THE TEMPERATURE DEPENDENCE OF ELECTRON EMISSION UNDER HIGH FIELDS

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Abstract

An expression showing the temperature dependence of electron emission under high fields is secured by combining the results of Fowler and Nordheim with the Fermi distribution of velocities used in the Sommerfeld electron theory of metals. The result is similar to that obtained previously by considering the diminution of the work function by the field. The temperature variation is small and decreases as the external field increases. It is of the right order of magnitude to agree with the most recent observations.

IN CONNECTION with the Sommerfeld application of the Fermi statistics to the electron theory of metals it was shown that the very small temperature effect in the emission of electrons under intense electric fields could be expected from the degenerate nature of the electron gas.[1] At that time no attempt was made to study the effect of the field on the emission, and it was assumed merely that the external field decreased the work necessary for an electron to escape from the metal. To estimate the order of magnitude of the effect, Schottky's[2] method was used. Since then, Fowler and Nordheim,[3] and Oppenheimer[4] have made computations, with the wave mechanics, which show the effect of the field in helping electrons to escape. This work shows that electrons may escape even when their energy is less than that required classically to pass through the surface. Fowler and Nordheim recognized that their work, combined with the Fermi velocity distribution, would give a small temperature effect, although they did not determine it explicitly. Oppenheimer started with the picture of a single atom, rather than that of an electron gas, so that a temperature effect was not so directly evident. It is the purpose of this note to give the first approximation to the temperature variation by combining the results of Fowler and Nordheim with the distribution of electron velocities used by Sommerfeld.

Sommerfeld[5] gives the number of electrons per unit volume whose velocity components lie in the element $d\xi d\eta d\zeta$ as

[1] W. V. Houston, Zeits f. Physik **47,** 33 (1928).

[2] W. Schottky, Zeits f. Physik **14,** 63 (1923). A. T. Waterman, Proc. Roy. Soc. **A121,** 28 (1928), considers that an additional diminution of the work function would be produced by the surface charge present under the external field. However, this question is rather difficult to handle because of our inadequate knowledge of the work function itself. He also points out a number of difficulties in the way of this type of interpretation of the field currents. Some of these are met by the treatment of Nordheim and Fowler.

[3] R. H. Fowler and L. Nordheim, Proc. Roy. Soc. **A119,** 173 (1928).

[4] J. R. Oppenheimer, Proc. Nat. Acad. **14,** 363 (1928).

[5] A. Sommerfeld, Zeits f. Physik **47,** 1 (1928) Eq. (19).

$$dn=2\left(\frac{m}{h}\right)^3\frac{d\xi d\eta d\zeta}{(1/A)e^{mv^2/2kT}+1} \tag{1}$$

where $\log A=W_i/kT$ and $W_i=(h^2/2m)(3n/8\pi)^{2/3}$.

Here n is the total number of free electrons per unit volume, while v represents the total velocity of the electrons. To find the total number of electrons with a velocity in the x direction which lies between ξ and $\xi+d\xi$ we may change to cylindrical velocity coordinates ξ, ρ, ϕ. Equation (1) then gives

$$\frac{\partial n}{\partial \xi}d\xi=4\pi\left(\frac{m}{h}\right)^3 d\xi\int_0^\infty\frac{\rho d\rho}{(1/B)e^{m\rho^2/2kT}+1} \tag{2}$$

where $B=Ae^{-m\xi^2/2kT}$. Let $m\rho^2/2kT=v$ and $e^v=w$. Then

$$\frac{\partial n}{\partial \xi}d\xi=\frac{4\pi m^2kT}{h^3}d\xi\int_1^\infty\frac{dw}{w}\,\frac{B}{w+B}=\frac{4\pi m^2kT}{h^3}\log\left(1+Ae^{-m\xi^2/2kT}\right)d\xi \tag{3}$$

From Eq. (3) we may write the current emitted by a surface normal to the x axis, due to electrons whose velocities in the x direction are between ξ and $\xi+d\xi$, in the form

$$dI=(4\pi mekT/h^3)\log\left(1+Ae^{-W/kT}\right)D(W)dW \tag{4}$$

where $W=m\xi^2/2$. $D(W)$ represents the fraction of the electrons of kinetic energy W, incident on the surface, which escape from the metal. Fowler and Nordheim give for this (reference 3, Eq. 18)

$$D(W)=(4/W_a)W^{1/2}(W_a-W)^{1/2}e^{-4\kappa(W_a-W)^{3/2}/3F} \tag{5}$$

In this expression W_a is the potential jump at the surface of the metal in Sommerfeld's notation, F is the applied external field expressed in ergs per cm, and $\kappa^2=8\pi^2m/h^2$. The current coming from the metal is then

$$I=(16\pi mekT/W_ah^3)\int_0^{W_a}\log(1+Ae^{-W/kT})W^{1/2}(W_a-W)^{1/2}e^{-4\kappa(W_a-W)^{3/2}/3F}dW \tag{6}$$

The integral is from $W=0$ to $W=W_a$, since (5) is valid only in this range. The current due to electrons for which $W>W_a$ is the ordinary thermionic current and is negligible for fields at which (5) is valid.

Fowler and Nordheim give the integral of I when $T=0$. To expand in terms of T we may take the derivative with respect to the temperature. Since this derivative is different from zero only in a small region around $W=W_i$, we may remove the terms independent of T from the integral sign and write

$$\frac{\partial I}{\partial T}=\frac{16\pi mek}{W_ah^3}W_i^{1/2}(W_a-W_i)^{1/2}e^{-4\kappa(W_a-W_i)^{3/2}/3F}$$
$$\int_0^\infty\left\{\log\left(1+Ae^{-W/kT}\right)+\frac{(W-W_i)/kT}{(1/A)e^{W/kT}+1}\right\}dW \tag{7}$$

This can be integrated by the methods given by Sommerfeld[5] and gives

$$\frac{\partial I}{\partial T}=\frac{16\pi^3}{3}\frac{mek^2T}{W_ah^3}W_i^{1/2}(W_a-W_i)^{1/2}e^{-4\kappa(W_a-W_i)^{3/2}/3F}. \tag{8}$$

The second derivative was found to be negligible, at least up to temperatures of 2000°. The expression for the current may then be written as

$$I=\frac{e}{2\pi h}\frac{W_i^{1/2}}{W_a(W_a-W_i)^{1/2}}e^{-4\kappa(W_a-W_i)^{3/2}/3F}\left\{F^2+\frac{32\pi^4mk^2}{3h^2}(W_a-W_i)T^2\right\} \tag{9}$$

This equation shows the same type of temperature variation as the treatment previously given. The temperature variation is small and decreases as the external field increases. As in the previous treatment there is no term containing the first power of the temperature. The coefficients are of the right order of magnitude to agree with the most recent observations.[6] Thus we may repeat the principal conclusion of the former work, that the Sommerfeld electron theory of metals definitely predicts that the emission of electrons under very high fields shall be almost independent of temperature, and that the temperature dependence shall decrease as the field increases.

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[6] Mr. Lauritsen has kindly told me that his most recent observations in this laboratory definitely show a temperature dependence of the order of magnitude given in Eq. (9). N. A. de Bruyne, Proc. Camb. Phil. Soc. **24,** 518 (1928) interprets his data as showing that the electron emission under high fields is entirely independent of temperature. However, the predicted temperature dependence is of the same order of magnitude as the dispersion of his measurements.