Active Thermal Extraction of Near-field Thermal Radiation: Supplementary Material

I. INTRODUCTION

In this supplementary material (SM), we outline the derivation of Eq. (8) of the manuscript in Section II of the SM and include a figure of how spectral near-field energy density varies with distance from the substrate in Section III of the SM.

II. DERIVATION OF NEAR-FIELD ABSORPTION COEFFICIENT

In this section, we derive the expression for the near-field absorption coefficient, given by Eq. (8) in the paper.

The absorption rate is the same as the stimulated emission rate in the absence of any degeneracy of the energy levels. The interaction with the near-field is being treated in Archambault et al. [35] where he outlines how to get the spontaneous and stimulated rates of a near-field excitation of a dipole close to the surface. We will first outline how to obtain the isotropic stimulated emission rate in Eq. (29) of [35] and then move on to show how we adopted this formulation to obtain Eq. (8) in the text.

From Eq. (C1) to (C3) of [35], the stimulated emission rate for each mode with wave vector $\mathbf{K}$ is given by

$$
\gamma_{\text{stimulated}}^\mathbf{K} = \frac{2\pi}{\hbar} \delta(E_j - E_i - \hbar\omega) \frac{\hbar n_{\mathbf{K}}}{2\epsilon_m \epsilon_0 S} |\mathbf{D}_{ij} \cdot \mathbf{u}(\mathbf{K}, z, \omega)|^2
$$

where $n_{\mathbf{K}}/\hbar\omega$ is the total energy of a single mode, $E_j$ and $E_i$ are the excited and ground state energy levels, $S$ is the normalization area, $\mathbf{D}_{ij}$ is the matrix element of the dipole moment operator $\hat{\mathbf{D}}_{ij}$ and the vector $\mathbf{u}(\mathbf{K}, z, \omega) = \exp(i\gamma_m z)(\hat{\mathbf{K}} - K/\gamma_m \hat{\mathbf{z}})/\sqrt{L(\omega)}$ according to Eq. (5) of [35]. The subscript $p$ denotes the region for $\gamma$ where $p = m$ for the region $z > 0$ and $p = s$ for the region inside the substrate where $z < 0$ so that $\gamma_p^2 = \epsilon_p k_0^2 - K^2$ where $k_0 = \omega/c$ and $K = |\mathbf{K}|$. $L(\omega)$ is the normalization of each mode defined by Eq. (B5) of [35] and has the dimension of length. $\hat{\mathbf{K}}$ and $\hat{\mathbf{z}}$ are unit vectors along $\mathbf{K}$ and $z$ axis respectively.
Substituting Eq. (26) and Eq. (C5) of [35] into Eq. 1

\[ \gamma_{\text{stimulated}} = \frac{2\pi}{\hbar^2} \int d\omega (h\delta (E_2 - E_1 - h\omega)) \frac{1}{2\epsilon_m \epsilon_0} |D_{ij} \cdot u(K, z, \omega)|^2 \langle W(\omega) \rangle \]

\[ = \frac{2\pi}{\hbar^2} \int d\omega \delta(\omega_0 - \omega) \frac{1}{2\epsilon_m \epsilon_0} |D_{ij} \cdot u(K, z, \omega)|^2 \langle W(\omega) \rangle \]

\[ = \frac{\pi |D_{ij}|^2 \exp(2i\gamma_m z)}{\epsilon_m \epsilon_0 h^2 L(\omega_0)} \left( |d_{ij,||} \cdot \hat{K}|^2 + |d_{ij,z} \frac{K}{\gamma_m}|^2 + 2\text{Re}(d_{ij,||} \cdot \hat{K} d_{ij,z}^* \frac{K}{\gamma_m}) \right) \langle W(\omega_0) \rangle \quad (2) \]

where \( d_{ij} = D_{ij}/|D_{ij}| \) and \( \langle W(\omega_0) \rangle \) is the energy density per unit surface [35].

We need to average Eq. 2 over different orientations of \( K \) for different dipole orientations. Also, we need to take into account of contributions from different frequencies and wave vectors \( K \).

First, let us consider averaging over different orientations of \( K \) for the case of a parallel orientation of the dipole (i.e. \( d_{ij,||} = 1, d_{ij,z} = 0 \)) averaged in the x-y plane.

\[ \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} |d_{ij,||} \cdot \hat{K}|^2 + |d_{ij,z} \frac{K}{\gamma_m}|^2 + 2\text{Re}(d_{ij,||} \cdot \hat{K} d_{ij,z}^* \frac{K}{\gamma_m}) \text{d}\theta \text{d}\phi \]

\[ = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \text{cos}(\theta) \text{cos}(\phi) + \text{sin}(\theta) \text{sin}(\phi))^2 \text{d}\theta \text{d}\phi = \frac{1}{2} \quad (3) \]

Likewise, one can derive an expression averaged over different orientations of \( K \) for the case of perpendicular orientation of the dipole (i.e. \( d_{ij,||} = 0, d_{ij,z} = 1 \)).

\[ \frac{1}{2\pi} \int_0^{2\pi} |d_{ij,||} \cdot \hat{K}|^2 + |d_{ij,z} \frac{K}{\gamma_m}|^2 + 2\text{Re}(d_{ij,||} \cdot \hat{K} d_{ij,z}^* \frac{K}{\gamma_m}) \text{d}\theta \]

\[ = \frac{1}{2\pi} \int_0^{2\pi} \frac{K}{\sqrt{\epsilon_m(\omega)k_0^2 - K^2}} \text{d}\theta \]

\[ = \left| \frac{K}{\sqrt{\epsilon_m(\omega)k_0^2 - K^2}} \right|^2 \quad (4) \]

If we consider the case for the surface plasmon dispersion in Eq. (4) of [35] such that \( \epsilon_m(\omega) = 1 \), we obtain

\[ \left| \frac{K}{\sqrt{\epsilon_m(\omega)k_0^2 - K^2}} \right|^2 \]

\[ = \left| \frac{k_0^2}{\epsilon_s(\omega) + \epsilon_m(\omega)} \right|^2 \]

\[ = \left| \frac{k_0^2}{\epsilon_s(\omega) + \epsilon_m(\omega)} \right|^2 = |\epsilon_s(\omega)| \quad (5) \]
In [34], two-third of the contribution to the decay rate is attributed to the parallel case and one-third to the perpendicular case for isotropic averaging. These weights can be understood intuitively as two of the axes lies in the x-y plane and one in the perpendicular direction. If we use these weights to average Eq. 2 with results from Eq. 3 and 5, we get Eq. (29) of [35]

\[ \gamma_{\text{stimulated}} = \pi |D_{ij}|^2 \exp(2i\gamma_1z) \frac{1}{3\epsilon_m\epsilon_0\hbar L(\omega_0)} (1 - \epsilon_s(\omega_0)) \langle W(\omega_0) \rangle \] (6)

Note that Archambault et al. [35] obtained Eq. (29) by integrating over possible angles of \( \hat{K} \) and \( \hat{d}_{ij} \) yields the same result as above.

For the present work, this formula has to be modified in a few aspects. Firstly, our near-field energy density from the formulation in [28,34] has a different normalization in per unit volume instead of per unit area for \( \langle W(\omega_0) \rangle \) in Eq. 2. The spectral near-field energy density defined as \( I(\omega) \) is plotted in Fig. 1 of Section II below. To reconcile this difference, we combine the term \( \exp(2i\gamma_1z) \) together with \( \langle W(\omega_0) \rangle \) from Eq. 2 to form \( I(\omega,k) = k \langle W(\omega_0) \rangle \exp(2i\gamma_1z)/L(\omega_0) \). Here, we define \( K = k_0k \) and decompose \( I(\omega) = \int_0^\infty I(\omega,k)dk \). Secondly, we sum the decay rate over all frequencies with respect to a normalized lineshape function \( g(\omega,\omega_0) = \frac{\Delta\omega}{(\omega-\omega_0)^2+(\Delta\omega/2)^2} \) instead of a delta function as of Eq. 1. Here \( \Delta\omega \) is the linewidth of the transition. In our case, Eq. 2 becomes

\[ W_{ij,\text{near-field}} = \frac{2\pi}{\hbar^2} \int d\omega g(\omega,\omega_0) \frac{1}{4\pi\epsilon_m\epsilon_0} \int kd\theta |D_{ij}|^2 |\mathbf{u}(K,z,\omega)|^2 \langle W(\omega) \rangle \] (7)

Using the above results, Eqs. 3 and 4, for isotropic orientation of emitters, we can simplify Eq. 7 as

\[ W_{ij,\text{near-field}} = \int d\omega g(\omega,\omega_0) \int dk \frac{\pi |D_{ij}|^2 \exp(2i\gamma_1z)}{2\epsilon_m\epsilon_0^2\hbar^2 L(\omega_0)} \langle W(\omega) \rangle k \left( 1 + \left| \frac{k}{\sqrt{\epsilon_m(\omega) - k^2}} \right|^2 \right) \]

\[ = \frac{\pi |D_{ij}|^2}{6\epsilon_m\epsilon_0^2\hbar^2} \int_0^\infty dk \int_{-\infty}^{\infty} d\omega \left( 1 + \left| \frac{k}{\sqrt{\epsilon_m(\omega)} - k^2} \right|^2 \right) I(|\omega|,k)g(\omega,\omega_0) \] (8)

such that we sum over contributions from all \( k \). Note that the factor of half is to account for integration from \(-\infty\) to \(\infty\) for frequency \( \omega \). If we substitute \( \gamma_{ij}^0 = \omega_0^3 |D_{ij}|^2 / (3\pi\epsilon_m\epsilon_0\hbar^3) \) from [35] into Eq. 8, we obtain Eq. (8) in the text.

\[ W_{ij,\text{near-field}} = \frac{\gamma_{ij}^0 |D_{ij}|^2}{2\hbar \omega_0^3} \int_{-\infty}^{\infty} \int_{0}^{\infty} (1 + \left| \frac{k}{\sqrt{\epsilon_{\text{medium}} - k^2}} \right|^2) I(|\omega|,k)g(\omega)dkd\omega \] (9)
III. NEAR-FIELD ENERGY OF SUBSTRATE

In this section, we plot the calculated near-field energy density $I(\omega)$ as a function of frequency for the substrate used in our model in the main text. The near-field energy density can be obtained from Eq. (45) with the Fresnel coefficients defined in Eq. (A8) of [34].

![Plot of near-field energy density][1]

**FIG. 1:** Energy density at different distances $d$ from the surface of the substrate with the permittivity $\epsilon(\omega) = \epsilon_\infty(\omega_T^2 - \omega^2 - i\gamma\omega)/(\omega_T^2 - \omega^2 - i\gamma\omega)$ where $\epsilon_\infty = 5.3$, $\omega_T = 388.4 \times 10^{12}$ s$^{-1}$, $\omega_L = 559.3 \times 10^{12}$ s$^{-1}$ and $\gamma = 0.9 \times 10^{12}$ s$^{-1}$ as described in the text. The top medium is GLS chalcogenide glass. The near-monochromatic nature of the near-field as distance is reduced is consistent with [28,34].

[1]: image.png