

New Constraint on Open Cold-Dark-Matter Models

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We calculate the large-angle cross correlation between the cosmic-microwave-background temperature and the x-ray-background intensity expected in an open universe with cold dark matter (CDM) and a nearly scale-invariant spectrum of adiabatic density perturbations. Results are presented as a function of the nonrelativistic-matter density Ω_0 and the x-ray bias b_x for both an open universe and a flat cosmological-constant universe. Recent experimental upper limits to the amplitude of this cross correlation provide a new constraint to the Ω_0 - b_x parameter space that open-CDM models (and the open-inflation models that produce them) must satisfy. [S0031-9007(99)09256-X]

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Determination of the matter density, Ω in units of the critical density, has long been one of the central goals of cosmology. The simplest and most attractive models of inflation [1], the leading paradigm for understanding the remarkable smoothness of the Universe and the origin of its large-scale structure, have for many years predicted the Einstein-de Sitter value, $\Omega = 1$. However, a variety of observations seem to suggest that nonrelativistic matter contributes a much smaller fraction, $\Omega_0 \approx 0.3$. One possible resolution is that the difference implies the existence of a cosmological constant (Λ) that contributes a fraction $\Omega_\Lambda \approx 0.7$ of the critical density (so that the Universe remains flat, $\Omega = \Omega_0 + \Omega_\Lambda = 1$). Another is that some (much more complicated) models of inflation may have produced an open universe with $\Omega = \Omega_0 = 0.3$ [2].

Like ordinary inflation, open inflation produces primordial adiabatic density perturbations that, with the presence of cold dark matter (CDM), give rise to the large-scale structure observed in the Universe today. When the amplitude of density perturbations is normalized to the cosmic background explorer (COBE) map of the cosmic microwave background (CMB), such "open-CDM" models are found to be consistent with the amplitude and shape of the galaxy power spectrum [3,4].

In an Einstein-de Sitter universe, large-angle CMB anisotropies are produced by gravitational-potential differences induced by density perturbations at the surface of last scatter at a redshift $z \approx 1100$ via the Sachs-Wolfe (SW) effect [5]. In a flat Λ universe [6], or in an open universe [3], additional anisotropies are produced by density perturbations at lower redshifts along the line of sight via the integrated Sachs-Wolfe (ISW) effect. Reference [7] thus argued that in a flat Λ universe, there should be some cross correlation between the CMB and a tracer of the mass distribution at low redshifts; a similar effect also occurs in an open universe [8].

The x-ray background (XRB) currently offers perhaps the best tracer of the mass distribution out to redshifts of a few. Reference [9] determined an upper limit to the amplitude of the cross correlation between the

COBE CMB map and the first High-Energy Astrophysical Observatory (HEAO I) map of the 2–20 keV XRB and used it to constrain $\Omega_0 \gtrsim 0.3$ (with some assumptions) in a flat Λ universe.

The ISW effect contributes a much larger fraction of the large-angle anisotropy in an open universe than in a flat universe with the same Ω_0 (see Ref. [3] and Fig. 1 in Ref. [8]), so one would expect the cross correlation to be much (~ 5 times for $\Omega_0 \sim 0.3$) larger in an open universe. However, the XRB probes only the matter distribution out to a redshift of $z \sim 4$, so the relevant quantity is the fraction of the CMB anisotropy produced by the ISW effect at these low redshifts. Although virtually all of the ISW effect comes from these low redshifts in a flat Λ universe, the range of redshifts over which the ISW effect contributes in an open universe is much broader (cf. Fig. 2 in Ref. [8]). Thus, a detailed calculation is necessary to apply the results of Ref. [9] to an open universe.

In this paper, we generalize the calculation of Ref. [7] to an open universe. We present results as a function of Ω_0 and a currently uncertain bias b_x of x-ray sources for an open and a flat Λ universe. An experimental upper limit [9] is used to constrain the Ω_0 - b_x parameter space for open-CDM and flat Λ CDM models. We show that these constraints depend only weakly on the Hubble constant, spectral index, uncertainties in the large-scale power spectrum, and uncertainties in the XRB redshift distribution. If $\Omega_0 \approx 0.3$ – 0.4 , then x-ray sources can be no more than weakly biased tracers of the mass distribution. We discuss how to apply these results to models of evolving x-ray bias.

We now detail our calculation: The fractional perturbation to the temperature in a direction $\hat{\mathbf{n}}$ is

$$\begin{aligned} \frac{\Delta T}{T}(\hat{\mathbf{n}}) &= \frac{1}{3} \Phi[(\eta_0 - \eta_{ls})\hat{\mathbf{n}}; \eta_{ls}] \\ &+ 2 \int_{\eta_{ls}}^{\eta_0} \frac{d\Phi[(\Delta\eta)\hat{\mathbf{n}}; \eta]}{d\eta} d\eta \\ &= (\Delta T/T)_{SW}(\hat{\mathbf{n}}) + (\Delta T/T)_{ISW}(\hat{\mathbf{n}}), \quad (1) \end{aligned}$$

where $\Delta\eta = \eta_0 - \eta$, η is the conformal time, the subscript “0” denotes the value of a given parameter today, and the subscript “ls” denotes the value at the surface of last scatter at $z \approx 1100$. In the second equality, we have split up the temperature anisotropy into a term due to potential perturbations at the surface of last scatter (SW) and one due to potential fluctuations along the line of sight (ISW). Here, $\Phi(\mathbf{x}; \eta)$ is the gravitational potential at position \mathbf{x} at conformal time η . The potential is related to the density perturbation, $\delta(\mathbf{x}) = [\rho(\mathbf{x}) - \bar{\rho}]/\bar{\rho}$, where $\rho(\mathbf{x})$ is the density at \mathbf{x} and $\bar{\rho}$ is the mean density, through the Poisson equation [10]. Throughout, we choose the scale factor to be $a_0 = H_0^{-1}(1 - \Omega_0)^{-1/2}$.

The fractional perturbation to the XRB intensity in direction $\hat{\mathbf{n}}$ is

$$\frac{\Delta X}{X}(\hat{\mathbf{n}}) = \int_{\eta_{ls}}^{\eta_0} g(\eta) \delta_x[(\Delta\eta)\hat{\mathbf{n}}; \eta] d\eta, \quad (2)$$

where $\delta_x(\mathbf{x}; \eta) = b_x \delta(\mathbf{x}; \eta)$ is the fractional perturbation to the luminosity density of x-ray sources, and we surmise

$$\tilde{\Theta}_\ell^T(k) = \tilde{\Theta}_\ell^{SW}(k) + \tilde{\Theta}_\ell^{ISW}(k) = \left[\frac{3\Omega_0}{2(1 - \Omega_0)(k^2 + 4)} \right] \left[\frac{1}{3} \Phi_k^\ell(\eta_0 - \eta_{ls}) F(\eta_{ls}) + 2 \int_{\eta_{ls}}^{\eta_0} F'(\tilde{\eta}) \Phi_k^\ell(\eta_0 - \tilde{\eta}) d\tilde{\eta} \right], \quad (5)$$

and the function $F(\eta)$ describes the time evolution of potential perturbations; it is given in terms of the well-known linear-theory growth factor $D(z)$ for density perturbations [11] by $F(z) = (1 + z)D(z)/D(0)$. The XRB analog is [12]

$$\tilde{\Theta}_\ell^X(k) = b_x \int_{\eta_{ls}}^{\eta_0} g(\tilde{\eta}) D(\tilde{\eta}) \Phi_k^\ell(\eta_0 - \tilde{\eta}) d\tilde{\eta}, \quad (6)$$

and for $k \gg 1$ (scales smaller than the curvature scale), $P(k) \propto k^n T^2(k)$ (with $n \approx 1$) is the power spectrum for the mass distribution with $T(k)$ the transfer function [13]. The functions $\Phi_k^\ell(\eta)$ are the radial harmonics for a space of constant negative curvature [3,14], the curved-space analog of spherical Bessel functions.

If the correlation functions are measured using beams with Gaussian profiles of FWHM θ_{FWHM} , then factors $W_\ell^A(\theta)W_\ell^B(\theta)$ should be included in the sum in Eq. (3), where $W_\ell(\theta) = \exp[-\ell(\ell + 1)\sigma_b^2/2]$ is the window function, and $\sigma_b = 0.00742(\theta_{\text{FWHM}}/1^\circ)$.

Since we compare the results of our calculation to the experimental limit of Ref. [9], we simply use the redshift distribution used by Ref. [9]. This model assumes that the universal x-ray luminosity evolves, increasing with increasing redshift, from $z = 0$ until $z_c = 2.25$, and is thereafter constant up to a maximum redshift $z_f = 4$, beyond which the x-ray luminosity is zero [15]. The fraction of the local x-ray flux that comes from any given differential redshift interval is obtained in the standard way (see, e.g., Ref. [11]), and $g(\eta)$ is also obtained in the standard way. To assess the effects of uncertainties

that this is equal to some bias factor b_x times the matter-density perturbation. Here $g(\eta)$ is the selection function that determines the fraction of the XRB intensity that comes from a conformal time η . It is related to the XRB redshift distribution to be discussed below.

The CMB/XRB angular auto- and cross-correlation functions are defined by

$$C^{AB}(\alpha) = \langle [\Delta A(\hat{\mathbf{m}})/A][\Delta B(\hat{\mathbf{n}})/B] \rangle_{\hat{\mathbf{m}} \cdot \hat{\mathbf{n}} = \cos\alpha} \\ = \sum_\ell \frac{2\ell + 1}{4\pi} C_\ell^{AB} P_\ell(\cos\alpha), \quad (3)$$

where $\{A, B\} = \{T, X\}$ are the fractional CMB/XRB intensity perturbations, $P_\ell(\cos\alpha)$ are Legendre polynomials, and the angle brackets denote an average over all pairs of lines of sight $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ separated by an angle α . Predictions for the multipole moments are given by

$$C_\ell^{AB} = \frac{2}{\pi} \int k^2 dk \tilde{\Theta}_\ell^A(k) \tilde{\Theta}_\ell^B(k) P(k), \quad (4)$$

where the CMB weight functions are [3]

in the XRB redshift distribution on the final results, we also consider two alternative redshift distributions. In the first, we simply scale our canonical redshift distribution so that it extends to a redshift $z_f = 5$, instead of 4 (so the evolution cutoff is at $z_c = 2.81$). In the second, we scale the canonical distribution so that it extends out only to $z_f = 3$ (so $z_c = 1.69$).

The power spectrum $P(k)$ is normalized so that the rms fluctuation, $\sigma_T = [C^{TT}(0)]^{1/2}$ [calculated from Eqs. (3)–(5) and smoothed with a Gaussian beam with $\theta_{\text{FWHM}} = 10^\circ$], matches that measured by COBE [16]. The x-ray bias b_x must then be chosen so that the predicted rms XRB fluctuation [calculated from Eqs. (3), (4), and (6) and smoothed with $\theta_{\text{FWHM}} = 3.6^\circ$] matches the empirical value. The experimental result for the rms XRB fluctuation from HEAO I is $\sigma_x^{\text{HEAO}} = 0.024$ [9]. However, some fraction of this measured fluctuation amplitude must come from Poisson fluctuations in the (currently uncertain) number density of sources that give rise to the diffuse extragalactic XRB. The rest is due to the large-scale mass inhomogeneities, as traced by x-ray sources, and is what we are interested in. Thus, $(\sigma_x^{\text{HEAO}})^2 = (\sigma_x)^2 + (\sigma_x^{\text{Poisson}})^2$, so $\sigma_x \leq 0.024$.

To proceed, we first determine the x-ray bias that would be needed if all of the measured XRB fluctuation amplitude were due to density perturbations (i.e., if we assumed $\sigma_x = \sigma_x^{\text{HEAO}}$), and then calculate the zero-lag cross-correlation amplitude $C^{XT}(0)$. If some fraction of the fluctuation amplitude is due to Poisson fluctuations, then the fluctuation due to density perturbations must

be smaller. The x-ray bias required to explain the observations must therefore also be accordingly smaller. Since it is proportional to the x-ray bias, the predicted cross-correlation amplitude must also be smaller by the same factor.

Figure 1 shows our results for the scaled zero-lag CMB/XRB cross-correlation amplitude. (See the figure caption for a description of the curves.) We have checked that our flat-universe calculation agrees with that of Ref. [9]. The decrease at small Ω_0 for an open universe occurs because of the additional ISW contributions to σ_T from redshifts $z \geq 4$. The predictions obtained using the alternative higher- and lower- z redshift distributions indicate that even fairly dramatic changes to the XRB redshift distribution have little ($\leq 10\%$) effect on the predicted cross-correlation amplitude.

Figure 1 shows results for two extreme assumptions about the contribution of Poisson fluctuations, or equivalently, the x-ray bias. The heavy curves show predictions obtained by assuming that all of the measured XRB fluctuation amplitude is due to density perturbations (i.e., no Poisson fluctuations), which implies the largest possible bias (and cross correlation). The lighter curves show results obtained if we assume the x-ray bias is $b_x = 1$; these curves provide a lower limit to the cross-correlation amplitude as long as x-ray sources are not antibiased. Since

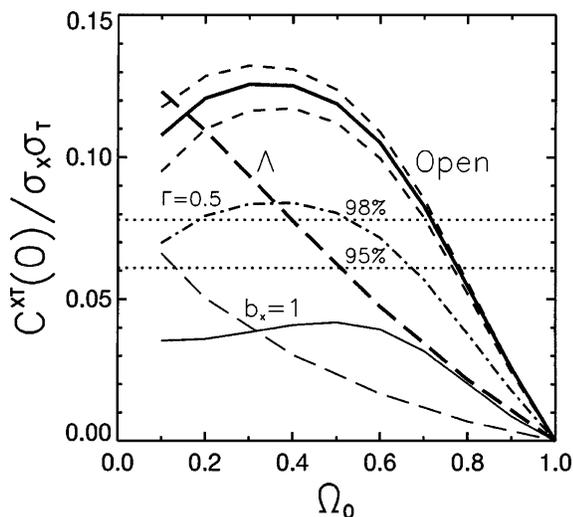


FIG. 1. Results for the scaled zero-lag CMB/XRB cross-correlation amplitude as a function of Ω_0 for an open (heavy solid curve) and a flat Λ (heavy dashed curve) universe, both with $\Gamma = 0.25$, assuming a constant x-ray bias and that the x-ray fluctuations are due entirely to perturbations in the density of x-ray sources. The upper (lower) lighter short-dashed curve shows the result for an open universe obtained with an alternative redshift distribution that extends to larger (smaller) redshifts. The dot-dashed curve shows the result for an open universe with $\Gamma = 0.5$. The lower lighter solid and long-dashed curves are the results for an open and a flat Universe, respectively, assuming that $b_x = 1$. The two dotted lines are 95% and 98% confidence-level (C.L.) upper limits from Ref. [9].

the scaled cross-correlation amplitude is proportional to the bias, the x-ray bias inferred if we assume no Poisson fluctuations can be obtained by taking the ratio of the maximal prediction to the $b_x = 1$ prediction. Similarly, we can obtain the predicted cross-correlation amplitude for any x-ray bias by interpolating between the $b_x = 1$ and maximal predictions. Doing so, we obtain constraints to the Ω_0 - b_x parameter space shown in Fig. 2, which illustrates our central results.

We now detail how our result depends on certain model parameters. We used a flat $n = 1$ primordial power spectrum. A smaller value of n will increase the cross-correlation amplitude relative to σ_T , but it will also increase σ_x . Numerically, if n is decreased to 0.7, the scaled cross-correlation amplitude increases by 15% for $\Omega_0 = 0.4$, and conversely for larger n .

Although the generalization of a power-law primordial power spectrum to an open universe is not well defined for low k [3], this uncertainty affects only the lowest CMB multipole moments. The effect on the XRB is small because the spectrum of density fluctuations leans much more to smaller scales (because of the Poisson equation) than that for the potential perturbations that give rise to CMB anisotropies. Numerical calculations show that the curves in Fig. 1 are changed by only a few percent (for $\Omega_0 \approx 0.4$) if alternative low- k power spectra from open-inflation models are used. If, however, a fraction f of the CMB variance is due to gravitational waves, then the lower bounds in Fig. 2 are increased by a factor of $(1 - f)^{-1}$.

There is some ambiguity concerning the smallest redshift at which the diffuse XRB begins (since nearby sources are subtracted). In our calculations, we assumed that the XRB distribution extends all the way down to

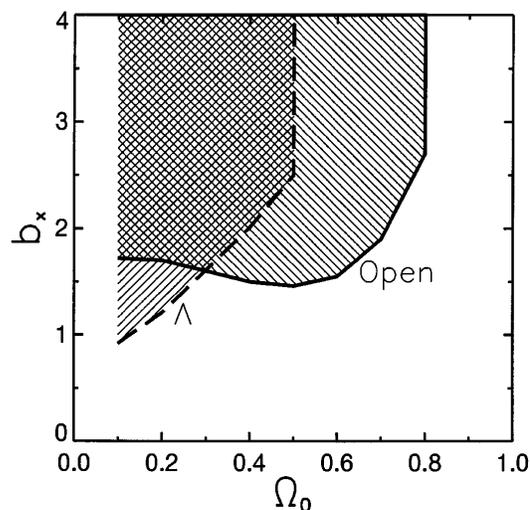


FIG. 2. Constraints to the Ω_0 - b_x parameter space for both an open universe and a flat Λ universe. The shaded regions are ruled out at the 95% C.L. In evolving-bias models, the b_x shown here is the value of the x-ray bias at $z \approx 1$.

$z = 0$. If the theoretical predictions are repeated assuming the XRB distribution vanishes below $z = 0.1$, the predicted cross-correlation amplitude would increase quite significantly, but the x-ray bias would also increase by roughly the same amount. Thus, this uncertainty would similarly have no significant effect on the results shown in Fig. 2.

How do the results depend on the parameter $\Gamma = \Omega_0 h$ that determines the peak of the present-epoch power spectrum? If we take a very conservative upper limit of $h \lesssim 1$, then $\Gamma \lesssim 0.5$ for $\Omega_0 \lesssim 0.5$. The dot-dashed curve in Fig. 1 shows the result for $\Gamma = 0.5$. Moreover, the reduction in the cross correlation from the $\Gamma = 0.25$ curve can be attributed almost entirely to the reduction in the x-ray bias. Thus, Fig. 2 will look roughly the same for any other reasonable value of Γ . Since the baryon density has only a weak effect on the power spectrum, the results are similarly independent of the baryon density.

So far, we have taken the x-ray bias to be constant, but it has been suggested that the x-ray bias is evolving [12,17]. We have repeated our calculations for a variety of evolving-bias models. In each case, our results can be reproduced by identifying our b_x with the value of the x-ray bias at a redshift $z \approx 1$ in evolving-bias models.

To conclude, we have carried out the first calculation of the amplitude of the CMB/XRB cross-correlation function in open-CDM models and used an experimental upper limit to place new constraints to the Ω_0 - b_x parameter space in both open CDM and Λ CDM models. In models with evolving x-ray bias, our b_x is the bias at a redshift $z \approx 1$. We have shown that the excluded regions of this parameter space are no more than weakly affected by uncertainties in the XRB redshift distribution, power spectrum, Hubble constant, or baryon density. Figure 2 shows that if $\Omega_0 \approx 0.3$, then the sources that give rise to the XRB can be no more than weakly biased tracers of the mass distribution (unless there is a significant gravitational-wave background). If the high-redshift active galactic nuclei that give rise to the XRB have biases $b_x \sim 3$ like other high-redshift populations such as clusters, radio sources [18], or Lyman-break galaxies [19], then low-density CDM models will be in trouble.

The limiting factor in providing a model-independent constraint to Ω_0 is currently the uncertain Poisson contribution to the XRB fluctuation amplitude, or equivalently, the uncertain x-ray bias. Ideally, one would remove this uncertainty by identifying all of the sources that contribute to the diffuse extragalactic XRB. Fortunately, data from forthcoming satellite experiments should help make progress toward this goal.

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