Weak and Electromagnetic Interactions of the Hadrons in Bootstrap Theory*†

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This is the first of a series of papers on the properties of the weak and electromagnetic currents of the hadrons in the bootstrap theory of strong interactions. In a bootstrap theory, there are many self-consistency conditions relating these weak and electromagnetic parameters to each other. We develop a formalism designed to take the fullest advantage of such bootstrap-like relations. In fact, we conjecture that the weak and electromagnetic properties of the hadrons are determined to a large extent, and perhaps completely, by self-consistency requirements. Some simple calculations of the weak and electromagnetic parameters pertaining to the octet of baryons and decuplet of resonances are given. The comparison of the results of these calculations with the experimental numbers indicates that the above conjecture holds, at least in this case.

I. INTRODUCTION

THIS is the first of a series of papers in which we investigate the role played by the weak and electromagnetic currents of the hadrons in a bootstrap theory of the strong interactions. The motivation for this program comes from the following observations.

(i) Although it has yet to be given a precise mathematical formulation, the bootstrap represents an unusually attractive approach to strong-interaction dynamics. It is clearly of interest, then, to see how the weak and electromagnetic interactions of hadrons can be fit conceptually into a bootstrap framework.

(ii) Even if the future theory of strong interactions turns out to have nonbootstrap elements, there is ample evidence that bootstrap-like requirements of self-consistency play an important role in determining the properties of at least the low-lying hadron states. For this reason, it seems evident that any practical means of calculating parameters associated with low-mass hadrons will necessarily contain many elements of a bootstrap theory. This will apply, in particular, to the weak and electromagnetic properties of hadrons.

(iii) Recently, it has become apparent that strong-interaction symmetries are closely related to algebraic properties of weak and electromagnetic currents.[1] One of the main goals of the present program is to show that such a situation is likely to arise in a bootstrap theory of hadrons. It is this aspect of our work that is likely to enjoy the most immediate interest. The study of current algebras in a bootstrap framework will, we feel, lead to a better understanding of the content of algebraic relations among currents and their connection to dynamics.

In the present paper, we develop a mathematical framework with which one can discuss the properties of currents in a bootstrap theory. We also present some arguments designed to make it seem plausible that the weak and electromagnetic properties of hadrons are determined, at least to a fairly large extent, by bootstrap-like self-consistency conditions.[2] The comparison of the results of some simple calculations, included here, with the corresponding experimental numbers lends support to this hypothesis. The algebraic properties of currents and their connection with symmetry are taken up in the following paper. As a by-product of this investigation, we have found a connection between the behavior of certain currents in a bootstrap theory and the stability of the strong-interaction bootstrap equations themselves. This result and some interesting applications of it will be discussed in a third paper.

We will use, almost exclusively, S-matrix theory as a mathematical vehicle. There is no apparent reason to believe, however, that our results would be drastically altered if we were to express the bootstrap notion in some other mathematical language.[3]

Actually, in the present paper, we do not completely restrict ourselves to a world with no elementary particles. Technically, that is, we allow for undetermined subtraction terms in our dispersion relations. Our reason for doing this is that it enables one to see how, even in a theory which contains elementary particles, bootstrap-like requirements of self-consistency will still play a major role in shaping the weak and electromagnetic properties of hadrons.

The organization and content of this paper can be summarized as follows:

In the next section, we discuss the various kinds of dispersion relations which are relevant in the study of weak and electromagnetic properties of hadrons. The most familiar kind is the dispersion relations for form factors. We reiterate, in the next section, the known fact that the Omnes equations to which these dispersion relations lead have, in general, a large number of solu-

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962). More recent references will be given in the following paper of this series.

[2] A somewhat related approach linking together the weak and strong interactions has been espoused by E. McCliment and K. Nishijima [Phys. Rev. **128**, 1970 (1962)].

[3] We have in mind here the possibilities suggested by various authors, that the bootstrap could be defined in terms of something like the Bethe-Salpeter equation or a field theory with vanishing renormalization constants.

TABLE I. Eigenvalues of X near one for $BB\theta$ and $BB^*\theta$ couplings in $SU(3)$.

Properties of θ: K	$SU(3)$ representation	Details of couplings	Physical examples
1	8	(i) $BB\theta$ couplings $\approx D+\frac{2}{3}F$ (ii) $g^{BB\theta}/g^{BB^*\theta}=g^{BB\Pi}/g^{BB^*\Pi}$	(i) magnetic moments ($S=1$, $L=1$, $K=1$) (ii) axial currents ($K=1$ with various S and L) (iii) induced pseudoscalar term ($S=0$, $L=1$, $K=1$)
0	8	$BB\theta$ couplings $\approx F+\frac{1}{4}D$	(i) electric form factors ($S=1$, $L=1$, $K=0$) (ii) weak vector form factors ($S=1$, $L=1$, $K=0$)
0	1		none

tions. Because of the ambiguity in the solution to form-factor equations, one is led to investigate other types of dispersion relations. To see what direction one might take, let us recall the original Chew-Low theory of photoproduction,[4] one of the landmarks of dispersion theory. In substance, their calculation can be considered as a computation of the γNN^* vertex in terms of the γNN vertex. They did this calculation not by the possibly more obvious method of looking at dispersion relations for N-N^* form factors, but by computing the residue of the N^* pole in the amplitude $\gamma+N\rightarrow\pi+N$, keeping all particles on the mass shell. The use of this sort of dispersion technique for calculating weak and electromagnetic properties of hadrons turns out to be the key to a bootstrap approach to the currents. The formalism for this approach is laid down in the remainder of Sec. II.

Section III is devoted to a discussion of the self-consistency requirements implicit in the many dispersion relations connecting the weak and electromagnetic parameters of hadrons. We show there how, quite independently of the existence of elementary particles, the observed weak and electromagnetic properties of hadrons may be largely determined by self-consistency. In Sec. IV we discuss some calculations which indicate that the effects of self-consistency are, in fact, of dominant importance. What we do here is to look at the usual static theory of baryons and resonances, both in $SU(2)$ and $SU(3)$, and ask ourselves what types of weak and electromagnetic interactions would be self-consistent. We find that all the observed weak and electromagnetic interactions of the baryons are included in the solutions to this problem. Furthermore, we calculate a number of ratios between parameters like magnetic moments and find the results to be in good agreement with experiment (see Table I).

Our final topic, taken up in Sec. V, is a discussion of currents with abnormal CP properties. Such currents may be of interest in connection with possible CP violations in the weak or electromagnetic interactions of hadrons. The appearance of these currents would not be unnatural in a bootstrap theory, and in our simple model they are self-consistent.

[4] See, for example, G. Chew, M. Goldberger, F. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

The reader who is primarily interested in current algebras may not find it necessary to study the present paper in any great detail before proceeding to the following paper on current algebras and symmetries. We give there a summary of the principle conclusions reached here.

II. WEAK AND ELECTROMAGNETIC CURRENTS IN S-MATRIX THEORY

In this section we analyze the properties of some dispersion-theoretic approaches to the weak and electromagnetic parameters of hadrons. Let us begin with a look at the usual Omnes equations for form factors. To be specific, we can consider the electromagnetic properties of hadrons, working to first order in the electric charge e, but, so far as we are able, treating the strong interaction exactly.

The essential properties of the general Omnes equation will be present in a problem containing many coupled two-body channels; three and higher body channels can be thought of as a continuum of two-body channels.[5] To this end, we consider N two-body channels labeled by the index i, $i=1\cdots N$, all of which have the same J^P and charge conjugation as the photon. A typical choice for the channels would be $\pi^+\pi^-$ in a P wave, $N\bar{N}$ in a $^3S_1$ state, and $N\bar{N}$ in a $^3D_1$ state. For each channel, the amplitude for $\gamma\rightarrow$ (particles in channel i) with a virtual photon of mass $\sqrt{(q^2)}$ is given by a form factor $F_i(q^2)$. The form factors are, of course, analytic functions of q^2 with only a right-hand cut, along which unitarity gives, schematically,

$$\mathrm{Im}F_i(q^2)\propto\textstyle\sum_j T_{ij}(q^2)F_j^*(q^2)\,, \tag{1}$$

where T_{ij} is the N-channel scattering amplitude. If one writes T in the form $T=ND^{-1}$ (N and D are matrices), it is known[6] that the general solution to (1) is

$$F_i(q^2)=\textstyle\sum_j [D^{-1}(q^2)]_{ji}P_j(q^2)\,, \tag{2}$$

where the P_j's are *arbitrary* polynomials. Now if we happen to know the high-energy behavior of D^{-1} and have some principle which determines the asymptotic

[5] The explicit generalization of the conclusions of this paragraph to three-body channels is contained in a recent report by S. Mandelstam (unpublished).

[6] J. Bjorken, Phys. Rev. Letters 4, 473 (1960).

behavior of the F's, then we might be able to conclude, for example, that all the P_i's are constants; i.e., zero-order polynomials. In any case, however, there will be at least N unknown constants in solution (2) and since $\det(D)$ must be nonzero[7] at $q^2=0$, we can always choose the $F_i(0)$, $i=1\cdots N$, to be whatever we wish. To see what this means physically, consider the two $N\bar{N}$ channels mentioned above. Now the $F_i(0)$ for the ${}^3S_1$ and ${}^3D_1$ channels are different linear combinations of the nucleon's charge and magnetic moment, so that one can obtain the form factor once the charge and magnetic moment are given, but this method is not capable, in principle, of predicting the magnetic moment in terms of the charge. We would like to stress that this inability of the form-factor dispersion relations to predict a magnetic moment is not simply a question of subtracted or unsubtracted dispersion relations for the F's; it is due to the fact that the dispersion relation obtained from (1) leads to an integral equation for the F_i's which does not have a unique solution.

While the form-factor equations cannot, by themselves, provide an exact calculation of a magnetic moment, it is easy to see how they could be used with reasonable further assumptions to obtain significant approximate results. Suppose that there is a low-lying stable particle with mass m and the quantum numbers of the photon. In that case, D^{-1} will have a pole at $q^2=m^2$, and using the fact that T_{ij} goes like $g_ig_j/(q^2-m^2)$ near $q^2=m^2$ where g_i is the coupling of the particle to channel i, we find that F_i has a pole of the form

$$F_i(q^2)\sim g_iR/(q^2-m^2),$$

where

$$R=\sum_{ij} g_iP_j(m^2)N_{ij}^{-1}(m^2).$$

Then if (2) is dominated by the pole for small q^2, we have $F_i(0)/F_j(0)\approx g_i/g_j$.

Of course, this still leaves unanswered the question of how to make an exact and unambiguous calculation of a magnetic moment. Evidently, if one is to use only the form-factor equations, he would have to invoke some new principle to determine the P_i's. Instead, we would like in the remainder of this section to show how, by moving from the form-factor equations to the full framework of S-matrix theory, one can determine the P's *without* introducing any new principle.

Basically, the reason why the form-factor equations leave so many parameters undetermined is that they use only analyticity and unitarity in one variable and do not take full advantage of crossing or analyticity in other variables. In order to use these other tools, we shall fix the mass of the photon at some convenient value $\sqrt{(q^2)}$ and, taking a process like $\gamma+N\rightarrow\pi+N$, write dispersion relations in the usual energy variables.

To see how this goes, let us consider a simple but physically interesting example. We will study the $J=\frac{1}{2}^+$ partial wave for the reaction $\gamma+N\rightarrow\pi+N$ with a real photon ($q^2=0$). The photon can be considered as a particle with isospin either one or zero; for simplicity, we choose the latter case. The partial-wave amplitude $h(W)$ is defined by $h(W)=g(W)M_{1-}{}^0/kq$, where W is the total c.m. energy, the magnetic dipole amplitude $M_{1-}{}^0/kq$ is defined as in CGLN,[4] and $g(W)$ is a factor which removes any kinematic singularities of relativistic origin. Denoting the nucleon mass by M, and taking $g(M)=1$, we find that h has a direct-channel pole with residue $-\frac{1}{2}(\mu_p+\mu_n)f$ at $W=M$, where f is the pion-nucleon coupling constant ($f^2\approx0.08$) and μ_p and μ_n are the total nucleon magnetic moments. We now want to write a dispersion relation which will give us $(\mu_p+\mu_n)$.[8] To do so, we multiply $h(W)$ by the denominator function $D(W)$ for π-N scattering in the $J=\frac{1}{2}^+$, $I=\frac{1}{2}$ state, noting[9] that $\lim_{W\rightarrow M}D(W)h(W)=-\frac{1}{2}D'(M)(\mu_p+\mu_n)f$ and that $\mathrm{Im}(Dh)=0$ along the elastic part of the right-hand cut in h, since h has the same phase as πN scattering in this region. Then, assuming that Dh obeys an unsubtracted dispersion relation, we obtain

$$h(W)=\frac{1}{D(W)}\left[\frac{1}{2\pi i}\int_L\frac{D(W')h(W')}{W'-W}dW' +\frac{1}{\pi}\int_{\text{inel. thresh.}}^{\infty}\frac{\mathrm{Im}(D(W')h(W'))}{W'-W}dW'\right] \quad (3)$$

and

$$\mu_p+\mu_n=-\frac{1}{D'(M)}\frac{2}{f}\left[\frac{1}{2\pi i}\int_L\frac{D(W')h(W')}{W'-M}dW' +\frac{1}{\pi}\int_{\text{inel. thresh.}}^{\infty}\frac{\mathrm{Im}(D(W')h(W'))}{W'-M}dW'\right], \quad (4)$$

where the contour L runs around the left cuts[9] in h. Thus we now have a dispersion relation with which to evaluate the isoscalar magnetic moment of the nucleon.

To completely evaluate (4), of course, we have to know all the left cuts in h as well as the inelastic right cut, so that, in principle, we must solve an infinite number of coupled equations like (3) and (4). It is the solution of the whole set of coupled equations which determines the magnetic moment and related parameters. Naturally, we cannot really prove whether or not a unique solution exists, and it might turn out that we have to include some undetermined subtraction constants. Nevertheless, we can discuss some general properties the set of coupled equations would have.

[7] If $\det(D)=0$ at $q^2=0$, there would be a hadron with zero mass.

[8] Here we are using methods developed by R. Dashen and S. Frautschi [Phys. Rev. 135, B1190 (1964) and *ibid.* 137, B1318 (1965)].

[9] Since the photon is massless, baryon exchange in the u channel also produces a pole at $W=M$. The residue of this exchange pole is $-\frac{1}{6}(\mu_p+\mu_n)f$ so that h actually has a pole with total residue $-\frac{2}{3}(\mu_p+\mu_n)f$ at $W=M$. For the purpose of separating direct- and crossed-channel effects, we have imagined that the crossed pole is slightly displaced to the left; it must, of course, be included in the integration over left-hand singularities in Eq. (4).

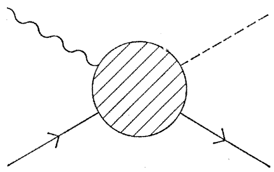

FIG. 1. The photoproduction amplitude $J_{N,\pi N}(0)$. Here, the solid lines are nucleons, the dotted line is a pion, and the wiggly line represents the photon.

To begin with, it is clear that we could have considered the reaction $\gamma+N\to\pi+N$ with a virtual γ of mass $\sqrt{(q^2)}$. Then the left-hand side of (4) would be the isoscalar magnetic form factor of the nucleon instead of its $q^2=0$ value, the magnetic moment. Evidently, for each fixed q^2, we can write a set of equations like (4). Next we need some new notation. Any scattering amplitude for a process like (photon+hadrons) $\to$ (hadrons) is really a matrix element of the electromagnetic current $J^\nu(x)$. For example, the amplitude for $\gamma+N\to\pi+N$ is proportional[10] to $\langle N|J^\nu(0)|\pi N\rangle$ and the nucleon magnetic moment is contained in $\langle N|J^\nu(0)|N\rangle$. Let us, then, denote the amplitude for an arbitrary process $\gamma+a\to b$ by $J_{ab}{}^\nu(q^2)$ which will, apart from kinematic factors, be equal to $\langle b|J^\nu(0)|a\rangle$. We allow a and b to represent any configuration of strongly interacting particles and, as usual, $\sqrt{(q^2)}=[(p_b-p_a)^2]^{1/2}$ is the mass of the (in general, virtual) photon.

Now let us suppose that we know enough about dispersion relations so that, keeping q^2 *fixed* and thinking of $J_{ab}(q^2)$ as a scattering amplitude, we are able to write some kind of generalized Mandelstam representation and/or generalized ND^{-1} equations for all the amplitudes $J_{ab}{}^\nu(q^2)$. The spectral functions or discontinuities in the dispersion relations will be determined by unitarity requirements on J^ν. Since we are working to first order in electromagnetism, the unitarity equation is always *linear* in J. Thus, it is clear that the result of writing all the (fixed q^2) dispersion relations for J will be a set of coupled linear equations which we symbolize by

$$J_{ab}{}^\nu(q^2)=\sum_{cd} X_{ab,cd}(q^2)J_{cd}{}^\nu(q^2)+C_{ab}{}^\nu(q^2)\,, \tag{5}$$

where the C's represent the possibility that our dispersion relations contain undetermined subtraction constants. In Eq. (5), one is to think of the left-hand side as the amplitude for which a dispersion relation is written, while the $J_{cd}{}^\nu$ appearing on the right are amplitudes which, because of crossing and unitarity, appear inside the dispersion integral. In most applications, J_{ab} will refer to residues of a direct-channel pole, like $\mu_p+\mu_n$ in Eq. (4), and $\sum X_{ab,cd}J_{cd}$ will be dominated by the contribution of one-particle exchange singularities, such as nucleon or ρ-meson exchange in the example of Eq. (4).

To take a specific example, the familiar dispersion-theoretic calculations of photoproduction are just the application of a truncated version of (5). In the present language, these calculations correspond to taking $J_{ab}{}^\nu=J_{N,\pi N}{}^\nu$ which represents the process $\gamma+N\to\pi+N$ (Fig. 1), and then truncating the sum over cd to include, say, $J_{NN}{}^\nu$, $J_{N,\pi N}{}^\nu$, and $J_{\pi,\pi\pi}{}^\nu$ which show up when one applies unitarity in the cross channels, as displayed in Fig. 2. On the other hand, the residue of the direct-channel nucleon pole (Fig. 3), which contains the nucleon magnetic moment, is not fed into the left side of the equation but is *determined* by the integration over the crossed-channel singularities. The fact that dispersion relations for scattering amplitudes can be used to determine pole parameters is a familiar circumstance in purely strong-interaction calculations and is, indeed, the basis of the whole bootstrap hypothesis. In the present paper we wish to emphasize that similar methods can be used to determine the weak and electromagnetic parameters of hadrons.

Since we can determine the pole parameters for the weak and electromagnetic interactions of hadrons, we now have a means for determining all the J's for any fixed q^2. Specifically, if we put $q^2=0$, we have achieved our goal of writing an equation which will determine parameters like the nucleon magnetic moments. What is not determined by the linear Eq. (5) is the over-all scale of J at each q^2. However, the ordinary form-factor equations fix the value of J for all q^2 given the J's at, say, $q^2=0$; thus we can determine all the amplitudes $J_{ab}(q^2)$ up to one constant scale factor.

Now it may appear that we have not really determined the magnetic moments at all, because of the "subtraction terms" $C_{ab}(q^2)$ which we have for the time being included on the right-hand side of (5). With respect to this problem, we shall argue in the present section that the C's may vanish if the strong interactions were fully bootstrapped, and we shall show in the next section that the C's are not important anyway for fixing ***ratios*** of moments if the X matrix, as seems to be the case, has an eigenvalue near one.

In discussing the subtraction terms, the first point to keep in mind is that we are keeping the photon mass fixed and treating the amplitude $J_{ab}{}^\nu$ for $\gamma+a\to b$ in much the same way that one treats a purely strong-interaction amplitude; essentially, the only difference is our case of a linear unitarity condition. A typical amplitude would be $\gamma+N\to\pi+N$, and the subtrac-

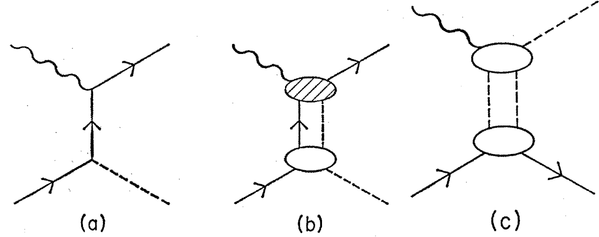

FIG. 2. Contributions to the right-hand side of Eq. (5) in a calculation of $J_{N,\pi N}(0)$.

[10] Translational invariance requires that $\langle a|J(x)|b\rangle$ is related to $\langle a|J(0)|b\rangle$ by a phase according to $e^{i(p_a-p_b)\cdot x}\langle a|J(0)|b\rangle=\langle a|J(x)|b\rangle$. It is conventional to work with $\langle a|J(0)|b\rangle$.

tions will as usual be determined by the high-energy behavior. Now the high-energy behavior of a scattering amplitude is largely determined by the spin properties of internal particles (i.e., particles which can be exchanged in the reaction). Since we are working to first order in electric charge, all the internal particles in our amplitudes are hadrons and it would seem, then, that the high-energy behavior of $J_{ab}{}^{\nu}$ should be similar to that of a strong-interaction amplitude.

There is, however, an apparent loophole in this argument which we must consider. Mandelstam[11] has shown that an elementary external particle with nonzero spin can sometimes lead to undetermined subtraction terms. In particular, an elementary photon might lead to an undetermined constant term in the amplitude. We argue in Appendix A, however, that the undetermined subtraction does not appear if the dispersion relations for the form factors of the photon couplings converge. Once again, then, the high-energy behavior of photon emission is related to the high-energy behavior of the hadron amplitudes.

Thus one would expect to have undetermined subtractions[12] C_{ab} to more or less the same extent as in the purely strong-interaction bootstrap. If continuation in angular momenta or some other device allows us to remove all the subtractions in the strong interactions, then it should be possible to remove all, or most of the C's in Eq. (5). In this connection we might remark that in the most extensively studied case, namely the photoproduction amplitude $J_{N,\pi N}$, subtractions do not seem to be important, at least at low energies.

We conclude this section with some comments about what kinds of terms appear in J, and which of them are interrelated by X. To begin with, we need not, of course, restrict ourselves to vector currents like the electromagnetic case described above; the treatment of scalar, tensor, and higher rank currents would proceed in the same way. In some cases, a current will contain more than one "spin," i.e., it may contain two or more pieces which transform separately under Lorentz transformation. For example, the weak axial-vector current $J_5{}^{\nu}$ can be broken up into spin-zero and spin-one parts by projecting out the divergence according to $J_5{}^{\nu}=J_5{}^{\nu}-(q^{\nu}/q^2)(q^{\mu}J_5{}^{\mu})+(q^{\nu}/q^2)(q^{\mu}J_5{}^{\mu})=J_{5,1}{}^{\nu}+(q^{\nu}/q^2)J_{5,0}$. As long as $q^2\neq0$ and the above decomposition is meaningful, it is clear that since crossing and unitarity do not mix $J_{5,1}{}^{\nu}$ and $J_{5,0}$, X will not have any elements connecting $J_{5,1}{}^{\nu}$ and $J_{5,0}$. In general, then, one can break an arbitrary current up into pieces with definite "spin" and write a separate X-matrix equation for each piece, except for $q^2=0$ where special care may be necessary.[13]

[11] S. Mandelstam, Nuovo Cimento 30, 1113, 1127 (1963).

[12] In this connection, it should be understood that as a matrix, X contains continuous as well as discrete indices and the sum over cd actually implies integrations over momenta. It can turn out that some of the integrations appear to diverge, which means the equation would need a subtraction C, but that actually the divergence can be removed and C determined by a suitable analytic continuation. In this case, we would not write an explicit term C on the right-hand side of (5), but include the continuation process in the definition of X. [Note that in any circumstance (5) always remains a linear equation for J.]

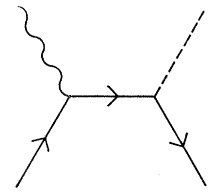

FIG. 3. The direct-channel nucleon pole which appears on the left-hand side of Eq. (5) in a calculation of $J_{N,\pi N}(0)$.

Since in our linear formalism we are including no weak or electromagnetic effects in X, the X matrix has all the symmetries characteristic of the strong interaction; e.g., X is a scalar under parity and charge conjugation as well as isospin rotation, and it conserves strangeness. Thus, if we can assign to currents quantum numbers like C, P, total isospin I, I_3, and so on, which are conserved by the strong interaction, X will not connect currents with different quantum numbers. To the extent that $SU(3)$ is a good symmetry of hadrons, X will also not connect currents which belong to different $SU(3)$ representations. Generally speaking, then, one can label currents according to the quantum numbers, "spin," parity, strangeness, etc., characteristic of a strongly interacting particle (q^2 corresponds to the mass of the particle) and treat currents with different quantum numbers separately. Furthermore, since X is an isotopic scalar, the X matrices for currents which differ only by an isotopic rotation will be identical. Similarly, X will be approximately the same for currents which differ only by an $SU(3)$ rotation.

III. THE EIGENVALUES OF X AND THEIR EFFECT ON WEAK AND ELECTROMAGNETIC CURRENTS

In theoretical studies of the properties of currents, as well as in practical calculations, the eigenvalues of X are of particular interest. To begin with, it is obvious that the C's cannot be zero unless X has an eigenvalue exactly equal to one for all q^2. Let us, however, work with the situation when $C\neq0$; as we proceed, it will be clear what happens in the limit $C=0$.

For simplicity, let us suppose that we are dealing with a scalar current J and suppose that the X matrix associated with J has a complete set of eigenvectors $f_{ab,\alpha}(q^2)$, $\alpha=1, 2, \cdots$ satisfying

$$\sum_{cd} X_{ab,cd}(q^2)f_{cd,}\ (q^2)=X_{\alpha}(q^2)f_{ab,\alpha}(q^2).$$

Now X is not, in general, symmetric so the f's need not be orthogonal, but if they form a complete set[14] we can always find a set of vectors $g_{ab,\beta}(q^2)$, $\beta=1, 2, \cdots$ which have the property that $\sum_{ab} g_{ab,\beta}f_{ab,\alpha}=\delta_{\alpha\beta}$ and $\sum_{\alpha} g_{ab,\alpha}f_{cd,\alpha}=\delta_{ac}\delta_{bd}$. Then, defining

$$J_{\alpha}(q^2)=\sum_{ab} g_{ab,}\ (q^2)J_{ab}(q^2)$$

[13] The special problems which arise in the case $q^2=0$ will be discussed in a future paper.

and

$$C_\alpha(q^2)=\sum_{ab} g_{ab,\alpha}(q^2)C_{ab}(q^2),$$

Eq. (5) becomes

$$J_\alpha(q^2)=x_\alpha(q^2)J_\alpha(q^2)+C_\alpha(q^2) \tag{6}$$

with the solution

$$J_\alpha(q^2)=[1-X_\alpha(q^2)]^{-1}C_\alpha(q^2), \tag{7}$$

$$J_{ab}(q^2)=\sum_\alpha f_{ab,\alpha}(q^2)J_\alpha(q^2). \tag{8}$$

From the form of (7), it is clear that eigenvalues X_α which are near unity are particularly interesting; the corresponding J_α's will be large and J_{ab} will be, approximately, a linear combination of these enhanced eigenvectors, more or less independently of how the C's are introduced. For example, if for some value of q^2, say q_0^2, X_1 is nearly equal to one and all the other X's are far from unity, we will have

$$J_{ab}(q_0^2)\approx f_{ab,1}(q_0^2)[1-X_1(q_0^2)]^{-1}C_1(q_0^2) \tag{9}$$

unless, for some reason, $C_1(q_0^2)$ happens to be very small compared to the remaining C's.

From the above discussion, one sees that if there are one or more eigenvalues X_i sufficiently close to unity, then the self-consistency properties of the strong interaction, as expressed in the X matrix, come close to determining the matrix elements of the current J. The extreme situation appears if the C's are zero. In this case, there must be at least one X exactly equal to unity, and self-consistency completely determines the possible J's.

From a philosophical point of view, there is a discontinuous transition between the two possibilities of $C\neq 0$ and $C=0$. However, the methods presently available for the study of strong interactions are totally incapable of determining whether or not some X's are exactly 1. From a practical point of view, then, there is a continuous transition from $X=1$ and $C=0$ to $X\approx 1$ and $C\neq 0$ (but small compared to J). In the former case, the current J is completely determined by self-consistency while in the latter case, self-consistency determines the current, loosely speaking, up to terms of order $(1-X)$ with $X\approx 1$. (We ignore, as unlikely and ugly, the possibility that the C's are chosen such that the enhanced C_α's vanish.)

All this discussion is, of course, academic unless we know that X does have some eigenvalues near one. In the next section we will show, in very approximate but physically reasonable models, that there are X's near one, so it seems likely that self-consistency plays a major role in shaping the weak and electromagnetic interactions of hadrons.

At this point it is perhaps worth emphasizing that since $X(q^2)$ varies with q^2, so will the eigenvalues $X_\alpha(q^2)$ and eigenvectors $f_\alpha(q^2)$. If we have $C=0$, then some $X(q^2)$ must be one *for all* q^2, but even in this case the corresponding eigenvector will vary with q^2.

In Sec. II it was pointed out that if a meson state produces a strong pole in the form factors associated with a current J, then one knows that near the pole $J_{ab}(q^2)$ follows a pattern determined by the meson couplings. We would now like to show how this follows directly from the properties of X. To this end, we suppose that there is a hadron, call it Z, with the same quantum numbers as the current under consideration. The amplitudes $J_{ab}(q^2)$ will then have a pole of the form $\lambda_Z t_{ab,Z}/(q^2-m^2)$ at $q^2=m^2$ where $t_{ab,Z}$ is the strong-interaction scattering amplitude for $a\to b+Z$ and λ_Z is a scale factor which measures the strength of the transition $Z\to$ (current). Now the X matrix, which represents the effect of one J on another, remains finite as $q^2\to m^2$, and we shall suppose that there are no poles in the subtractions C; the latter would correspond to the introduction of a "bare" or "elementary" transition mass for a process like $\rho^0\to\gamma$. Then letting q^2 approach m^2 in (5) and picking out the pole, we find

$$t_{ab,Z}=\sum_{cd} X_{ab,cd}(m^2)t_{cd,Z}, \tag{10}$$

so that $t_{ab,Z}$ is an eigenvector of $X(m^2)$ with eigenvalue exactly equal to one.

For $q^2\neq m^2$, $t_{ab,Z}$ is, of course, *not* an exact eigenvector of $X(q^2)$, but for $q^2\approx m^2$ there must be, for continuity reasons, an eigenvalue of $X(q^2)$ either equal or very nearly equal to unity (equality is required if $C=0$) and the associated eigenvector must be, approximately, proportional to $t_{ab,Z}$. Thus, when q^2 is near m^2, the solution to (5) will have the form

$$J_{ab}(q^2)\approx j(q^2)t_{ab,Z}, \tag{11}$$

where $j(q^2)$ is a proportionality factor which will be approximately equal to $\lambda_Z(q^2-m^2)^{-1}$. We see, then, that the "pole dominance" methods which are often used to study the weak and electromagnetic properties of hadrons are equivalent to finding *some* of the eigenvalues of X which are near unity. It is, however, not obvious that an eigenvalue close to unity must necessarily come from a pole in $J(q^2)$ (see, for example, the treatment of magnetic moments in the next section), so the direct study of X should be a more general and powerful method.

IV. SOME EVIDENCE THAT X HAS EIGENVALUES CLOSE TO UNITY

This section is devoted to the results of some very approximate but highly encouraging calculations based on the formalism developed in the preceding section. In our first example, we discuss the magnetic moments of baryons and the magnetic and quadrupole parts of the $(J=\frac{3}{2}^+)$ resonance-baryon-photon vertex. In both $SU(2)$ and $SU(3)$ symmetric calculations, it turns out that the experimental magnetic and quadrupole moments closely follow the pattern suggested by the enhanced eigenvector, indicating that self-consistency of the strong interactions does play a major role in shaping the electromagnetic interactions of hadrons. Then we generalize by considering, instead of just the electric current, the couplings of arbitrary currents to the baryons and resonances. Here it turns out that all the observed weak and electromagnetic couplings of these particles correspond to eigenvalues of X near unity.

In order that the general pattern of the results presented here will not be obscured by a cloud of computational details, we will, in this section, simply state the results with a few brief discussions of the techniques and approximations involved in the calculations. The essential ingredients of the computations and some discussion of the approximations are given in Appendix B.

We now proceed to our examples.

Consider the piece of X which connects the amplitudes $J_{NN}{}^{\nu}(0)$ and $J_{NN^*}{}^{\nu}(0)$ where J^{ν} is the electromagnetic current and N and N^* are the nucleon and (3,3) resonances. We have four unknowns: μ_s and μ_v, the isoscalar and isovector total magnetic moments of the nucleon, and μ^* and q^*, the magnetic dipole and electric quadrupole parts of the amplitude $J_{NN^*}{}^{\nu}(0)$. (In the static approximation which we will employ, the charge coupling contained in J_{NN} completely decouples from these other quantities, and we shall not consider it.)

To calculate the relevant elements of X, we consider N and N^* as bound and resonant states in the πN system and write an equation like (4) for each of the quantities μ_s, μ_v, μ^*, and q^*. Assuming that the dispersion integrals converge rapidly, we keep only the nearest singularities, which are the "short cuts" (i.e., static poles) due to N and N^* exchange in the cross channels. Then, assuming that the D functions are roughly linear in the region $|W-M|\ll M$, we obtain the equations

$$\begin{pmatrix}\mu_s\\ \mu_v\\ \mu^*\\ q^*\end{pmatrix}=(X)\begin{pmatrix}\mu_s\\ \mu_v\\ \mu^*\\ q^*\end{pmatrix}=\begin{pmatrix}-\frac{1}{3} & 0 & 0 & 0\\ 0 & \frac{1}{9} & 16r/9 & 0\\ 0 & 4/9r & \frac{1}{9} & 0\\ 0 & 0 & 0 & \frac{1}{3}\end{pmatrix}\begin{pmatrix}\mu_s\\ \mu_v\\ \mu^*\\ q^*\end{pmatrix}, \quad (12)$$

where r is the ratio f^*/f of the πNN^* and πNN couplings. For r, we take the static-model result $r=(\sqrt{2})^{-1}$ which agrees well with experiment. The absence of terms connecting μ_s to μ_v, μ^*, and q^*, is of course, due to the fact that μ_s has $I=0$ while the latter quantities have $I=1$. The terms connecting q^* to μ^* and μ_v are zero only in the present approximation of keeping only the "short cuts."

It is now trivial to find the eigenvectors f_i and eigenvalues x_i, $i=1\cdots 4$, which are

$$\begin{aligned} &x_1=-\tfrac{1}{3},\quad f_1=\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\quad x_2=\tfrac{1}{3},\quad f_2=\begin{pmatrix}0\\0\\0\\1\end{pmatrix},\\ &x_3=1,\quad f_3=\begin{pmatrix}0\\2r\\1\\0\end{pmatrix},\quad x_4=-7/9,\quad f_4=\begin{pmatrix}0\\2r\\-1\\0\end{pmatrix}.\end{aligned}\quad (13)$$

Evidently X has, in this approximation, one eigenvalue X_3 exactly equal to unity. In a better treatment of the problem, we would expect the eigenvalue spectrum of X to remain qualitatively the same as that given in (13), i.e., have a single eigenvalue near one and the remaining eigenvalues small or negative. We can then predict that $J_{NN}{}^{\nu}(0)$ and $J_{NN^*}{}^{\nu}(0)$ will lie mostly along f_3, which gives $\mu \approx\sqrt{2}\mu^*$, $\mu_s\ll\mu$, and $q^*\ll\mu_v$. All these predictions are in agreement with experiment[15]: the relations $\mu_v=\sqrt{2}\mu^*$ and $q^*\ll\mu_v$ are in good agreement with photoproduction data, and μ_s/μ_v is experimentally about 20%.

In deriving the above results, we completely ignored the contribution to the dispersion integrals for μ_s, μ_v, μ^*, and q^* arising from processes such as ω-meson exchange. This is the same, of course, as ignoring the elements of X which couple J_{NN} and J_{NN^*} to amplitudes like $J_{\omega\pi}$. In Appendix B, it is argued that the inclusion of terms like those connecting J_{NN} to $J_{\omega\pi}$ would not greatly change the results obtained above.

Next we can try extending our calculation to the $SU(3)$ problem of calculating the D and F magnetic moments μ_D and μ_F of the baryon octet B and the couplings u^* and q^* which appear at the γBB^* (B^*= decuplet) vertex.[16] (Here we are assuming that the electromagnetic current is part of an octet.) We consider B and B^* as bound and resonant states in the ΠB channels ($\Pi=0^-$ octet) and proceed as before. This time, however, the problem is complicated by the fact that the two octet channels in which B appears form a coupled two-channel problem and the results depend, to some extent, on how one wishes to treat the coupled channels. Methods for dealing with such multichannel problems are given in Appendix B and in Refs. 9 and 17; here we simply state the results found in Appendix B. For any choice of the F/D ratio at the strong $BB\Pi$ vertex in the usual range of $\frac{1}{3}$ to $\frac{2}{3}$, and more or less independently of how one chooses to handle the two-channel octet problem, the X matrix connecting μ_D, μ_F, μ^*, and q^* has one eigenvalue near unity, lying between about 0.8 and 1.2, while the remaining eigenvalues are far from one. The eigenvector associated with the $x\approx 1$, gives: (i) $q^*\approx 0$, (ii) μ_F/μ_D between about $\frac{1}{2}$ and $\frac{3}{4}$, in agreement with the ratio of the neutron and proton magnetic moment, and (iii) $\mu^*/(\mu_D+\mu_F)=\frac{4}{5}$ to 1 which is about what is needed to keep intact our previous relation between the nucleon moment and NN^* transition moment.

[14] Some asymmetric matrices do not possess a complete set of eigenvectors. However, if X is in this category, one can convince himself that conclusions essentially the same as those in the text would still obtain.

[15] In a recent unpublished report, A. Abarbanel, C. Callan, and D. Sharp discuss a calculation of μ_s and μ_v using methods similar to those advocated here. Whereas we are basing our conclusions on rough self-consistency arguments, these authors carry out a detailed evaluation of the dispersion relations for μ_s and μ_v. Their conclusions are in substantial agreement with ours.

[16] This approach to the μ_D/μ_F ratio and the resulting conclusions have been previously discussed in R. Dashen, Phys. Letters **11**, 89 (1964).

The results of all these simple calculations are very encouraging. Clearly, we cannot produce highly accurate predictions to compare with experiment, but there does seem to be an indication that self-consistency, as expressed by the eigenvalues of X, plays a major role in determining the electromagnetic parameters of the low-lying baryons and resonances.[17]

Next let us try something more daring: We will now investigate, in the same approximation, the X matrix which connects the BB and BB^* matrix elements of an *arbitrary* current. Our aim is to see if all the known weak and electromagnetic interactions of these particles correspond to enhanced eigenvectors of X.

We are led, then, to consider the B-B and B-B^* matrix elements of an arbitrary current. We will assume that $\sqrt{(q^2)}$ is nonzero but is small compared to the mass of the baryons, and that the current has a definite spin S and belongs to a definite representation of $SU(3)$ denoted by N, $N=1, 8, \overline{10}, \cdots$ [throughout this subsection we assume $SU(3)$ symmetry for the strong interactions]. In our dispersion-theoretic approach it is convenient to imagine that a particle, call it θ, is coupled weakly to the current under consideration; we can then think of the B-B and B-B^* matrix elements of the currents as amplitudes for $B\to B+\theta$ and $B^*\to B+\theta$ where θ has mass $\sqrt{(q^2)}$ and spin S. In general, there will be several different kinds of couplings at the $BB\theta$ and $B^*B\theta$ vertices corresponding to the various ways of coupling the spins and orbital angular momentum of the particles; we denote these by G_{SLK} for $B\to B+\theta$ and G_{SLK}^* for $B^*\to B+\theta$, where S is the spin of θ, L is the orbital angular momentum of θ in the center-of-mass frame, L and S are coupled together to give angular momentum K, and K is in turn coupled to the spin of the baryon to give a total angular momentum of $\frac{1}{2}$ for G and $\frac{3}{2}$ for G^*.

Conservation of angular momentum requires $K=0$ or 1 for G and $K=1$ or 2 for G^*. Similarly, we consider only L values such that, taking account of the parity of the current, the process $B(B^*)\to B+\theta$ is parity-allowed. Finally, we add a superscript N corresponding to the $SU(3)$ representation to which θ belongs, obtaining[18] G_{SLK}^{*N}, $N=8, 27, \cdots$ and G_{SLK}^{N}, $N=1, 8_D, 8_F, 27, \cdots$, where we use the notation 8_D and 8_F to specify the D and F couplings of an octet θ at the vertex $BB\theta$. As an example of this notation, the magnetic and quadrupole matrix elements of the electromagnetic current discussed above are given by $\mu_D=G_{111}^{8D}$, $\mu_F=G_{111}^{8F}$, $\mu^*=G_{111}^{*8}$, and $q^*=G_{112}^{*8}$.

Now let us calculate the X matrix in the same approximation as before. For a given N and S, we consider the reaction $\theta+B\to B+\Pi$, and projecting out the partial wave corresponding to a given L and K, we write a dispersion integral just like (4) for each of the couplings. Again we assume that the dispersion integrals converge rapidly, and keep only the "short cuts" or "static poles" corresponding to B and B^* exchange in the u channel. As indicated in Appendix B, the approximate X matrix obtained in this manner does not connect G's with different L or K, and recalling that K must be 0, 1, or 2, we obtain

$$\begin{pmatrix} G_{SL1}^{*N} \\ G_{SL1}^{N} \end{pmatrix} = (X_1^N)\begin{pmatrix} G_{SL1}^{N} \\ G_{SL1}^{*N} \end{pmatrix}, \quad N\neq 8 \quad K=1 \tag{14a}$$

$$\begin{bmatrix} G_{SL1}^{*8} \\ G_{SL1}^{8D} \\ G_{SL1}^{8F} \end{bmatrix} = (X_1^8)\begin{bmatrix} G_{SL1}^{*8} \\ G_{SL1}^{8D} \\ G_{SL1}^{8F} \end{bmatrix}, \quad N=8 \quad K=1 \tag{14b}$$

$$G_{SL2}^{*N}=X_2^N G_{SL2}^{*N}, \quad K=2 \tag{14c}$$

$$G_{SL0}^{N}=X_0^N G_{SL0}^{N}, \quad K=0 \quad N\neq 8 \tag{14d}$$

$$\begin{pmatrix} G_{SL0}^{8D} \\ G_{SL0}^{8F} \end{pmatrix} = (X_0^8)\begin{pmatrix} G_{SL0}^{8D} \\ G_{SL0}^{8F} \end{pmatrix}, \quad K=0 \quad N=8 \tag{14e}$$

where the matrices X_K^N have the dimension indicated by the vectors they act on and we have suppressed any subtractions C.

In the present approximation, the matrices X_K^N appearing in Eq. (14) are independent of S and K and, for q^2 small compared to the baryon mass, are independent of q^2. They do, however, depend on N and the F/D ratio λ for the strong $BB\Pi$ couplings. We take λ in the usual

[17] A further test of the hypothesis that electromagnetic interactions follow enhanced eigenvectors of X would be provided by calculations involving the electromagnetic parameters of mesons. Unfortunately, the presently available dynamical models of mesons are less reliable than the models employed here for the baryons. About all one can do under these circumstances is to look at the sign and order of magnitude of some typical contributions to X and see if conditions are favorable for generating eigenvalues near one. We have looked at two examples: (i) the $\rho\pi\gamma$ coupling μ_ρ obtained from the ρ pole in $\gamma+\pi\to\pi\pi$ with the cross-channel cuts coming from ρ exchange. Here, X is just a number, i.e., $\mu_\rho=X_{\rho\rho}\mu_\rho$, so the presence of an eigenvalue near one corresponds to $X_{\rho\rho}\approx 1$; (ii) the analog of (i) in $SU(3)$. In both these cases, the signs and order of magnitude suggest that there are again eigenvalues near unity.

[18] The relative normalization of the couplings G_{SLK}^{N} and G_{SLK}^{N*} are defined as follows. We set the amplitude for the process $B^*\to B+\theta$ equal to

$$G^{*N}\begin{pmatrix} 8 & N & 10 \\ B & \theta & B^* \end{pmatrix} C(\tfrac{3}{2}m_{B^*},\tfrac{1}{2}m_B,Km_\theta)$$

where

$$\begin{pmatrix} 8 & N & 10 \\ B & \theta & B^* \end{pmatrix}$$

is an $SU(3)$ Clebsch-Gordan coefficient as defined by de Swart, and $C(\frac{3}{2}m_{B^*},\frac{1}{2}m_B,Km_\theta)$ is the angular momentum Clebsch-Gordan coefficient for coupling K and $\frac{1}{2}$ to $\frac{3}{2}$ with magnetic quantum numbers m_θ, m_B, and m_{B^*}. For $N\neq 8$, the amplitude for $B'\to B+\theta$ is given by

$$G^N\begin{pmatrix} 8 & N & 8 \\ B & \theta & B' \end{pmatrix} C(\tfrac{1}{2}m_{B'};\tfrac{1}{2}m_B,Km_\theta)$$

and for $N=8$ the amplitude for $B'\to B+\theta$ is defined as

$$\left[2(5/3)^{1/2}G^D\begin{pmatrix} 8 & 8 & 8_s \\ B & \theta & B' \end{pmatrix}+2\sqrt{3}G^F\begin{pmatrix} 8 & 8 & 8_A \\ B & \theta & B' \end{pmatrix}\right] \times C(\tfrac{1}{2}m_{B'};\tfrac{1}{2}m_B,Km_\theta).$$

With these definitions, one can verify that the magnetic moments satisfy

$$\mu_V:\mu_{NN^*}{}^* = (G_{111}^D+G_{111}^F):G_{111}^{*8}/\sqrt{2}.$$

range $\frac{1}{3}$ to $\frac{2}{3}$, and use the methods outlined in Appendix B for calculating these matrices.

It turns out that the one-by-one matrices $X_2{}^N$ for all N and $X_0{}^N$ for $N\neq 1$ or 8 are never close to unity, and the two-by-two matrix $X_1{}^N$ for $N\neq 8$ does not have eigenvalues close to one. On the other hand, $X_0{}^1$ is about one and the matrix $X_0{}^8$ has an eigenvalue somewhere around 0.7 to 0.85 whose associated eigenvector corresponds to $G_{SL0}{}^{8D}/G_{SL0}{}^{8F}=\frac{1}{4}$ to $\frac{1}{5}$. Finally, the 3×3 matrix $X_1{}^8$ is the same as the matrix discussed in the magnetic moment example and we know that it has a single eigenvalue near unity corresponding to $G_{SL1}{}^{8D}/G_{SL1}{}^{8F}=1.5$ to 2.0 and $G_{SL}{}^{*8}/(G_{SL1}{}^{8D}+G_{SL1}{}^{8F})\approx\frac{4}{5}$ to 1. Thus, we conclude that for any S and L, and more or less independently of how the subtractions C might be inserted in our dispersion integrals, the following pattern of coupling will emerge: (i) For $N\neq 8$, all the G's will be small except for $G_{SL0}{}^1$ which can be large; (ii) For $N=8$, we will have $G_{SL2}{}^{*8}\approx 0$ and $G_{SL0}{}^8$, $G_{SL1}{}^8$, $G_{SL1}{}^{*8}$ large and in the ratios $G_{SL0}{}^{8D}/G_{SL0}{}^{8F}\approx\frac{1}{4}$, $G_{SL1}{}^{8D}/G_{SL1}{}^{8F}\approx 1.5$ to 2.0, and $G_{SL1}{}^{*8}/(G_{SL1}{}^{8D}+G_{SL1}{}^{8F})\approx\frac{4}{5}$ to 1. These results are summarized in Table I.

Now let us see how well these predictions agree with experiment. In the first place, the calculation predicts that aside from the special case of the $G_{SL0}{}^1$ coupling, currents with $N\neq 8$ should, if they exist, have small matrix elements. This is in agreement with the fact that the observed weak and electromagnetic currents seem to belong to octets.[19] Next, with regard to the electric current, we recall that the predictions for the quadrupoles and magnetic moments are in good agreement with experience. Also, for the electric current, where the couplings $G_{110}{}^{8D}$ and $G_{110}{}^{8F}$ correspond to the electric form factors $G_E(q^2)$ of baryons, the result that $G_{SL0}{}^{8D}$ is small compared to $G_{SL0}{}^{8F}$ agrees with the fact that the electric form factor of the neutron seems to be very small. Similar conclusions hold for the vector part of the weak current. To see how our conclusions work for the axial-vector part of the weak current, let us recall that the general form of the B-B matrix element of a $(CP=+1)$ axial-vector current is $\bar{u}(\gamma_5\gamma^\nu F_1(q^2)+q^\nu\gamma_5 F_2(q^2))u$, where F_1 and F_2 are form factors. In the present notation, F_2 corresponds to a term $G_{011}{}^8$ and F_1 is a combination of the couplings $G_{011}{}^8$, $G_{101}{}^8$, and $G_{121}{}^8$. Since K is the same for both F's, we predict that the D/F ratio for either of the couplings F_1 and F_2 will be about 1.5 to 2.0, in agreement with recent experimental analyses of the beta decay of hyperons. We can also predict the matrix elements of the axial-vector current between B and B^*. Here there are four couplings $G_{011}{}^{*8}$, $G_{101}{}^{*8}$, $G_{121}{}^{*8}$, and $G_{122}{}^{*8}$, and we have the relations $G_{122}{}^{*8}\approx 0$ and $G_{SL1}{}^{*8}\approx(G_{SL1}{}^{8D}+G_{SL1}{}^{8F})$ for $S=0$, $L=1$, and $S=1$, $L=0$, 2. These relations can be tested[20] in leptonic decay of the Ω^- and in reactions like $\nu+p\to N^*+\mu$. Experimental tests of these predictions will provide a good check of our hypothesis that the weak interaction follows an enhanced eigenvector of X.

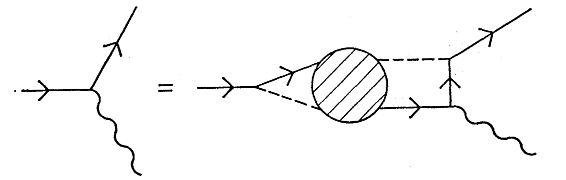

FIG. 4. Diagramatic representation of Eq. (14) involving $X_0{}^N$. The solid lines are baryons, the dashed line is a pseudoscalar meson, and the wiggly line represents any θ particle.

One will note that our predicted F/D ratio for the B-B magnetic moments and for the B-B matrix elements of the axial-vector currents is, for practical purposes, the same as the F/D ratio λ for the strong ΠBB vertex which we used to calculate the X matrix. The reason for this is easily explained. As pointed out in the last section, the amplitudes $t_{ab,\Pi}$ for the process $a\to b+\Pi$ satisfy $t_{ab,\Pi}=\sum_{cd}X_{ab,cd}(m_\Pi{}^2)t_{cd,\Pi}$; thus if our model of B and B^* as bound and resonant states of ΠB is self-consistent, the ΠBB and ΠBB^* couplings must form an eigenvector of $X(m_\Pi{}^2)$ whose eigenvalue is exactly one.

Noting that the processes $B\to B+\Pi$ and $B^*\to B+\Pi$ occur with the Π in an $L=1$ state, we label the Π couplings with the suggestive notation $g_{011}{}^{8D}$, $g_{011}{}^{8F}$, and $g_{011}{}^{*8}$, and conclude that if our model is consistent we must have

$$\begin{pmatrix} g_{011}{}^{8D} \\ g_{011}{}^{8F} \\ g_{011}{}^{*8} \end{pmatrix} = (X_1{}^8)\begin{pmatrix} g_{011}{}^{8D} \\ g_{011}{}^{8F} \\ g_{011}{}^{*8} \end{pmatrix}, \tag{15}$$

where we have used the facts that in our approximation X is independent of S, L, and q^2. In fact, the model is self-consistent and (15) will hold identically if one takes full account of the complications introduced by the circumstance that the $J=\frac{1}{2}^+$ octet channels in $\Pi B\to\Pi B$, where the baryons appear as bound states, form a true two-channel problem.[16] When the coupled-channel aspects of the problem are neglected, Eq. (15) is no longer exactly true, but the enhanced eigenvector of $X_1{}^8$ lies mostly along the vector formed from the Π couplings. It is clear, then, that for all S and L the ratios among the enhanced couplings $G_{SL1}{}^{8D}$, $G_{SL1}{}^{8F}$, and $G_{SL1}{}^{*8}$ will be the same as the ratios of the respective Π couplings. For the special case of B-B matrix elements of the axial-vector current, one could have easily obtained this result from an $SU(3)$ generalization of the Goldberger-

[19] In some models based on more than one fundamental triplet, the electromagnetic current has an $SU(3)$ singlet piece. It has been pointed out [M. Nauenberg, Phys. Rev. 135, B1047 (1964)] that this would introduce an $SU(3)$ singlet piece into the baryon magnetic moments. However, since we have an eigenvalue of $X\approx 1$ for the octet but not for the singlet moments, the octet term will dominate dynamically in any case and the singlet term will be small.

[20] Using similar methods, G. Chapline (to be published) and Y. Dothan (to be published) have carried out more detailed studies of these experimental situations.

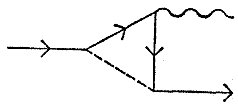

FIG. 5. A triangle diagram which has the same group-theoretic properties as the right-hand side of Fig. 4.

Treiman relation.[21] However, it is not clear how a similar argument based on "pole dominance" of form factors could give a simple derivation of the fact that the magnetic moments have the same D/F ratio as the Π couplings.

In the preceding paragraph, we "explained" the unit eigenvalue of X_1^8 by relating it to the unit eigenvalue of X associated with the pseudoscalar meson couplings. The near-unity eigenvalue of X_0^8 can also be related to another phenomena, namely octet enhancement in the baryon mass splittings.[22] To see this, it is easiest to think of Eq. (14e) graphically as shown in Fig. 4, where the solid lines are baryons, the dashed lines are mesons, the blob represents ΠB scattering in the $J=\frac{1}{2}^+$ octet states, and the wiggly line is the current which, because it has $K=0$, couples to the baryons like a scalar particle. Since the ΠB scattering blob conserves spin and $SU(3)$, the diagram on the right-hand side of Fig. 4 will (for fixed q^2) be a constant times the simple triangle diagram shown in Fig. 5, where the triangle now represents only a sum over $SU(3)$ Clebsch-Gordan coefficients.

In the present model, the proportionality constant relating the right-hand sides of Fig. 4 and the diagram in Fig. 5 is about one and we find that the X-matrix equation (14e) can be expressed diagramatically as in Fig. 6(a). This equation clearly has the same appearance as that shown in Fig. 6(b) which represents the change in the mass of a baryon due to changes in the masses of the baryons of which it is composed.

In the terminology of our previous paper on the mass splittings in the baryon octet and $J=\frac{3}{2}^+$ decuplet,[23] the "bubble diagram" in Fig. 6(b) is, apart from a dynamical factor, the same as the piece of the A matrix which we called A_{ext}^{MM}. The eigenvalues of A_{ext}^{MM} for mass splittings which belong to $SU(3)$ representation N are then proportional to the eigenvalues of X_0^N. In the model with which we are working, the proportionality constant is roughly one and it turns out that the eigenvalues and eigenvectors of A_{ext}^{MM} largely determine the pattern of mass splittings in the baryon octets. Thus, the fact that X_0^N has no eigenvalues near unity unless $N=1$ or 8 is closely related to our previous result that self-consistent mass splittings in the baryon octet must have an octet rather than a **27** character ($N=1$ does not lead to mass splitting). Finally, our result that the near-unit eigenvalue of X_0^8 corresponds to mostly F coupling for the current is related to the fact that the mass splittings in the baryon octet are largely F, i.e.,

$$(M_\Sigma - M_\Lambda)/(M_\Xi - M_N) \ll 1\,.$$

V. $CP=\pm 1$ CURRENTS

In this section, we wish to make a few comments concerning $CP=-1$ vector and axial-vector currents, which may be of interest in connection with the recent observation of CP violations in K decays. In field-theory models based on fundamental spin-$\frac{1}{2}$ objects such as quarks, these currents often appear ugly or unnatural because they require derivative couplings. From the present point of view, however, a $CP=-1$ current would seem *a priori* to be on the same footing as one with $CP=+1$. While we are in no sense predicting the existence of these abnormal currents, we can make a few statements about the couplings to B and B^* which they would have if they do exist.

It turns out that the approximate calculations described above are independent of the CP properties of the current under consideration; all that counts here is the $SU(3)$ representation N of the current and the angular momentum K involved in the coupling to B and B^*. For example, the magnetic moment coupling to baryons has $C=-1$ and $P=-1$ whereas the coupling of a pseudoscalar meson to baryons corresponds to $C=+1$ and $P=-1$, but in our G_{SLK} notation they both have $K=1$, i.e., G_{011} for the pseudoscalar and G_{111} for the magnetic moment. Because X depends only on K and not S or L, both these couplings are enhanced in the same way, although clearly the photon and pseudoscalar meson have different CP properties. Since our X matrix does not depend on the CP of a current, we find that abnormal octet currents will be enhanced relative to ones which belong to 10, $\overline{10}$, or 27 and that the ratios among the B-B and B^*-B matrix elements of the octet currents will be the same as for normal $CP=+1$ currents.

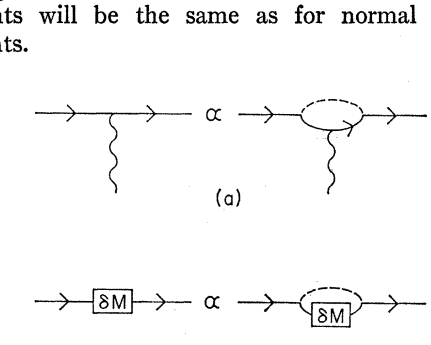

FIG. 6. A diagramatic illustration of the similarity between X_0^N and A_{ext}^{MM}. The upper line (a) is a schematic representation of Eq. (14) for G_{SL0}^N (the $BB\theta$ coupling) while the lower line represents the effect on a baryon mass of a small change in the masses of the baryons of which it is composed. The diagrams represent only products of Clebsch-Gordan coefficients. The proportionality signs indicate that the diagram is to be multiplied by a dynamical factor; in both cases, the dynamical factor is about one.

[21] The connection between the Goldberger-Treiman relation and the present methods was first pointed out to us by G. Chapline. Similar conclusions have been reached by Y. Hara, Phys. Rev. **137**, B1553 (1965), from a somewhat different point of view.

[22] This is a special case of a general connection, to be derived in the third paper of this series, between X^0 and A.

[23] R. Dashen and S. Frautschi, Phys. Rev. **137**, B1331 (1965).

VI. CONCLUSIONS

In this paper we calculated, in a rough approximation, the part of the X matrix which connects the B-B and B-B^* matrix elements of an arbitrary current. We found that for some types of currents, this approximate X matrix has an eigenvalue near unity. According to our hypothesis that strong interactions largely (quite possibly, completely) determine the weak and electromagnetic interactions of hadrons, we expect the observed couplings of B and B^* to the weak and electromagnetic current to follow the pattern set by the enhanced eigenvectors of X. As we have seen, this is indeed the case. It would seem, in fact, that our conclusions about the $SU(3)$ transformation properties of currents and our predictions concerning ratios of magnetic moments, quadrupole moments, electric form factors, and weak axial-vector couplings are sufficiently numerous and well-verified by experiment as to rule out the possibility that the agreement obtained is pure luck.

In this connection, we ought to make a few comments about the reliability of the approximation scheme which we have used here. Clearly, it is not capable of producing highly accurate quantitative predictions; any agreement to better than, say, 10 to 20% should be considered as accidental. On the other hand, we feel that for the purpose of determining the general structure of the eigenvalues and eigenvectors of X, our methods are adequate and reliable. Theoretical reasons for trusting the method have been discussed in the Appendix and in Ref. 23.

Another reason is pragmatic: The method has been consistently successful in diverse applications concerning the deviations from $SU(3)$ symmetry in the B octet and B^* decuplet, the parity-violating nonleptonic decays of hyperons, and the neutron-proton mass difference. The results of all these calculations fit together with the results of the present calculation to give an internally consistent picture of many properties of the baryons and resonances. In this respect, it is important to note that the assumption that a dispersion integral like (4) is dominated by low-mass singularities is distinct from the question of whether the actual binding of Π and B to make B or B^* can be understood in terms of nearby singularities, i.e., long-range forces. It may well be that the actual mechanism which binds B and B^* is

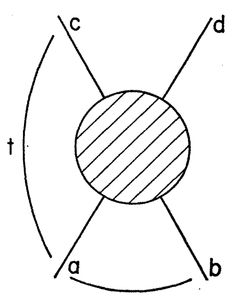

FIG. 7. The scattering process discussed in the text in connection with the effect of external particle spin on the asymptotic behavior of the amplitudes. Particles a, b, and c are spinless while d is assumed to have nonzero spin.

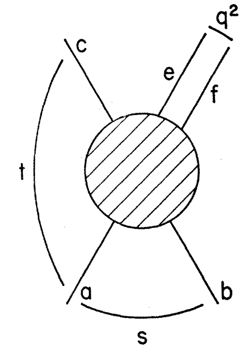

FIG. 8. A production amplitude. If particle d of Fig. 7 is an e-f bound state, its spin does not cause subtractions in the amplitude of Fig. 7.

sensitive to short-range effects, but it seems that the pattern of mass shifts, weak decays, magnetic moments, and so forth is determined mostly by the long-range forces.

In conclusion, we would like to point out again that there is a very attractive possibility that the C's are all zero so that the weak and electromagnetic properties of hadrons are completely determined by bootstrap-like self-consistency conditions. From a technical point of view, the role of subtractions C in the dispersion relations for the matrix elements of currents should be, as was pointed out in Sec. II, more or less the same as the role of subtractions in purely strong-interaction phenomena. If the strong interactions are free of arbitrary subtractions, it would seem most natural, then, for X-matrix equations to also be free of arbitrary constants. The fact that in our approximate calculations the observed currents follow the eigenvectors of X whose eigenvalues are near unity may well be an indication that, in an exact calculation, these eigenvalues would be exactly equal to one and the C's would then be zero.

APPENDIX A

In the text we argued that the asymptotic behavior of the amplitudes for a processes like $a \to b+\theta$ should be the same as that of a purely strong-interaction amplitude. Since all the "internal" particles in the amplitude are hadrons, it is evident that any difference in asymptotic behavior can only have its origin in the fact that the external particle θ is being treated as an elementary object with fixed spin. We shall show here that fixed θ spin will cause no trouble if all form factors vanish as $q^2 \to \infty$.

The manner in which the spin of an external particle can affect the asymptotic behavior of a scattering amplitude has been studied in detail by Mandelstam.[11] Our arguments will be very similar to his.

Let us first review Mandelstam's conclusions, which can be summarized as follows. Consider the reaction pictured in Fig. 7. The particles a, b, and c are, for simplicity, assumed to have spin zero while particle d is taken to have a spin $\sigma > 0$. For the time being, we suppose that all four particles are hadrons; the spinning particle d will later correspond to θ. Now, according to

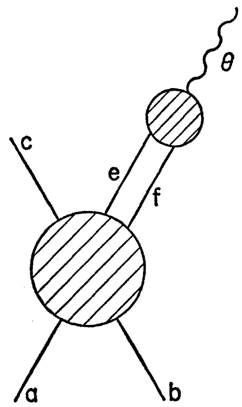

FIG. 9. The e-f intermediate state in the form factor for $J_{ab,c}$.

Ref. 11, if d is an *elementary* hadron, the amplitude will for large s contain a non-Regge term which behaves like $s^{\sigma-1}$ independently of t; consequently, if $\sigma\neq 0$, a subtraction will be necessary in the dispersion relations. In Regge-pole language, this subtraction would come about because the spin of d can bring one of the well-known fixed singularities at $l=-1$ up to a physical value of $J=l+\sigma\geq 0$. Mandelstam shows, however, that if d is not elementary this subtraction will not, in fact, be present. To see this, one considers the production amplitude shown in Fig. 8, where, again for simplicity, we suppose that e and f are spinless. The production amplitude depends on five variables which can be chosen to be s and t as shown in Fig. 8, $q^2=(p_e+p_f)^2$, $\cos\beta$ = cosine of the angle between $\mathbf{p}_c$ and $\mathbf{p}_e$ in the center of mass of e and f, and ϕ = an azimuthal angle between $\mathbf{p}_b$ and $\mathbf{p}_e$. Mandelstam argues that for fixed t, q^2, $\cos\beta$, and ϕ, the production amplitude goes, apart from Regge terms, like s^{-1} for large s. This behavior is independent of whether or not the two-particle system e-f has a bound state. If we suppose then that d is not elementary but appears as a bound state in the e-f system, we conclude, by picking the d pole out of the production amplitude (Fig. 8), that the $s^{\sigma-1}$ term does not appear in the amplitude for $a+b\to c+d$, even though d may have spin.

We now turn to the behavior of the amplitude for $a+b\to c+\theta$, in the case that the weakly coupled particle θ has spin. Since we are treating θ as elementary, we cannot directly use Mandelstam's arguments to show that the $s^{\sigma-1}$ term (σ = spin of θ) does not occur. A modified argument does, however, suffice. Let us denote the amplitude for $a+b\to c+\theta$ as $J_{(ab),c}(q^2)$ where, as usual, q^2 is the mass of θ. We assume that $J_{(ab),c}$ satisfies an unsubtracted dispersion relation in q^2; i.e., we write

$$J_{(ab),c}(q^2)=\int\frac{\mathrm{Im}J_{(ab),c}(q^{2\prime})}{q^{2\prime}-q^2}dq^{2\prime}.$$

A typical contribution to $\mathrm{Im}J_{(ab),c}$ would be the e-f intermediate state shown in Fig. 9. Keeping, for simplicity, only the e-f contribution to $\mathrm{Im}J$ yields

$$J_{(ab),c}(q^2,s,t)=\int\frac{T_{ab,cef}(s,t,q^{2\prime},\cos\beta,\phi)f_{ef}^*(q^{2\prime})P(\cos\beta,\phi)dq^{2\prime}d\cos\beta d\phi}{q^{2\prime}-q^2},$$

where T is the strong-interaction amplitude for $a+b\to c+e+f$, f_{ef} is the amplitude for $e+f\to\theta$, $P(\cos\beta,\phi)$ is the function which projects out the $J=\sigma$ partial wave for the e-f system, and we have explicitly shown how J, T, f, and P depend on the different variables. Now, as pointed out above, for fixed t, $q^{2\prime}$, $\cos\beta$, and ϕ, T goes like s^{-1} plus Regge terms for large s, and since f and P are independent of s one finds $J_{ab,c}\sim s^{-1}$ plus Regge terms as $s\to\infty$, independently of the spin of θ.

The same argument can be repeated for any other intermediate state. We thus conclude that if all form factors vanish for $q^2\to\infty$, so that the J's obey unsubtracted disperion relations in q^2, then the spin of θ cannot change the asymptotic behavior of amplitudes.

APPENDIX B

In this Appendix, we list some formulas and treat some points which were used in deriving the results discussed in Sec. IV. References 9 and 17 cover the general mathematical scheme which we use here, and some further discussion concerning calculations of this sort can be found in an interesting paper by Hara.[20]

The notation used here is explained in Sec. IV.

(1) Dispersion Integrals for the Couplings G and G^*

The dispersion integrals which are used to calculate the couplings G_{SLK}^{*N} and G_{SLK}^{N} are essentially the same as those in Eq. (4). Here we sketch the generalizations to $SU(3)$ and to currents with arbitrary spin.

First we give the equation for G_{SLK}^{*N}. Let A_{SLK}^{*N} be the amplitude for $\Pi+B\to B+\theta$ in the $J^P=\frac{3}{2}^+$, **10** channel where N is the $SU(3)$ representation to which θ belongs and, as in Sec. IV, S is the spin of θ, L is its orbital angular momentum in the center-of-mass system, $\mathbf{K}=\mathbf{S}+\mathbf{L}$, and K is coupled with the spin of B to give total $J=\frac{3}{2}^+$.

Using the methods of Ref. 9, one finds that G^* is given by

$$G_{SLK}^{*N}=-\frac{1}{2\pi i}\int_L\frac{F^*(W')A_{SLK}^{*N}(W')}{W'-M^*}\frac{1}{\pi}\times\int_{\text{inel. thresh.}}^{\infty}\frac{\mathrm{Im}(F^*(W')A_{SLK}^{*N}(W'))dW'}{W'-M^*},\quad(16)$$

where M^* is the B^* mass, W is the total c.m. energy, the contour L runs around the left cuts in A^*, and the second integral is the contribution of inelastic intermediate states like $B+\Pi+\Pi$ to the right-hand cut. The function $F^*(W)$ is equal to

$$F^*(W')=\left[\gamma^*\frac{d}{dW}D^*(W)\big|_{W=W^*}\right]^{-1}D^*(W'),\quad (17)$$

where D^* is the denominator function for ΠB scattering in the $J=\frac{3}{2}^+$, **10** channel and $-(\gamma^*)^2$ is the residue of the (direct-channel) B^* pole in the ΠB partial-wave amplitude.

The equation for $G_{SLK}{}^N$ is somewhat more complicated because of the fact that B couples to two ΠB channels, namely, 8_D and 8_F. This time we let $A_{SLK}{}^{iN}$, $i=8_D$, 8_F be the amplitude for $\Pi+B\rightarrow B+\theta$ in the $J^P=\frac{1}{2}^+$, $i=8_D$ or 8_F channel. If θ belongs to an octet, we denote, as before, the two octet channels of $B+\theta$ by $N=8_D$ and $N=8_F$. Again using the techniques of Ref. 9, one can show that

$$G_{SLK}{}^N=-\sum_{i=8_F,8_D}\left[\frac{1}{2\pi i}\int_L\frac{F_i(W')A_{SLK}{}^{iN}(W')}{W'-M}dW'\right.$$
$$\left.+\frac{1}{\pi}\int\frac{\mathrm{Im}(F_i(W')A_{SLK}{}^{iN}(W'))}{W'-M}dW'\right],\quad (18)$$

where M is now the baryon mass. In this case, we have

$$F_i(W)=(\sum_K\gamma_K{}^2)^{-1}\sum_{lm}\gamma_l\Delta_{lm}D_{mi}(W),$$
$$\Delta_{lm}=\lim_{W\rightarrow M}(W-M)(D^{-1}(W))_{lm},\quad (19)$$

where D_{ij}, $i=8_F$, 8_D, $j=8_F$, 8_D, is the D matrix for ΠB scattering in the $J=\frac{1}{2}^+$ octet states and $-\gamma_i\gamma_j$ is the residue matrix of the baryon pole. Note that the F_i's as well as F^* of Eq. (17) are independent of S, L, K, and N.

(2) Convergence of the Dispersion Integrals

Equations (16) and (18) are, as they stand, *exact*. In order to evaluate the integrals in practice, however, we assume that they converge rapidly so that the dominant contributions to (16) and (18) are from the nearby singularities. A detailed discussion of the expected convergence properties of dispersion integrals similar to (16) and (18) is given in Ref. 23. There, it is concluded that if B and B^* are *not* elementary particles, so that the denominator functions D and D^* will, for large W, go something like a constant or $\ln W$, then dispersion integrals like (16) and (18) should converge rapidly.

(3) The "Nearby" Singularities in $\Pi+B\rightarrow B+\theta$

If we take, for simplicity, $\sqrt{(q^2)}$ equal to the Π mass, the analytic structure of A and A^* will be the same as in the familiar case of ΠB elastic scattering. The dominant nearby singularities, i.e., those lying in the region $|W-M|$ somewhat less than M, come from: (i) "short cuts" due to B and B^* exchange in the u channel (because an orbital state L crosses only to the same L in the static limit,[24] the "short cuts" associated with the exchange of higher mass ΠB resonances with J^P other than $\frac{1}{2}^+$ or $\frac{3}{2}^+$ are very weak), (ii) exchange of light objects such as Π in the t channel. For reasons to be discussed below, we shall not concern ourselves with the t-channel cuts.

We should add a word about the far-away singularities. It is true that B and B^* exchanges also produce "distant" cuts running along the imaginary W axis, but we do not feel that the presently available methods for including these cuts would be likely to improve our estimate for the G's. The reasons for this are: First, the rather poorly understood Regge behavior of the particles will be important in this region, and secondly, there is little reason to believe that B and B^* exchange dominates the cut along the imaginary axis. We prefer to consider the contribution of all the distant cuts as an effective "C" which gives, schematically, $G=XG+$"C."

(4) Use of the Static Crossing Relations

The B and B^* short cuts are well approximated by poles whose residues are given in terms of those of the direct-channel poles by the static crossing relations, i.e., the crossing relations that would hold in the limit $M\rightarrow\infty$. In this static limit, the transition from the s channel to the u channel is obtained by: (i) replacing Π and θ by their antiparticles $\bar{\Pi}$ and $\bar{\theta}$; (ii) taking S_Z, L_Z, and K_Z to $-S_Z$, $-L_Z$, and $-K_Z$, with the squares of these angular momenta remaining the same; and (iii) taking $\omega=W-M$ to $-\omega$. We will not concern ourselves here with the isotopic [$SU(3)$] part of the crossing relations; $SU(3)$ crossing matrices have been amply discussed in the literature. Neglecting, then, the isotopics and denoting the $J=\frac{3}{2}^+$ amplitudes by $A_{SL1}{}^*$ and $A_{SL2}{}^*$ and the $J=\frac{1}{2}^+$ amplitudes by A_{SL0} and A_{SL1}, one easily verifies that

$$A_{SL2}{}^*(-\omega)=A_{SL2}{}^*(\omega),\quad (20)$$

$$A_{SL0}(-\omega)=A_{SL0}(\omega),\quad (21)$$

and

$$\begin{pmatrix}A_{SL1}(-\omega)\\A_{SL1}{}^*(-\omega)\end{pmatrix}=\begin{pmatrix}-\frac{1}{3}&\frac{4}{3}\\\frac{2}{3}&\frac{1}{3}\end{pmatrix}\begin{pmatrix}A_{SL1}(\omega)\\A_{SL1}{}^*(\omega)\end{pmatrix},\quad (22)$$

which hold for $|\omega|\ll M$. We note that the residues of the cross-channel poles in the integral for a coupling G_{SLK} or $G_{SLK}{}^*$ will contain only couplings with the same S, L, and K. The reason for this is that **S**, as the spin of an external particle, crosses only to $-$**S**, **L** crosses only to $-$**L** in the static limit,[24] and thus **K**=**S**+**L** crosses only to $-$**K**. Adding to (20), (21), (22) the $SU(3)$ crossing

[24] To see this, note that $t=-2q_s{}^2(1-\cos\theta_s)=-2q_u{}^2(1-\cos\theta_u)$. Now in crossing from the s to u channel, t does not change, and in the static region $q_u{}^2\approx q_s{}^2$, which implies that $\cos\theta_s\approx\cos\theta_u$ and therefore $P_L(\cos\theta_s)\approx P_L(\cos\theta_u)$.

relations completes the determination of the residues of the "crossed poles" in terms of the direct-channel pole.

(5) Linear D Functions

Since we only need the F's in Eqs. (16) and (18) in the region $|W-M|\ll M$ where the short cuts appear, we can approximate $D^*(W)$ and $D_{ij}(W)$ by linear functions which have zeros at the masses of B^* and B [actually $\det(D_{ij})=0$ for the coupled-channel B problem]. In order to complete the determination of the matrix D_{ij}, one can either: (I) assume that D_{ij} can be diagonalized independently of W in the region $|W-M|\ll M$ which reduces the two-channel problem to an effective one-channel problem, or else (II) determine a self-consistent two-channel D matrix as in Ref. 17. The results of the calculations presented in Sec. IV are not sensitive to which of these methods is chosen, nor are they significantly changed if the D functions are given a reasonable curvature (see, in this connection, Ref. 23).

(6) Calculation of $SU(2)$ X Matrix

Before calculating the X matrix for full $SU(3)$, it is instructive to work out the $SU(2)$ X matrix, Eq. (12) of the text. Here it is entirely appropriate to use one-channel linear D functions. The spin crossing for the magnetic moments goes like $K=1$ [Eq. (22)], and the spin crossing for the quadrupole moment goes like $K=2$ [Eq. (20)]. For an isoscalar photon, the isospin crossing factor is 1, and of course relates only $I=\frac{1}{2}$ amplitudes; combining this with the spin-crossing factor of Eq. (22) gives the X-matrix element of $-\frac{1}{3}$ connecting μ_s to μ_s. For an isovector photon, the isospin crossing factor is well-known to be the same as the spin crossing for $K=1$; combining spin and isospin factors produces the remaining elements of the X matrix shown in Eq. (12).

(7) $SU(3)$ Calculation of X_0^N and X_2^N

In these cases we use the trivial spin-crossing relations (20) and (21), together with the usual $SU(3)$ crossing relations, and it is easiest to assume (I) for D_{ij}. It is then a straightforward matter to show that $|X_2^N|\ll 1$ for all N and that X_0^N has eigenvalues near unity only for $N=1$ and $N=8$, as discussed in the text.

(8) $SU(3)$ Calculation of X_1^N

In this case it is easiest to use the self-consistent D matrix (II). Since this matrix makes the Π couplings self-consistent, we know that X_1^8 has an eigenvalue exactly equal to one. It is then straightforward to verify that X_1^N for $N\neq 8$ has no eigenvalues near one.

(9) t-Channel Cuts and The Asymmetry of X

We did not include the nearby part of the t-channel cuts in evaluating (16) and (18). There are several reasons why this may be a reasonable approximation. Suppose, for example, that in calculating $G_{SL0}{}^{8F}$ and $G_{SL0}{}^{8D}$ we had kept, say, the t-channel cut due to vector-meson V exchange. Denoting by g_V the relevant coupling for $V\to\Pi+\theta$, we would write an expanded X-matrix equation like

$$\begin{pmatrix} G_{SL0}{}^{8F} \\ G_{SL0}{}^{8D} \\ g_V \end{pmatrix} = \begin{pmatrix} & & a \\ & X_0{}^8 & b \\ a' & b' & c \end{pmatrix} \begin{pmatrix} G_{SL0}{}^{8F} \\ G_{SL0}{}^{8D} \\ g_V \end{pmatrix} + \begin{pmatrix} C_F \\ C_D \\ C_V \end{pmatrix}. \quad (23)$$

Now if the V mesons are composed largely of $\Pi\Pi$, $V\Pi$, and possibly VV, then we do not expect the θBB couplings to have a large effect on g_V. In this case, a' and b' will be small, and neglecting these terms the eigenvalues x of the expanded X matrix of Eq. (23) are given by the solutions of $(c-x)[\det(X_0{}^8-xI)]=0$. One sees, then, that even if g_V has a large effect on the G's, i.e., large a and b, the expanded X matrix still has the near-unit eigenvalue of $X_0{}^8$ and it is easy to verify that the enhanced eigenvector corresponds to the same ratio of G^{8F} and G^{8D} as before.[25] Actually, estimates indicate that a and b are themselves somewhat smaller than the effects of B and B^* exchange, which further strengthens the likelihood that V exchange terms can be neglected in first approximation.

Note added in proof. After this paper was written we learned that B. Diu and R. P. Van Royen had independently arrived at some of the conclusions listed in Sec. IV [B. Diu and R. P. Van Royen (to be published)].

[25] This is a particular example of how the asymmetry of the X matrix plays an important role in many practical calculations. A detailed discussion of the properties of asymmetric matrices within a similar context can be found in Ref. 23.