I. INTRODUCTION

Now that a Higgs boson has been observed with properties similar to those predicted by the Standard Model (SM), the next critical task is a program of precision measurements of its properties. Studies of the Higgs mass, coupling strengths, and production and decay channels are well advanced. However, less attention has been paid to the possibility of observing $CP$ violation in the Higgs sector and the purpose of the present work is to explore such a possibility.

If the 125 GeV boson is measured to be a mixture of $CP$ even and odd states, it immediately indicates that there must be new physics not far above the electroweak scale. One of the most straightforward ways is to extend the scalar sector of the SM, and the simplest case is the complex version of the 2 doublet model (C2HDM) [1–6]. The presence of $CP$ violation leads to observable changes in Higgs production and decay rates, as well as contributions to low energy observables such as electric dipole moments (EDMs) [7]. It has also been pointed out that some of the heavy scalar decay channels could be sensitive to $CP$ violation [8], and could be probed at the LHC and future colliders. The complementarity of LHC and low energy measurements for constraining Higgs $CP$ violation has been explored in Refs. [7–10].

Alternatively, it is also possible to extend the fermion sector of the SM while keeping the scalar sector minimal. The simplest case is to introduce a chiral fourth generation or mirror family, but they are strongly disfavored after the discovery of the Higgs boson, since they would lead to a large enhancement in the Higgs production rate. The next simplest extension is to introduce vectorlike quarks (VLQs). Vectorlike fermions are defined as having the same gauge quantum numbers for both left- and right-handed fermion pairs, and thus they do not generate chiral anomalies and their effects decouple in Higgs physics. They are the ingredients of beyond the SM frameworks like the little Higgs [11] or composite Higgs models [12,13], and theories of extra dimensions [14–16] or extended supersymmetry [17,18]. VLQs have also been discussed recently in light of modified Higgs couplings such as that to two photons [19,20]. The current LHC lower limit on the masses of VLQs from direct searches is around 800 GeV [24] regardless of the decay modes. The indirect effects of VLQs in electroweak precision measurements and flavor physics have also been extensively explored in the literature [25–29].

In this work, we consider the $CP$ violating aspects of the VLQ models, which have been less studied. Our goal is to examine their impact on the $CP$ nature and interactions of the Higgs boson, as well as the low energy constraints [30,31]. For simplicity, we focus on the simple cases where VLQs are in a single representation of $SU(3)_c \times SU(2)_L \times U(1)_Y$. We consider only fermion representations that can have new Yukawa couplings with the SM quarks and Higgs doublet. These new Yukawa couplings could provide a new source of $CP$ violation at the weak scale. Under these assumptions, we find that only the case where the VLQ lies in the $(3, 2, 1/3)$ representation can generate significant $CP$ violation in Higgs physics.

We study $CP$ violating phenomenology in the doublet VLQ model using an effective theory language where the heavy VLQs have been integrated out. Then the $CP$ violating effects manifest themselves through a new right-handed charged-current interaction mediated by the $W$-boson. We clarify the source of $CP$ violation in this model in Sec. II. In Sec. III, we calculate the loop induced $CP$ violating Higgs interactions with gauge bosons. Interestingly, we find that $CP$ violation only exists in the $hWW$ coupling, but not in the $hZZ$ [32], $h\gamma\gamma$, or $hZ\gamma$ [33] ones. In Sec. IV, we explore the current constraints on this coupling from low energy measurements, including electric dipole moments and $B$ physics, and comment on the future prospects.

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1There are also recent studies which extend both scalar and fermion sectors of the SM [21–23].
TABLE I. Models of vectorlike quarks and their representations under $SU(3)_c \times SU(2)_L \times U(1)_Y$, together with the possibilities of introducing new physical $CP$ violating phases.

<table>
<thead>
<tr>
<th>VLQ models</th>
<th>Representation</th>
<th>$CP$ violation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{L,R}$</td>
<td>$(3, 1, 4/3)$</td>
<td>No</td>
</tr>
<tr>
<td>$B_{L,R}$</td>
<td>$(3, 1, -2/3)$</td>
<td>No</td>
</tr>
<tr>
<td>$(T, B)_{L,R}$</td>
<td>$(3, 2, 1/3)$</td>
<td>Yes</td>
</tr>
<tr>
<td>$(X, T)_{L,R}$</td>
<td>$(3, 2, 7/3)$</td>
<td>No</td>
</tr>
<tr>
<td>$(B, Y)_{L,R}$</td>
<td>$(3, 2, -5/3)$</td>
<td>No</td>
</tr>
<tr>
<td>$(X, T, Y)_{L,R}$</td>
<td>$(3, 3, 4/3)$</td>
<td>No</td>
</tr>
<tr>
<td>$(T, B)_{L,R}$</td>
<td>$(3, 3, -2/3)$</td>
<td>No</td>
</tr>
</tbody>
</table>

II. $CP$ VIOLATION FROM VECTORLIKE QUARKS

The vectorlike quark representations that allow new Yukawa couplings with SM quarks are summarized in Table I.\(^2\)

The key point we want to make is that only the doublet $(T, B)$ model can offer nonzero (unsuppressed) $CP$ violation for the Higgs boson, through the Yukawa coupling to third generation quarks.\(^3\) To see this, we write the Yukawa sector of this model,

$$
\mathcal{L}_Y = -y_t\bar{Q}_{3L}\tilde{H}t_R - y_b\bar{Q}_{3L}b_R - M\bar{Q}_{3L}^c Q_R' - M'\bar{Q}_{3L}^c Q'_{3L},
$$

where $H = (\phi^+, \phi_0)^T$ is the SM Higgs doublet, $\tilde{H} = i\sigma_2 H^*$, $Q_3^L = (t_L, b_L)$ is the third generation left-handed quark doublet and $Q_{3L}' = (T, B)_{L,R}$ are the vectorlike quark doublets.

In general, the SM gauge invariance permits us to generalize the fields above, $(Q_{3L}, t_R, b_R)$, to linear combinations of all three generations. In our study, we assume the fields of Eq. (1) are dominantly composed of third generation fermions, because they have the largest Yukawa couplings and thus have the strongest impact on $CP$ violation in Higgs physics, which is the motivation of this work. To be concrete, we can take advantage of the hierarchical structure of the Cabibbo-Kobayashi-Maskawa matrix, and define Eq. (1) in the basis where the SM $3 \times 3$ blocks of the up- and down-type Yukawa matrices are close to diagonal up to Cabibbo-Kobayashi-Maskawa-like rotations.\(^4\) This helps to suppress the mixing between heavy VLQs and the quarks in the first and second generations and minimize the low energy flavor changing effects in the spirit of next-to-minimal $CP$ violation.[34]

From now on, we will focus on the mixing between VLQs and the third generation quarks. Since $Q_{3L}$ and $Q_{3L}'$ have the same quantum numbers, one can always redefine fields and set the parameter $M' = 0$. After electroweak symmetry breaking, the quark mass matrices take the form,

$$
\mathcal{L}_m = -\left(\bar{t}_L, \bar{T}_L\right) \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & M \end{pmatrix} \begin{pmatrix} t_R \\ T_R \end{pmatrix} - \left(\bar{b}_L, \bar{B}_L\right) \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & M \end{pmatrix} \begin{pmatrix} b_R \\ B_R \end{pmatrix} + H.c. \quad (2)
$$

In general the parameters are all complex, and one can remove unphysical phases by redefining the phases of the fields. Under the gauge invariant transformations, $Q_R \rightarrow Q_R e^{i\alpha}$, $Q_{3L}' \rightarrow Q_{3L}' e^{i\alpha}$, $Q_{3L} \rightarrow Q_{3L} e^{i\delta}$, $t_R \rightarrow t_R e^{i\delta}$, $b_R \rightarrow b_R e^{i\delta}$, the parameters change to

$$
y_t \rightarrow y_t e^{i(\delta - \gamma)}, \quad \lambda_t \rightarrow \lambda_t e^{i(\delta - \gamma)},
$$

$$
y_b \rightarrow y_b e^{i(\delta - \gamma)}, \quad \lambda_b \rightarrow \lambda_b e^{i(\delta - \gamma)},
$$

$$
M \rightarrow M' e^{i(\delta - \gamma)}. \quad (3)
$$

Clearly, what is invariant is the combination of phases of the parameters $\arg(y_b) + \arg(\lambda_t) - \arg(y_t) - \arg(\lambda_b)$, or the quantity,

$$
\text{Im}(y_t\lambda_t y_b\lambda_b) = |y_t y_b \lambda_t \lambda_b| \sin\theta. \quad (4)
$$

This is the only new source of $CP$ violation in this model that can enter into physical processes. It is convenient to use the above rephasing freedom to rotate the phase $\theta$ into $\lambda_t$ and make the other parameters real. In this case, from the quark mass terms, $y_q (v + h) q / \sqrt{2}$, we can first assign the Higgs boson to be $CP$ even. Then any coupling between $h$ and $CP$ odd operators induced by Eq. (4) indicates $CP$ violation in the Higgs sector.

We diagonalize the mass matrices in Eq. (2), and obtain the mass eigenstates

$$
i_R = \cos \theta_R t_R + \sin \theta_R e^{-i\theta} T_R, \quad \hat{i}_R = \cos \theta_R t_R + \sin \theta_R e^{i\theta} T_R,
$$

$$
\hat{b}_R = \cos \theta_R b_R + \sin \theta_R B_R. \quad (5)
$$

where $\hat{T}_R, \hat{B}_R$ are orthogonal to $\hat{i}_R, \hat{b}_R$, respectively. There are similar mixings among the left-handed fields, parametrized by angles $\theta^L_1$ and $\theta^L_2$. The mixing angles among the right-handed quarks satisfy
The angle $\theta_R$ denotes the mixing between the $SU(2)_L$ singlet $b_R$ and doublet $B_R$, and is constrained by electroweak precision measurements such as $Z \to b\bar{b}$ [25,35,36]. The mixings among left-handed quark fields only appear at order $(v/M)^2$ and are much smaller [25,26].

The lower bound on the VLQ mass scale is around 800 GeV from direct searches at the LHC [24], which suggests that we can integrate them out when studying Higgs physics. Since $T_R, B_R$ lie in an $SU(2)_L$ doublet, integrating out the heavy vectorlike quarks yields an anomalous $Wtb$ interaction,

$$\mathcal{L}_{\text{eff}} = a_R \left( \frac{g}{\sqrt{2}} \right) \bar{t}_R P^\mu b_R W^\mu_\nu,$$

where

$$a_R = \sin \theta_R \sin \theta_R e^{i\phi} = \frac{|\lambda_b \lambda_d| v^2}{2M^2} e^{i\phi}. \tag{8}$$

As discussed above, in this model $CP$ violation must appear in physical processes through the combination of couplings, $\text{Im}(y_b \lambda_d y_b^\ast \lambda_d^\ast)$. The new right-handed $Wtb$ coupling $a_R$ obtained here is proportional to $\lambda_b \lambda_d^\ast$. Therefore, a physical process that makes the $CP$ violation manifest should involve both left- and right-handed currents in order to allow mass (Yukawa coupling $y_t, y_b$) insertions. We write the most general renormalizable $Wtb$ coupling as

$$\mathcal{L}_{\text{eff}} = \frac{g}{\sqrt{2}} \bar{t}_R P^\mu (a_L P_L + a_R P_R) b_R W^\mu_\nu + \text{H.c.} \tag{9}$$

We neglect the hat symbol for mass eigenstates hereafter. In the SM, $a_L = V_{tb} \approx 1$ and $a_R = 0$. For the vectorlike quark doublet model we consider, $a_R$ is given by Eq. (8), and the deviation of $a_L$ from 1 occurs only at higher order in $v/M$.

Following a similar reasoning, we have also examined other representations of VLQs. Interestingly, none of them can offer an irreducible $CP$ violating phase such as that in Eq. (4); i.e., under the same assumptions, the Higgs boson is essentially $CP$ even in those models. This observation places the VLQ doublet $(T, B)$ model in a unique place in the perspective of $CP$ violation. In the coming sections, we will explore the $(T, B)$ model as a theory of Higgs $CP$ violation, and study in detail its predictions in phenomenology and the constraints on the parameters of the model.

The Yukawa interactions between vectorlike and SM quarks introduce a new source of $CP$ violation. One of the consequences is that the Higgs boson will obtain $CP$ violating interactions with the other SM particles. As discussed in Eq. (4), in the model with a single VLQ doublet, $(T, B)$, the physical $CP$ violating phase has to appear via the combination of parameters, $\text{Im}(y_t \lambda_d y_b^\ast \lambda_d^\ast)$. This means that the diagram giving $CP$ violating interactions to the Higgs boson must involve both top and bottom quarks. As a result, the leading $CP$ violation in this model resides only in the $hW^+_\mu W^-\mu$ operator, generated at loop level as shown in the left diagram of Fig. 1. At this order, $CP$ violating tree level Higgs-quark or loop level Higgs-Z-boson and Higgs-photon interactions are absent. The direct probes of $CP$ violating $hWW$ interactions at the LHC have been discussed in the $h \to W \to \nu$ decay, the $h$ associated production channel and the $WW$ fusion channel [37,38].

In the heavy fermion limit, the gauge invariant operator generating the Higgs-gauge-boson $CP$ violation starts from dimension 8 in this model,

$$\mathcal{L}_8 = \frac{C_8}{\Lambda^4} (\epsilon_{ij} H^i (\sigma^a)_k H^k W^a_{\mu\nu}) [\epsilon_{mn} H^m (\sigma^b)_l H^l \tilde{W}^{b\mu\nu}], \tag{10}$$

where $\sigma^a$ are the Pauli matrices; $\tilde{W}^\mu_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} W^{\alpha\beta}$. and $i, j, k, l, m, n$ are $SU(2)_L$ indices. After electroweak symmetry breaking, $H^i = \phi^i = 0$ and $H^3 = \phi^0 = (v + h)/\sqrt{2}$ in the unitary gauge. This projects out the $CP$ violating $hW^+_\mu W^-\mu$ interaction

$$\mathcal{L}_8 \to \frac{2C_8 v^3}{\Lambda^4} h W^+_\mu \tilde{W}^{\mu\nu} \equiv a_L \frac{h}{v} W^+_\mu \tilde{W}^{\mu\nu}. \tag{11}$$

FIG. 1 (color online). Feynman diagrams generating $CP$ violating Higgs couplings. The label L (R) means a left-handed (right-handed) current interaction with the W-boson. The left diagram is in the full theory, and the right one is in the effective theory when the vectorlike quarks are integrated out. The right-handed current $Wtb$ vertex is derived in Eq. (9). For the $hW^+ W^-$ coupling, one of the $\phi^0$ fields is replaced by the electroweak vacuum expectation value, and the other is replaced by $h$. 

The perspective of enology and the constraints on the parameters of the model.
In the last step, we define the coefficient $a_3^W$ in the same notation as Eq. (1.15) in the Higgs Working Group Snowmass report [39].

In reality, because the top quark mass is comparable to the center-of-mass energy $\sqrt{s}$ in the Higgs production and decay processes, the coefficient $a_3^W$ becomes a form factor. We first calculate the form factor for $CP$ violating $W(q_1)h(p)$ associated production [40,41] via an off-shell $W^+(q_2)$. In this case, the momenta satisfy $q_1^2 = M_W^2$, $p^2 = m_b^2$, and $s = q_2^2 \geq (m_h + M_W)^2$. The leading contribution to the form factor $a_3^W$ can be conveniently calculated in the effective theory when the vectorlike quarks are integrated out, as shown in the right-hand side of Fig. 1. We find

$$a_3^W(\sqrt{q_2^2}) = \frac{\sqrt{2N_c G_F m_h}}{4 \pi} \text{Im}(a_1^L a_R)$$

$$\times \int_0^1 dx \int_0^{1-x} dy (1 - x - y) \left[ \frac{1}{\Delta_t} - \frac{1}{\Delta_b} \right],$$

(12)

where the denominators are

$$\Delta_t = (x + y) z_t^2 - xy z_h^2 + (x + y - 1)(x z_t^2 + y z_h^2 - z_b^2),$$

$$\Delta_b = (x + y) z_b^2 - xy z_h^2 + (x + y - 1)(x z_t^2 + y z_h^2 - z_b^2),$$

(13)

and $q_2^2$ are the off-shell momenta of the $W$ gauge bosons, $z_a = m_a/M_W$, $(a = t, h, b)$, $z_t^2 = q_t^2/M_W^2 = 1$, and $z_b^2 = q_b^2/M_W^2 = s/M_b^2$. The $1/\Delta_{t,b}$ terms correspond to the diagrams where the Higgs field is attached to the top (bottom) quark propagators. There is a minus sign between the two pieces in the integrand of the Feynman parameter integral. From the analysis of [38], only the real part of the form factor $a_3^W$ contributes to the final $CP$ violation observable, i.e., a phase shift in azimuthal angle. In Fig. 2, we plot the real part of $a_3^W$ as a function of $\sqrt{q_2^2}$. For $Wh$ associated production, the kinematics require $\sqrt{q_2^2} > m_h + M_W$; i.e., the physical region is the white region to the right of the shaded region in the plot.

We next examine the form factor in the decay process $h(p) \rightarrow W(q_1)W^+(q_2)$. In this case, the momenta satisfy $p^2 = m_b^2$, $q_1^2 = M_W^2$, $0 \leq q_2^2 \leq (m_b - M_W)^2$. The integral of the $1/\Delta_t$ term is real. On the other hand, the integral of the $1/\Delta_b$ term has an imaginary (absorptive) part, which is due to a pole in $y$ corresponding to the on-shell cut of the $b\bar{b}$ propagators. Numerically, we find the integral over $1/\Delta$, and dispersive part of the $1/\Delta_b$ integral almost cancel each other. The integral is dominated by the absorptive part of the $1/\Delta_1$ integral, which we find to be of order 1 for all values of $q_2^2$. Physically, it indicates that the $CP$ violating effect in the $h \rightarrow WW^*$ decay is dominated by processes where the Higgs boson first decays to $b\bar{b}$ and then the $b\bar{b}$ rescatter into $WW^*$.

The coefficient $a_3^W$ calculated above is proportional to the quantity $\text{Im}(a_1^L a_R)$ and is of the order $(v/M)^2$. There is another contribution obtained by changing the right-handed current to a left-handed one in the heavy quark vertex (left diagram of Fig. 1), and we have checked that this contribution is $\mathcal{O}(v/M)^4$ and is subdominant.

Using the central values of masses and constants from the PDG [42], we find in both processes the coefficient for the $CP$ violating $hWW$ interaction is

$$a_3^W = 10^{-3} \times \text{Im}(a_1^L a_R).$$

(14)

The first factor contains the usual loop factor and the bottom quark Yukawa coupling. The $CP$ violating parameter $\text{Im}(a_1^L a_R)$ depends on the model parameters $\lambda_b, \lambda_t$. The goal of the next section is to explore the current and future experimental constraints (sensitivities) to $\text{Im}(a_1^L a_R)$.

### IV. CONSTRAINTS

In this section, we explore phenomenological constraints on the parameter $\text{Im}(a_1^L a_R)$ relevant for the $CP$ violating $hWW$ coupling. We find the most relevant limits come from

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5In general a cut is not necessary for $CP$ violation to occur because the final state $W^+W^*$ is already an eigenstate under $CP$. The cancellation between the $1/\Delta_t$ and dispersive part of the $1/\Delta_b$ integrals seems accidental.
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the electric dipole moments and the rate and CP asymmetry of the rare B decay $b \to s\gamma$.

A. Electric dipole moments

Electric dipole moments are sensitive probes of new sources of CP violation. We first study the constraint from the electron EDM. The interactions Eq. (9) can contribute to the electron EDM through the two-loop Barr-Zee type diagrams as shown in Fig. 3. This contribution has been calculated analytically in Ref. [43],

$$d_e = \frac{-\alpha^2}{8\pi^2 \sin^2 \theta_W M_W} \frac{z_a z_b}{z_a + z_b} \frac{1}{2} x^2 \frac{1}{2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x(1-x)}{(g_b - x(1-x))} \log \frac{g_b - x(1-x)}{g_b - x(1-x)},$$

where $x = x_1 + x_2$, $z_a = m_a / M_W$, $(a = e, t, b)$; and $g_t = x(z_t^2 - z_t^2) + z_t^2$, $g_b = x(z_b^2 - z_b^2) + z_b^2$. Here $d_e$ is the coefficient of the effective EDM operator,

$$L_{\text{eff}} \supset d_e \left( -\frac{i}{2} \bar{e} \sigma_{\mu\nu} \gamma_5 e F^{\mu\nu} \right).$$

Numerically, we find

$$d_e \approx -1.58 \times 10^{-27} \text{Im}(a_t^* a_R)e \cdot \text{cm}. \quad (17)$$

The current experimental upper limit on the electron EDM is $|d_e| < 8.7 \times 10^{-29}e \cdot \text{cm}$ at 90% C.L. from the ACME experiment in 2013 [44]. This translates into the upper bound,

$$|\text{Im}(a_R)| < 0.055. \quad (18)$$

It turns out that the electron EDM constraint is weaker than the one from $B$ physics, as will be discussed in the next subsection, although the EDM constraint will become relevant if the current ACME limit is improved by only a factor of a few. In Fig. 5, the horizontal magenta dotted line shows the future exclusion if the limit reaches 10 times the ACME-2013 limit.

Next, we consider the EDMs of the neutron, the proton and the mercury atom. These constraints usually involve large hadronic and nuclear physics uncertainties. However, given the future prospects of these experiments, they could become relevant. There are several contributions to these observables. The first includes light quark EDMs, from a similar diagram as Fig. 3, with the replacement $(e, \nu) \to (u, d)$ or $(d, u)$. At $\mu = 1 \text{ GeV},$

$$d_n(\mu) = -2.3 \times 10^{-26} \eta_1 \text{Im}(a_t^* a_R)e \cdot \text{cm}, \quad (19)$$

where

$$d_n(\mu) = -2.3 \times 10^{-26} \eta_1 \text{Im}(a_t^* a_R)e \cdot \text{cm}. \quad (20)$$

Here the renormalization group (RG) running effect from the $M_W$ scale down to the GeV scale is taken into account,

$$\eta_1 = \frac{\alpha_s(M_W)}{\alpha_s(m_b)} \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \frac{\alpha_s(m_c)}{\alpha_s(1 \text{ GeV})} \approx 0.417. \quad (21)$$

Hereafter we have used the NLO values of $\alpha_s$ at various scales in the following table.

<table>
<thead>
<tr>
<th>$\alpha_s(M_W)$</th>
<th>$\alpha_s(m_b)$</th>
<th>$\alpha_s(m_c)$</th>
<th>$\alpha_s(1 \text{ GeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.120808</td>
<td>0.218894</td>
<td>0.382156</td>
<td>0.455862</td>
</tr>
</tbody>
</table>

The contribution of Eqs. (19), (20) to the neutron EDM, $d_n \sim -0.35 d_n(\mu) + 1.4 d_n(\mu)$, is too small to yield a competitive constraint on $\text{Im}(a_t^* a_R)$ in view of the current limit $|d_n| < 2.9 \times 10^{-26}e \cdot \text{cm}$.

There is no light quark chromo-EDM at one loop level in the VLQ model. Instead, there is a contribution to the chromo-EDM of the bottom quark, shown in Fig. 4. The bottom quark chromo-EDM can contribute to the EDMs via matching to the three-gluon Weinberg operator at a low scale. The effective Lagrangian for the two operators takes the form [45],

$$L_{\text{eff}} \supset i \frac{\Lambda^2}{\Lambda^2} g_s m_b \bar{b} T^a G^{\mu\nu}$$

$$\supset \frac{C_G}{\Lambda^2} g_s f^{abc} \epsilon_{\mu\nu\rho\sigma} G^{a}_{\mu\nu} G^{b}_{\rho\sigma}. \quad (22)$$

\text{Unlike Ref. [43], we find that the contributions of diagrams similar to Fig. 3 but with photon lines replaced by gluon ones and leptons replaced by light quarks vanish and do not give rise to chromo-EDMs.}
The calculation of the coefficient of the chromo-EDM operator is similar to that in the left-right symmetric model [46]. At the weak scale, \[ \frac{\tilde{\delta}_b(M_W)}{\Lambda^2} = -\frac{\sqrt{2}G_F m_b}{8\pi^2 m_b} \operatorname{Im}(a_L^b a_R) f(z_i), \] where \[ f(z_i) = [1 - \frac{3}{4} z_i^2 - \frac{1}{4} z_i^6 + \frac{3}{4} z_i^2 \log z_i^2] / (1 - z_i^2) \approx 0.35. \] Interestingly, there is an enhancement factor \( (m_b/m_t) \) [47]. At the scale \( m_b \), the matching condition is \( C_G(m_b) = \frac{1}{12}\frac{\alpha_s(m_b)}{\alpha_s(1 \text{ GeV})} \tilde{\delta}_b(m_b) \) [7]. Taking into account the RG running, the coefficient \( C_G \) at \( \mu = 1 \text{ GeV} \) is

\[ C_G(\mu) = \frac{\alpha_s(m_b)}{12\pi} \left[ \frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right] \frac{\tilde{\delta}_b(M_W)}{\Lambda^2} \times \left[ \frac{\alpha_s(m_e)}{\alpha_s(1 \text{ GeV})} \right] \frac{\tilde{\delta}_b(m_e)}{\Lambda^2} \approx -4.5 \times 10^{-3} \operatorname{Im}(a_L^b a_R) \text{ GeV}^{-2}. \]

The final contribution to the neutron EDM is dominated by the Weinberg operator [7,45],

\[ d_n = (2 \times 10^{-20} e \cdot \text{cm}) \left( \frac{v^2}{\Lambda^2} \right) C_G(\mu) \approx -5.5 \times 10^{-24} \operatorname{Im}(a_L^b a_R) e \cdot \text{cm}, \]

where we have used the hadronic matrix element given in Ref. [7]. The current limit on the neutron EDM, \( |d_n| < 2.9 \times 10^{-26} e \cdot \text{cm} \) at 95% C.L., translates into the upper bound,

\[ |\operatorname{Im}(a_L^b a_R)| < 0.5 \times 10^{-2}. \]

Because the three-gluon operator is an isospin singlet, the proton EDM in this model is equal to the neutron EDM. A possible future experiment measuring the proton EDM is expected to give a strong constraint [48,49].

The EDM of mercury \(^{199}\text{Hg}\) is also sensitive to the three-gluon operator, which contributes through the Schiff moment [7,45].
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The SM prediction has a central value $B(B \to X_s \gamma)_{\text{SM}} = 3.15 \times 10^{-3}$. The world average of the measurements is $B(B \to X_s \gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}$ [52].

In order to present this result as a limit, we note the fact that in the VLQ model the contribution to $a_R$ dominates over the deviation $a_L - 1$. Using results from the EDM discussions that $\text{Im}(a_L^* a_R)$ is already constrained to be less than $\sim 10^{-2}$, we conclude that

$$\text{Im}(a_L^* a_R) = \text{Im}(a_R).$$

In this case, the constraint from the $B \to X_s \gamma$ decay rate measurement is presented in the parameter space of $\text{Re}(a_R)$ versus $\text{Im}(a_R)$, as shown by the red shaded region in Fig. 5.

The direct $CP$ asymmetry in the $B \to X_s \gamma$ decay rate is [53]

$$A_{CP} = a_s(m_b) \left\{ \frac{40}{81} \text{Im} \left( \frac{c_2}{c_7} \right) - \frac{4}{9} \text{Im} \left( \frac{c_8}{c_7} \right) \right\} - \frac{40 \Lambda_c}{9 m_b} \text{Im} \left[ \left( 1 + c_s \right) \frac{c_2}{c_7} \right],$$

where $c_2 = 1.11$, $c_s = -0.007 + 0.018 i$, $\Lambda_c = 0.38$ GeV, and $a_s(m_b) = a_s(M_W)/\pi = 0.21$. The most stringent experimental measurement is from BABAR, $A_{CP} = (1.7 \pm 1.9 \pm 1.0)\%$ [54]. Again, we show this as a constraint in the $\text{Re}(a_R)$ versus $\text{Im}(a_R)$ plane in Fig. 5. The regions inside the yellow circles are excluded.

Summarizing the EDM and the $B$ physics constraints, we conclude that in the doublet VLQ model it is still possible to have $\text{Im}(a_R)$ as large as order 0.01. From Eq. (14), this implies the $CP$ violating $hWW$ coupling $a_3^W$ is currently constrained to be at most $10^{-5}$. The next generation EDM search is expected to further narrow down the allowed window of $\text{Im}(a_R)$. In the case of discovery, this would trigger an exciting interplay between the studies of $CP$ violation in a future $B$ factory and a future Higgs factory.

V. CONCLUSION

In this work, we have studied the possibility of introducing $CP$ violating interactions to the 125 GeV Higgs boson by extending the fermion sector of the SM with vectorlike quarks. We examined the simplest class of models where VLQs arise from a single representation under the SM gauge group. There are seven possible representations where the VLQs have Yukawa interactions with the SM Higgs doublet and third generation quarks. The new complex Yukawa couplings could accommodate new sources of $CP$ violation. Among them, we find that an irreducible $CP$ phase shows up only for one representation of VLQ, which is a doublet under $SU(2)$ and carries hypercharge $1/3$. For the other representations all the phases can be rotated away and are unphysical.

We study the $CP$ violating phenomenology in the doublet VLQ model. Since the VLQs are already constrained to be heavier than 800 GeV by the LHC, we integrate them out and study the effective theory, where $CP$ violation manifests itself through a new right-handed charged current mediated by the SM $W$-boson. We have calculated the $CP$ violating Higgs interactions with SM gauge bosons, generated at one loop level involving both top and bottom quarks. This corresponds to a dimension 8 operator in the heavy top/bottom quark limit. As a consequence, only the $hWW$ coupling is $CP$ violating, while the $hZZ$, $h\gamma\gamma$, and $hZ\gamma$ couplings are essentially $CP$ conserving at this order. The strength of the $CP$ violating $hWW$ coupling is proportional to the quantity $\text{Im}(a_L^* a_R)$, where $a_{L,R}$ are the coefficients of left- and right-handed current $Wtb$ interactions, respectively. At low energy, we find the most relevant constraints on $\text{Im}(a_L^* a_R)$ come from the electric dipole moments and the $b \to s\gamma$ decay rate and $CP$ asymmetry, which are complementary to each other. The current experimental constraints require $\text{Im}(a_L^* a_R) \lesssim 0.01$. They in turn imply that the coefficient of the $CP$ violating $hWW$ interaction, $a_3^W$, cannot be larger than of order $10^{-5}$, and, as we stress again, only in the $hWW$ channel. We expect exciting interplays of various experimental searches in the future to probe and distinguish new sources of $CP$ violation near the electroweak scale.
ACKNOWLEDGMENTS

We thank JiJi Fan, Enrico Lunghi, and Miha Nemevsek for useful discussions. The work of C.-Y. Chen and S. Dawson is supported by the U.S. Department of Energy (DOE) under Grants No. DE-AC02-98CH10886 and Contract No. DE-AC02-76SF00515. This work of L. Randall and R. Sundrum, A Large Mass Hierarchy from a Small Extra Dimension, Phys. Rev. Lett. 83, 3370 (1999).


[33] Y. Zhang is supported by the Gordon and Betty Moore Foundation through Grant No 776 to the Caltech Moore Center for Theoretical Cosmology and Physics, and by the DOE Grant No. DE-FG02-92ER40701, and also by a DOE Early Career Award under Grant No. DE-SC0010255.


