the difference of these two equations and neglecting the small quantities of the second order gives

$$\lambda' = \lambda(n_1 - n)/(n_1' - n') - n_1 d\lambda/(n_1' - n') + n_1' d\lambda'/(n_1' - n').$$

The neglect of the quantities of the second order is justified by the fact that the small quantities of the first order rarely exceed 1/500,000 of the whole, and the total corrections determined in this way are never over 0.002A.

By this method, fifteen values for 6678, taken as differences between ten photographic observations, gave a mean value of 6678.1484 I.A. with a mean deviation of 0.0008A. On the other line, twenty values taken as differences between fourteen observations, gave a mean of 4387.9290 I.A. with a mean deviation of 0.0030A.

The order of interference was then determined from these adopted wave-lengths and compared with the observed. This showed the necessity of a correction of +0.0144 for 6678 and −0.0186 for 4387. The sign is chosen so that the correction is to be applied to the measured order of interference to get the one which can be compared with the order of interference of the line 5015.6750 by means of the equation $n\lambda = n'\lambda'$.

To determine the correction for intermediate lines it was assumed that the variation with wave-length is linear. This was justified in the case of the line 6563, since it is close to 6678. In the cases of 4861 and 4686 there was not the same justification, but the fact that the whole correction is smaller than one millionth of the orders of interference used would have made an error of even ten percent in the amount of the correction, less than the errors of measurement.

Measurements of the Wave-lengths

The lines 6563 and 4861 were measured to determine the Rydberg constant for hydrogen. The hydrogen was present as an impurity in helium. The light was taken from the side of a tube, about 5 mm in diameter, immersed in liquid air. The tube was excited by a transformer giving from 20 to 50 ma, while the total pressure varied between 0.5 mm and 2.0 mm. The orders of interference were so adjusted that one component of the doublet was equally spaced between adjacent orders of the other component. This eliminated the errors due to the overlapping of the components. This adjustment can be made approximately for both lines at the same time, but not for the 4341 line. Hence this line was not included in the results. Table I gives the values determined for these two lines. They average about 0.003A

TABLE I
Wave-lengths of the hydrogen lines.

Plate	Wave-lengths			
1	6562.7129	6562.8477	4861.2815	4861.3605
2	.7088	.8460	.2782	.3561
3	.7136	.8483	.2800	.3563
4	.7088	.8464	.2786	.3555
5	.7108	.8480	.2816	.3607
Means	6562.7110	6562.8473	4861.2800	4861.3578
Mean deviation	.0018	.0009	.0013	.0022

less than the values previously published.[1] The difference is at least partly due to the difference in standards, since these values are surely correct, with reference to the standards used, within an amount of the order of the mean deviation.

The helium line 4686 was produced inside a hollow cathode about 10 cm long and 2.5 cm in diameter. A 1 cm copper rod extended from the cathode into a flask of liquid air outside the tube. This somewhat cooled the cathode and had some effect in making the lines narrower. The tube was excited by a direct current generator, and was operated at about 265 ma with a potential drop of about 800 volts.

The complicated structure of this line, 4686, introduces some difficulty into its measurement. The spread of the components is so great that it is impossible to prevent the overlapping of orders when any reasonable resolving power is used. However, the distribution of intensities makes it possible largely to avoid the difficulties introduced by the overlapping. As has been theoretically predicted by Sommerfeld and Unsöld,[2] and observed by Paschen[3] and others, the line consists of two strong components, one of intermediate strength, and others which are faint. Thus, if a short exposure is used, only the two strong components will appear. Of course, the other components will shift the positions of the maxima slightly, and the intermediate component will have a considerable effect, but this can be minimized by selecting suitable orders of interference. No plates were used except those on which the intermediate component was fairly symmetrically situated with respect to the stronger ones, and a series of orders of interference were used so that the effect of the other components would average out. That these effects were actually present can be seen in the variations in the values of the separation of the two strong components. Some of these

[1] Houston, Astrophys. J. **64,** 81 (1926).
[2] Sommerfeld and Unsöld, Zeits. f. Physik **38,** 237 (1926).
[3] Paschen, Ann. d. Physik **82,** 689 (1927) and references there.

are too high and some too low, but the mean is very near the theoretical value. It is also noticeable that the weaker component, 4685.8030, is the more strongly affected. Table II gives the results of these measurements.

TABLE II

Wave-lengths of the 4686 lines of ionized helium.

Plate	λ_1	λ_2	$\Delta\lambda$	$\Delta\nu_0$
11	4685.7012	4685.7967	.0955	.8439
12	.7050	.8010	.0960	.9221
13	.7008	.8022	.1014	.9502
14	.7039	.8051	.1012	1.0966
15	.7037	.8031	.0994	1.0964
16	.7035	.8065	.1030	1.1011
17	.7028	.8063	.1035	1.1598
Mean	4685.7030	4685.8030	.1000	
Mean deviation	.0012	.0026	.0026	

In this table the last column headed $\Delta\nu_0$ gives the separation of the orders in terms of wave-number units. This makes it possible to determine the distribution of the components among the overlapping orders.

These mean values are probably slightly low on account of the standards used, in the same way that the hydrogen lines are probably slightly low. If they are increased a little they lie between the values given by Paschen. In his first work Paschen[4] gives the separation between these two components as 0.106A, while later[3] he finds from a microphotometer curve that it is 0.098A. If the latter is taken as correct it is evident that at least one of the wave-lengths must be changed, and it seems reasonable to change the weaker by the larger amount. Furthermore, Paschen used the helium line 4713 as a standard. This line is now known to be double and unsymmetrical and therefore somewhat unsuitable for a standard. Considering these things, it may be concluded that these values are not in essential disagreement with the general scheme of Paschen's measurements.

While all these values may be slightly low with reference to the cadmium standard, the hydrogen and helium lines are correct with reference to each other within an amount of the order of the mean deviations. It is this relative accuracy which is necessary for the determination of e/m.

EVALUATION OF THE CONSTANTS

Throughout this work the relativity equation of Sommerfeld is assumed to apply exactly. This does not represent the neglect of the spinning electron theory or the new wave mechanics, because these theories together seem to give exactly the equation of Sommerfeld.[5] Furthermore, the agreement of the Rydberg constant, computed by this equation, from the first three lines of the Balmer series, shows the very exact experimental applicability of this equation, apart from the various theoretical ways of deriving it.[6]

[4] Paschen, Ann. d. Physik **50**, 913 (1916).

[5] Richter, Proc. Nat. Acad. **13**, 426 (1927), and references.

[6] Houston, Phys. Rev. **29**, 748 (1927).

To apply this equation, it is necessary to know the wave-lengths of the individual components of the fine structure. For this purpose the hydrogen line 6563 was examined with the compound interferometer.[7] Fig. 1 shows a microphotometer curve from a plate taken with this instrument. The

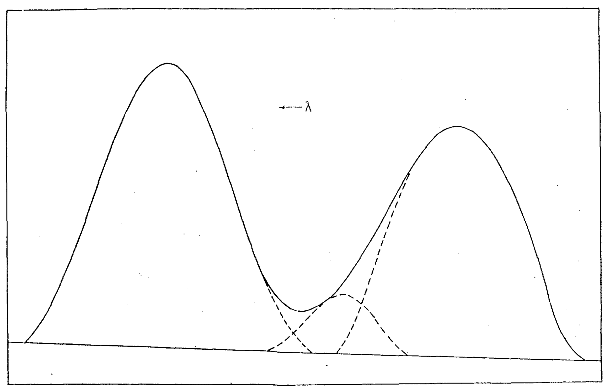

Fig. 1. Microphotometer curve for Hα.

asymmetry in the shorter wave-length component is so pronounced that it is possible to determine the position of the third component which is causing it. This is the component predicted by the theory of Sommerfeld and Unsöld, and which has previously been inferred from the displacement of the maximum.[1] The asymmetry has also been observed by Hansen,[8] and by Kent, Taylor, and Pearson.[9] Because of this asymmetry, only the long wave-length component is used in the computation of the Rydberg constant.

However, although this long wave-length component appears symmetrical in the figure, the theory requires that it also shall be multiple, and consist of one strong and two weak components. If we assume the distribution of intensities predicted, we can correct the position of the observed center of gravity to the position of the strongest component. Table III gives the results of this correction and the values of R_H. The uncertainty in the

TABLE III
Computation of R_H

λ(air)	ν(vac.)	Cor.	R_H	Wt.
6562.8475	15233.0888	+.0056	109677.754 ± .015	3
4861.3678	20564.6582	+.0024	109677.775 ± .050	1
		Mean	109677.759 ± .008	

individual values is equivalent to the mean deviation of the observed wave-lengths, while the uncertainty of the means is merely the mean deviation.

In He 4686 the two strong components were identified with the components Ia and IIb. With this identification the values of R_{He} are given in Table IV.

[7] Houston, Phys. Rev. **29**, 478 (1927).
[8] Hansen, Ann. der Physik **78**, 558 (1925).
[9] Kent, Taylor, and Pearson, Phys. Rev. **30**, 266 (1927).

TABLE IV

Computation of R_{He}

λ(air)	ν(vac.)	R_{He}	Wt.
4685.7030	21335.5622	109722.406 ± .028	2
4685.8030	21335.1070	109722.397 ± .061	1
	Mean	109722.403 ± .004	

These mean deviations can hardly be taken as representing the precision of the values of R, and yet it seems reasonable to believe that, relative to each other, these values are correct within 0.020. Of course, the actual error from the correct value based on the cadmium standard is probably larger, perhaps of the order of 0.050.

The difference between this value of R_{He} and that given by Paschen is due to the fact that his value gave more weight to his component 4685.809 than to his component 4685.703. From these two Rydberg constants it is possible to determine the ratio of the mass of the electron to the mass of the nucleus. We have

$$R_{He}=R_\infty/(1+m/m_{He}) \text{ and } R_H=R_\infty/(1+m/m_H)$$

where m is the mass of the electron and m_H and m_{He} are the masses of the hydrogen and helium nuclei respectively.

The atomic weight of helium is 4.0001[10] from which $m_{He}=3.9990$, and the atomic weight of hydrogen is 1.0077[11] from which $m_H=1.0072$. In these units the mass of the electron is $m=0.00054$. Using these values it can be shown that

$$m/m_H=(R_{He}-R_H)(m_{He}-m)/R_H(m_{He}-m_H)=1.33648(R_{He}-R_H)/R_H.$$

Then $e/m=(m_H/m)\ (F/m_H)$ where F is the Faraday constant which is 96470 absolute amperes per gram molecule. With these values we have

$$\mathbf{e/m=1.7606\pm.0010\times10^7\ \mathit{E.M.U.}\ per\ gram.}$$

This value agrees with that found by Babcock[12] from Zeeman effect measurements. However, the Zeeman effect method is subject to some criticism on account of the empirical nature of the Runge denominators.

From R_H and m/m_H we have $R=109737.424\pm.020$, from which, if $e=(4.774\pm.005)\times10^{-10}$, we have that $h=(6.557\pm.008)\times10^{-27}$. Most of this uncertainty in h is due to the uncertainty in e.

This value of h gives the value of the fine structure constant $\alpha^2=5.307\times10^{-5}$, and $\Delta\nu_H=0.3638$ cm^{-1}.

This precise determination of e/m, together with its close agreement with Dr. Babcock's value, makes this quantity the most accurately known factor in the Rydberg constant.

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[10] Baxter and Starkweather, Proc. Nat. Acad. **12,** 699 (1926).
[11] International Critical Tables.
[12] Babcock, Astro. J. **52,** 149 (1923).