Double-beta decay to excited 0^+ states: Decay of ^{100}Mo

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We develop a formalism for describing the 2ν mode of double-beta decay to an excited final state in the quasiparticle random phase approximation (QRPA). With this, we deduce the half-lives for the ^{100}Mo double-beta decays to both the ground and 1130 keV excited 0^+ states in ^{100}Ru. We predict the matrix elements of these two transitions to be of a similar magnitude. We also calculate the strengths of the associated single-beta decays from the ground state of the intermediate nucleus, ^{100}Tc, and compare them with experiment. We find that the QRPA cannot simultaneously reproduce all of the experimental quantities, and, in particular, the single-beta transition between the initial and intermediate nuclei is overestimated by the theory. In addition, we demonstrate that the influence of the particle-particle force is an important factor in the calculation of both β^- and β^+ transition strengths.

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I. INTRODUCTION

Double-beta decay is the process by which a nucleus with neutron and proton numbers (N,Z) undergoes a transition to the nucleus $(N-2,Z+2)$ when the single-beta decay to the intermediate nucleus $(N-1,Z+1)$ is energetically forbidden. When accompanied by the emission of two neutrinos, in addition to the two electrons, this mode of decay may occur in second order in the standard theory of weak interactions. There has been much work, both experimental and theoretical, to study these 2ν decays [1–3], which have focused so far upon transitions between the ground states of the initial and final even-even nuclei.

However, recently, an indication of the 2ν double-beta decay from the ground state of ^{100}Mo to the excited 1130 keV 0^+ state in ^{100}Ru has been reported [4]. This state, which we believe to be a member of a $0^+,2^+,4^+$ two-phonon vibrational triplet, promptly de-excites to the 540 keV 2^+ one-phonon vibrational state and finally on to the ground state. The observation of the emitted gamma ray cascade leads to a half-life of $1.78^{+1.85}_{-0.60}\times 10^{21}$ yr. This is to be compared with the half-life for the ground state to ground state 2ν double-beta decay of $1.16^{+0.34}_{-0.08}\times 10^{19}$ yr [5]. Difficulties in analyzing this latter decay have been described recently in Ref. [6].

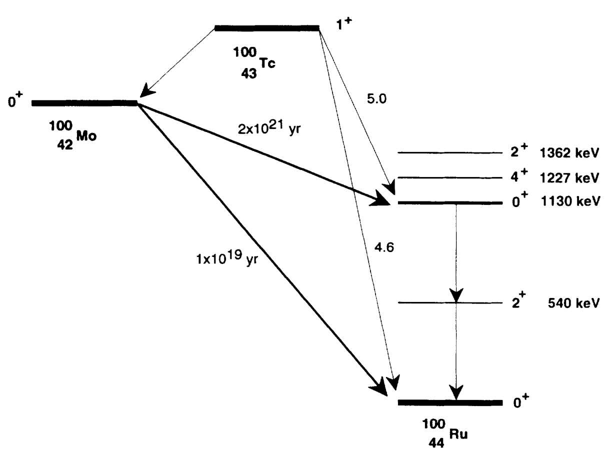

FIG. 1. Partial level scheme for the decay of ^{100}Mo. The experimental 2ν double-beta decay half-lives and $\log ft$ values for the relevant single-beta decays from the intermediate nucleus are indicated.

Since the phase space factor for the less energetic ground state to excited state transition is 56 times smaller than that of the corresponding ground state to ground state transition, these experimental results imply that the nuclear matrix elements governing these two modes of decay are of approximately equal magnitude.

It is the purpose of this paper to determine whether or not this can be understood theoretically. Since the ground state of the intermediate nucleus, ^{100}Tc, happens to be 1^+, the theoretical description should include evaluation of the single-beta decay amplitudes connecting this state with the ground states of the initial and final nuclei, as well as the 1130 keV 0^+ state in ^{100}Ru. Such a detailed description, involving five pieces of experimental data, as shown in Fig. 1, represents a very severe test of our ability to describe the Gamow-Teller transitions in a relatively heavy nucleus.

II. FORMALISM

We use the quasiparticle random phase approximation (QRPA) to model both the states in the intermediate odd-odd nucleus (proton-neutron QRPA) and the excited states in the final even-even nucleus (standard charge conserving QRPA). Following Refs. [7,8] we describe the ground state to ground state 2ν decay as follows.

States, labeled k, of spin J and projection M are generated in the intermediate odd-odd nucleus $(Z+1,N-1)$ using QRPA operators of the form

$$Q_k^{\dagger,JM}=\sum_{pn}\left[X_{pn}^J(k)b_{pn}^{\dagger,JM}-(-1)^M Y_{pn}^J(k)b_{pn}^{J-M}\right], \qquad (1)$$

where the proton-neutron quasiparticle pair operators are written

$$b_{pn}^{\dagger,JM}=\sum_{m_p,m_n}(j_p m_p j_n m_n|JM)\alpha_{j_p m_p}^{\dagger}\alpha_{j_n m_n}^{\dagger}\ , \tag{2}$$

and follow the usual boson commutation relations. The ground state 0_i^+ of the initial nucleus (Z,N) and the ground state $0_f^{+(g)}$ of the final nucleus $(Z+2,N-2)$ are the QRPA vacua for the operators Q_k^{JM}, and the amplitudes $X_{pn}^J(k)$ and $Y_{pn}^J(k)$ are obtained by solving the corresponding QRPA equations. The pairs $b_{pn}^{\dagger,JM}$ are allowed to interact in both the particle-hole and particle-particle channels.

If T^{JM} is the one-body operator that transforms a neutron into a proton, then, within the framework of the QPRA, T^{JM} may be expressed in form

$$T^{JM}=\sum_{pn}t_{pn}^{-,J}b_{pn}^{\dagger,JM}+(-1)^M t_{pn}^{+,J}b_{pn}^{J-M}\ , \tag{3}$$

where

$$t_{pn}^{-,J}=\frac{u_p v_n}{\sqrt{2J+1}}\langle p\|T^J\|n\rangle \tag{4}$$

and

$$t_{pn}^{+,J}=(-1)^J\frac{v_p u_n}{\sqrt{2J+1}}\langle p\|T^J\|n\rangle\ . \tag{5}$$

With these definitions we can now write down the relevant transition amplitudes for single-beta decays. Thus, for the "β^--like" transition from the ground state 0_i^+ of the initial nucleus (Z,N) to some state $|k;JM\rangle$ in the intermediate nucleus $(Z+1,N-1)$, we have in this approximation

$$\langle k;JM|T^{JM}|0_i^+\rangle=\sum_{pn}[t_{pn}^{-,J}X_{pn}^J(k)+t_{pn}^{+,J}Y_{pn}^J(k)]\ . \tag{6}$$

On the other hand, the "β^+-like" transition amplitude from the ground state $0_f^{+(g)}$ of the final nucleus $(Z+2,N-2)$ to the same intermediate state is written

$$(-1)^M\langle k;JM|T^{\dagger,J-M}|0_f^{+(g)}\rangle=\sum_{pn}[\bar{t}_{pn}^{-,J}\bar{Y}_{pn}^J(k)+\bar{t}_{pn}^{+,J}\bar{X}_{pn}^J(k)]\ , \tag{7}$$

where the quantities with an overbar refer to the pairing factors and X,Y amplitudes of the final nucleus. We define a "β^--like" transition to be one for which the conversion of a neutron into a proton increases the number of quasiparticles in the final state. A "β^+-like" transition, on the other hand, leads to a reduction in the total number of quasiparticles.

In order to obtain the decay rate for the 2ν ground state to ground state double-beta decay we assume that the transitions are allowed and are predominantly Gamow-Teller (GT) transitions. The inverse half-life for this process is then given by the formula

$$[T_{1/2}^{2\nu}(0_i^+\rightarrow 0_f^{+(g)})]^{-1}=G^{2\nu}(0_i^+\rightarrow 0_f^{+(g)})\times|M_{\mathrm{GT}}^{2\nu}(0_i^+\rightarrow 0_f^{+(g)})|^2\ , \tag{8}$$

where $G^{2\nu}$ is the calculable phase space factor and

$$M_{\mathrm{GT}}^{2\nu}(0_i^+\rightarrow 0_f^{+(g)})=\sum_{k,M}\frac{\langle k;1M|T^{1M}|0_i^+\rangle(-1)^M\langle k;1M|T^{\dagger,1-M}|0_f^{+(g)}\rangle}{E_k-(M_i+M_f)/2}\ . \tag{9}$$

Owing to the approximate nature of the QRPA, the intermediate states $|k,1M\rangle$ calculated in (6) using the pairing factors and amplitudes X,Y of the initial nucleus are not identical to those calculated in (7) using the pairing factors and amplitudes $\bar{X},\bar{Y}$ of the final nucleus. In order to circumvent this problem, we insert into (9) a QRPA overlap factor of the form [9]

$$[X_{pn}^1(k)\bar{X}_{\bar{p}\bar{n}}^1(\bar{k})-Y_{pn}^1(k)\bar{Y}_{\bar{p}\bar{n}}^1(\bar{k})]\ . \tag{10}$$

We now turn to the description of the 2ν ground state to excited state double-beta decay. While our derivation is valid for any state that in the Tamm-Dancoff approximation (TDA) is a superposition of four-quasiparticle states, we consider in detail the case relevant to ^{100}Mo where the final state $0_f^{+(e)}$ is the spin-zero member of the two-phonon triplet of states.

In the following we shall use two different forms of the QRPA. The proton-neutron QRPA, with phonons of Eq. (1), describes the charge-changing excitations built on the ground state (phonon vacuum) of an even-even nucleus. The p,n phonon states $|k;JM\rangle=Q_k^{\dagger,JM}|0^{+(g)}\rangle$ are states with angular momentum J in the odd-odd nuclei $(Z+1,N-1)$ and $(Z-1,N+1)$. The equations of the proton-neutron QRPA are given in Refs. [7,8].

The collective vibrational states in even-even nuclei are described by the standard, charge-conserving QRPA (see the monograph by Ring and Schuck [10] for details). The phonon operators, Eq. (12) below, contain two like-particle creation and annihilation operators, Eq. (13). Since we are considering transitions connecting states in odd-odd nuclei (pnQRPA) and excited states in even-even nuclei (standard QRPA), we have to use both forms of the method. In Eqs. (11)–(19) below we display the expressions we need.

Ideally, the same residual interaction should be used everywhere, in particular in both QRPA equations of motion relevant in our case. In practice, however, the renormalization effects in different channels are different and therefore we are forced to renormalize interaction strengths in each J^π channel, as described in the next section.

As stated earlier, the excited 0^+ state in the final nucleus is given by

$$|0_f^{+(e)}\rangle=\frac{1}{\sqrt{2}}[Q_1^{\dagger,2}Q_1^{\dagger,2}]^0|0_f^{+(g)}\rangle\ , \tag{11}$$

where $Q_1^{\dagger,2}$ is the creation operator of the first excited one-phonon 2^+ vibrational state, defined in analogy to (1) but in the standard QRPA as

$$Q_l^{\dagger,JM}=\sum_{rs=pp',nn'}[X_{rs}^J(l)b_{rs}^{\dagger,JM}-(-1)^M Y_{rs}^J(l)b_{rs}^{J-M}]\ , \tag{12}$$

with

$$b_{rs}^{\dagger,JM}=\sum_{m_r,m_s}(j_rm_rj_sm_s|JM)\alpha_{j_rm_r}^\dagger\alpha_{j_sm_s}^\dagger\ . \tag{13}$$

Naturally, we have to solve *another* set of QRPA equations to obtain the amplitudes X,Y occurring in Eq. (12). (Note that in the following these $J=2^+$ quadrupole amplitudes will be denoted X^2,Y^2 in contrast to the $J=1^+$ amplitudes X^1,Y^1 of the intermediate states.)

Having determined the nature of the final excited state, we proceed to calculate the transition amplitudes. The matrix elements for the "β^--like" transitions between the ground state of the initial nucleus and levels in the intermediate nucleus are unchanged and given by (6), while the matrix elements describing the transitions from the intermediate 1^+ levels to the final excited 0^+ state are given by

$$\begin{aligned}(-1)^M&\langle k;1M|T^{\dagger,1-M}|0_f^{+(e)}\rangle\\ &=(-1)^M\langle 0_f^{+(g)}|Q_k^{1M}T^{\dagger,1-M}[Q_1^{\dagger,2}Q_1^{\dagger,2}]^0|0_f^{+(g)}\rangle\\ &=-\frac{1}{\sqrt{3}}\langle 0_s^{+(g)}|[Q_k^1T^{\dagger,1}]^0[Q_1^{\dagger,2}Q_1^{\dagger,2}]^0|0_f^{+(g)}\rangle\ .\end{aligned} \tag{14}$$

For simplicity in writing we will develop this expression in the TDA ($Y\to 0$).

The normalized pp,nn contribution to $0_f^{+(e)}$ is

$$[Q_1^{\dagger,2}Q_1^{\dagger,2}]^0=\sqrt{2}\sum_{pp',nn'}X_{pp'}^2(1)X_{nn'}^2(1)\times\left[\frac{b_{pp'}^{\dagger,2}}{\sqrt{1+\delta_{pp'}}}\frac{b_{nn'}^{\dagger,2}}{\sqrt{1+\delta_{nn'}}}\right]^0\ , \tag{15}$$

where the pp and nn quadrupole pair creation operators $b_{pp'}^{\dagger,2M}$ and $b_{nn'}^{\dagger,2M}$ are defined in Eq. (13). It is easy to see that only such pp,nn components of the state (11) can be populated in double-beta decay.

On the other hand, the term

$$[Q_k^1T^{\dagger,1}]^0=\sum_{pn,p',n'}t_{p'n'}^{-,1}X_{pn}^1(k)[b_{pn}^{\dagger,1}b_{p'n'}^{\dagger,1}]^{\dagger,0} \tag{16}$$

is defined with respect to the pn 1^+ pair creation operators $b_{pn}^{\dagger,1M}$. We therefore recouple these to pp,nn quadrupole pairs using the relation

$$[b_{pn}^{\dagger,1}b_{p'n'}^{\dagger,1}]^0=-\sqrt{15}(-1)^{j_n+j_{p'}}\begin{Bmatrix}j_p & j_{n'} & 1\\ j_n & j_{p'} & 2\end{Bmatrix}[b_{pp'}^{\dagger,2}b_{nn'}^{\dagger,2}]^0\ , \tag{17}$$

and evaluate (14) to give

$$(-1)^M\langle k;1M|T^{\dagger,1-M}|0_f^{+(e)}\rangle=\sqrt{10}\sum_{pp'nn'}(-1)^{j_n+j_{p'}}\begin{Bmatrix}j_p & j_{n'} & 1\\ j_n & j_{p'} & 2\end{Bmatrix}\sqrt{1+\delta_{pp'}}\sqrt{1+\delta_{nn'}}t_{p'n'}^{-,1}X_{pn}^1(k)X_{pp'}^2(1)X_{nn'}^2(1)\ . \tag{18}$$

The neglected RPA terms can be reintroduced into this expression by means of the substitution

$$t_{p'n'}^{-,1}X_{pn}^1(k)X_{pp'}^2(1)X_{nn'}^2(1)\to t_{p'n'}^{-,1}X_{pn}^1(k)X_{pp'}^2(1)X_{nn'}^2(1)-t_{p'n'}^{+,1}Y_{pn}^1(k)Y_{pp'}^2(1)Y_{nn'}^2(1)\ . \tag{19}$$

In comparing this transition amplitude leading to the final excited state with the corresponding transition to the ground state we find an important difference. Namely, the combination of $t_{p'n'}^{-,1}$ with $X_{pn}^1(k)$ occurring in (19) is not the same as in (7), but instead resembles that of the transition amplitude (6). This implies that the excited state transition should exhibit the behavior of a "β^--like," rather than a "β^+-like," transition and therefore should *not* be strongly suppressed by the presence of the particle-particle interaction. We will discuss this dependence further in the following section.

Finally, the rate for the 2ν ground state to excited state double-beta decay rate is calculated from formulas analogous to (8) and (9), with the phase space factor $G^{2\nu}(0_i^+\to 0_f^{+(e)})$ and the energy denominators in the matrix element $M_{\rm GT}^{2\nu}(0_i^+\to 0_f^{+(e)})$ corrected to account for the increased mass of the final state.

III. PARAMETERS AND NUMERICAL RESULTS

In the extreme single particle limit, we would expect the 2ν double-beta decay transition in ^{100}Mo to be dominated by the intermediate nucleus spin-orbit partner configuration $[p1g_{9/2}n1g_{7/2}]^{1^+}$. The results are therefore very sensitive to the positions of these two orbitals, and in particular the $n1g_{7/2}$, with respect to their Fermi levels. Consequently, we choose the single particle energies to reproduce experimental reaction data as closely as possible. Thus for the "active" proton orbitals we assume single particle energies for the $3p_{1/2}$ and $1g_{9/2}$ orbitals of 0.0 and 0.9 MeV, respectively [11,12]. For the "active" neutron orbital we assume single particle energies for the $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1g_{7/2}$ of 0.0, 1.0, 2.0, and 2.7 MeV, respectively, as deduced from the ^{88}Sr(d,p) reaction [11]. In addition, we included all other subshells within 10 MeV of the Fermi level and their spin-orbit partners. The energies of these more distant states were taken from the Coulomb corrected Woods-Saxon potential advanced by Bertsch [13].

To obtain the quasiparticle energies and pairing factors u and v, we solve the BCS equations adopting a schematic δ force. In this way we were able to reproduce the experimental pairing energies, as deduced from the odd-even nuclear mass differences, using interaction strengths of $g_{\text{pair}}=-280$ and -260 MeV fm^3 for protons and neutrons, respectively.

The residual particle-hole and particle-particle interactions entering into the RPA calculations are also represented by a δ interaction. We choose the zero-range δ force for the ease of handling and treat the particle-hole and particle-particle coupling constants as independent parameters (see Ref. [8]). We obtain qualitatively similar results with a more realistic G-matrix based finite-range nucleon-nucleon force. The δ force is parametrized in terms of the strength parameters α_0 and α_1 for the isospin $T=1$ and 0 components of the particle-hole force, and similarly defined strengths α_0' and α_1' for the particle-particle force. (The notation of [8], used here, might appear confusing. However, since the δ force acts only in the $S,T=0,1$ and 1,0 states, the subscript i of α_i and α_i' characterizes S, not T.)

In the proton-neutron QRPA, the values of the two particle-hole parameters α_0 and α_1 largely determine the energies of the isobaric analogue state and the GT giant resonances. Fitting to these energies for a wide range of nuclei leads to the values $\alpha_0=-890$ MeV fm^3 and $\alpha_1=-1010$ MeV fm^3 [8].

With the strength of the particle-hole interaction thus fixed, we turn to the determination of the particle-particle strengths α_1' and α_0'. Previously, fits to the β^+/EC decay rates of a number of semimagic nuclei restricted α_1' to a window between -390 and -432 MeV fm^3 [8]. This range, under the assumption that the spin-singlet force is approximately 60% as strong as the spin-triplet force, leads to a value of α_0' which is consistent with the strength g_{pair} required to obtain the experimental pairing energies.

Finally, in the standard charge-conserving QRPA calculation, we were forced to reduce the strength of the residual pp and nn particle-hole force since ^{100}Ru is rather close to the QRPA collapse in the 2^+ channel. Again, this finding remains true even when the finite-range force is used. Thus, in order to reproduce the experimental energy of the first excited 2^+ state in ^{100}Ru we use the parameters $\alpha_0'\simeq g_{\text{pair}}$ and $\alpha_0=\alpha_1=-312$ MeV fm^3 (α_1' does not appear in this case since two like nucleons cannot be in a $T=0$ state). At the same time, the experimental $B(E2)$ value for the associated ground state gamma transition is given using a polarization charge of $e_{\text{pol}}=0.09$.

As mentioned earlier, the ground state of the intermediate nucleus ^{100}Tc being the spin 1^+ exhibits a number of GT single-beta transitions to the states in the neighboring nuclei (see Fig. 1). In particular, there are the transitions to the ground state of the initial nucleus, and the excited and ground states of the final nucleus, whose rates are directly proportional to the squares of the transition amplitudes given in Eqs. (6), (7), and (18), respectively. With a given set of strength parameters these single-beta decay rates should be reproduced correctly in order to have confidence in the quality of the calculated double-beta decay matrix elements.

We list the experimental logft values for these three decays in Table I. (In fact, the transition between the ground states of ^{100}Tc and ^{100}Mo has not been observed. We therefore assume that the rate is similar to that of the corresponding transitions in the adjacent nuclei, ^{98}Zr and ^{102}Mo, both of which have log$ft=4.2$.) Also given are the resulting $B(\text{GT})(0^+\rightarrow 1^+)$ values, calculated according to the formula

$$B(\text{GT})=\frac{6160}{g_A^2 ft}\ , \tag{20}$$

where we choose $g_A=1.0$ to take account of the effect of distant states responsible for the "missing strength" in the giant GT resonance. Note that in the previous section we adopted a convention in which the matrix elements correspond to transitions *from* 0^+ states in the even-even nuclei to 1^+ states in the intermediate nucleus. Where this is not the case experimentally, the B(GT) values have been adjusted by the required spin multiplicity factor. Finally, we tabulate the experimental lifetimes and the corresponding matrix elements for the 2ν double-beta decays.

The theoretical $B(\text{GT})(0^+\rightarrow 1^+)$ values for the three single-beta decays are plotted in Fig. 2 as a function of the particle-particle strength parameter α_1'.

Firstly, we will consider the strengths of the two "β^--like" transitions, namely, $[1_1^+\rightarrow 0_i^+]$ and $[1_1^+\rightarrow 0_f^{+(e)}]$, as derived from Eqs. (6) and (18), respectively. As illustrated in the figure, such transitions are enhanced by the action of particle-particle force. Comparing these values with the experimental results given in Table I, we see that the $[1_1^+\rightarrow 0_f^{+(e)}]$ transition is correctly predicted in the vicinity of $\alpha_1'=-450$ MeV fm^3, close to the window of values deduced in [8]. On the other hand, the $[1_1^+\rightarrow 0_i^+]$ transition is overestimated by a factor of more than five in the same window. (As mentioned previously, the calculations are sensitive to the adopted single particle level scheme, and in particular to the position of the $n\,1g_{7/2}$ subshell. In fact, the Woods-Saxon potential [13] predicts a slightly different level ordering from the set given

TABLE I. Experimental single-beta decay logft values and GT strength (upper part) and double-beta decay half-lives and GT matrix elements (lower part) for the transitions indicated in Fig. 1.

	logft	$B(\text{GT})(0^+\rightarrow 1^+)$
$\beta[0_i^+\rightarrow 1_1^+]$	4.2	0.39
$\beta[1_1^+\rightarrow 0_f^{+(g)}]$	4.6	0.46
$\beta[1_1^+\rightarrow 0_f^{+(e)}]$	5.0	0.18
	$T_{1/2}^{2\nu}$ (yr)	$M_{\text{GT}}^{2\nu}$ (MeV^{-1})
$\beta\beta[0_i^+\rightarrow 0_f^{+(g)}]$	$1.16(^{+0.34}_{-0.08})\times 10^{19}$	$\pm 0.30(^{+0.01}_{-0.04})$
$\beta\beta[0_i^+\rightarrow 0_f^{+(e)}]$	$1.78(^{+1.85}_{-0.60})\times 10^{21}$	$\pm 0.18(^{+0.04}_{-0.05})$

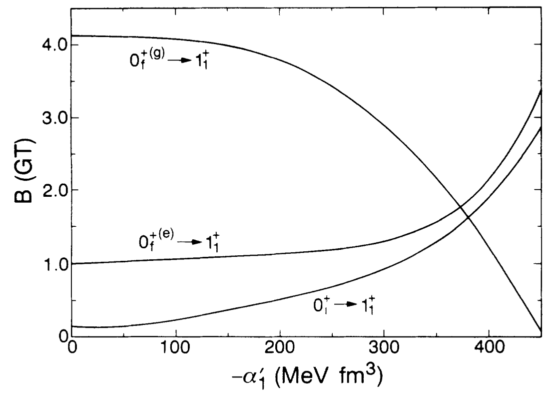

FIG. 2. The dependence of the B(GT) strengths for the single-beta decays upon the particle-particle interaction strength. The $0_f^{+(e)} \rightarrow 1_1^+$ curve is multiplied by a factor of 20.

above, placing this subshell close to the neutron Fermi level. However, this serves only to increase these amplitudes for a given value of the particle-particle force and therefore is not able to explain the experimental $[1_1^+ \rightarrow 0_i^+]$ transition strength.)

By contrast, the behavior of the "β^+-like" transition, $[1_1^+ \rightarrow 0_f^{(g)}]$, is significantly different. Here, the QRPA ground state correlations induced by the particle-particle force acts so as to suppress the amplitude (7), which ultimately can be made to pass through zero. An explanation of this effect has already been given in [8]. From Fig. 2 we see that the experimental transition strength is reproduced near $\alpha_1' = -430$ MeV fm^3.

Thus it would appear that, with the proposed window of values for the parameter α_1', the QRPA model is able to adequately explain the strengths of both single-beta transitions from ^{100}Tc to ^{100}Ru, but overestimates the (expected) strength of the transition from ^{100}Tc to ^{100}Mo.

We now turn to the calculated matrix elements $M_{\rm GT}^{2\nu}$ for the two double-beta transitions, shown in Fig. 3. As expected, the matrix element for the ground state to excited state transition is enhanced by the particle-particle force, depending as it does upon the product of two "β^--like" amplitudes, while the ground state to ground state transition, depending in part upon a "β^+-like" amplitude, is suppressed and may pass through zero. By inspection, we see that the experimental values listed in Table I may be reproduced at $\alpha_1' = -440$ MeV fm^3 for the ground state to ground state transition, while the ground state to excited state transition would appear to require little or no particle-particle interaction at all.

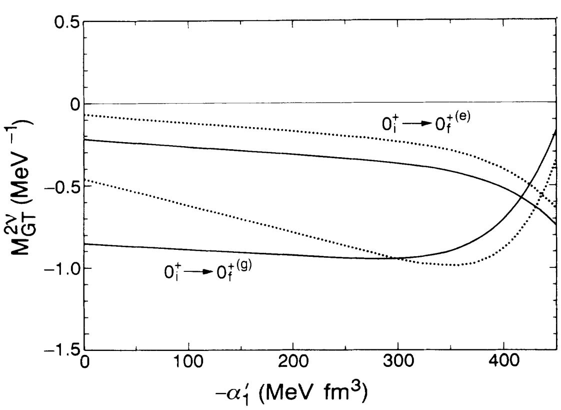

FIG. 3. The dependence of the 2ν ground state to ground state and ground state to excited state double-beta decay matrix elements upon the particle-particle interaction strength. The contributions to each from the lowest 1^+ intermediate (ground) states are shown dotted.

Also plotted in Fig. 3 as the dotted lines are the contributions to the total matrix elements from the lowest 1^+ (ground) state in the intermediate nucleus. As can be seen, these constitute a significant fraction of the net effect. This is only to be expected, since, as we stated before, the lowest intermediate state is dominated by the $[p\,1g_{9/2}n\,1g_{7/2}]^{1^+}$ configuration which is accompanied by strong GT transitions. In consequence, the matrix elements for double-beta decay rely strongly on the three single-beta transitions mentioned above. Thus, the 2ν decay rates and the three single-beta decay rates must be considered together.

We now discuss two possible scenarios in which the QRPA calculations can be made to resemble the experimental data. In both cases we are faced with the obvious problem that at least some of the calculated single-beta transitions are faster than in the experiment. To remedy (or parametrize) it we introduce a quenching factor affecting certain transitions.

Firstly, we can reduce the $[1_1^+ \rightarrow 0_i^+]$ transition amplitude by a factor of between 2 and 3. All three single-beta transitions will then have approximately their experimental values within the prescribed window for α_1'. In turn, this will reduce the contribution of the lowest intermediate state to the total double-beta decay matrix elements by the same factor. The effects of this renormalization can be estimated from Fig. 3. The experimental value is now reproduced in the region of $\alpha_1' = -390$ MeV fm^3. The matrix element $M_{\rm GT}^{2\nu}[0_i^+ \rightarrow 0_f^{+(e)}]$ of the ground state to excited state transition is also reduced in magnitude. While this improves the agreement with experiment, the theoretical value is still somewhat overestimated. For example, at $\alpha_1' = -390$ MeV fm^3 we calculate a matrix element of -0.29. Thus with this *ad hoc* quenching of the $[1_1^+ \rightarrow 0_i^+]$ transition amplitude it is possible to obtain reasonable experimental agreement for all the other quantities within the suggested range of α_1', although there is no single value which satisfies all conditions simultaneously.

As an alternative we consider the possibility that the QRPA overestimates all transitions proceeding through the lowest intermediate state by the same factor. However, there is no value of α_1' for which all three single-beta transitions are incorrectly predicted by identical factors. For example, at $\alpha_1' = -370$ MeV fm^3 we can reduce all transition amplitudes by a factor of one-half. Both the $[1_1^+ \rightarrow 0_i^+]$ and $[1_1^+ \rightarrow 0_f^{+(g)}]$ transition amplitudes are then correctly given, while the $[1_1^+ \rightarrow 0_f^{+(e)}]$ transition amplitude becomes a factor of 3 too small when compared to experiment. At the same time the contribution of the lowest intermediate state to the total double-beta decay matrix elements is reduced to a quarter of the values indicated in Fig. 2. The theoretical ground state to ground state matrix element thus becomes -0.11. Only the ground state to excited state matrix element, which we calculate to be -0.19, is close to the experimental value. This scenario is therefore somewhat less successful at explaining the experimental data than the preceding one.

It is clear that, in this case at least, the QRPA approach has a limited capability of reproducing all of the required experimental criteria. Nevertheless, our calculations clearly indicate that the most important contribution to the two double-beta decay rates comes from the ground state of the intermediate nucleus ^{100}Tc, for which the relevant single-beta decays are experimentally known. In fact, if we assume that this state is the *only* one to contribute to the double-beta decay we obtain ± 0.25 and ± 0.24 for the $M_{GT}^{2\nu}$ leading to the ground and excited states of ^{100}Ru, respectively, quite close to the experimental values in Table I. (A similar proposal was made earlier by Abad *et al.* [14].)

Given the inability of the QRPA to explain all of the pertinent experimental data in the present system, it is reasonable to inquire whether this problem is present in other double-beta decay candidates. There is in fact one other case, ^{128}Te, for which the intermediate nucleus, ^{128}I, has a 1^+ ground state (corresponding to the $[p2d_{5/2}n2d_{3/2}]^{1^+}$ configuration in the extreme single particle limit), which we shall now briefly discuss.

We consider first the "β^--like" $[1_1^+ \to 0_i^+]$ transition. The QRPA calculations predict B(GT) strengths ranging from 0.6 to 1.1 as the particle-particle interaction strength α_1' is varied between 0 and -450 MeV fm^3, respectively. This is to be compared with the experimental value of $B(\mathrm{GT})(0^+ \to 1^+) = 0.15$ ($\log ft = 5.1$). Thus the "β^--like" transition strength is overestimated by almost an order of magnitude within the preferred window of values for α_1', as indeed was the situation in ^{100}Mo.

For the "β^+-like" $[1_1^+ \to 0_f^{+(g)}]$ transition, on the other hand, the QRPA calculations predict strengths ranging from 0.05 to essentially 0 as the particle-particle interaction strength α_1' is varied between 0 and -450 MeV fm^3, respectively. We are therefore able to reproduce the experimental result of $B(\mathrm{GT})(0^+ \to 1^+) = 0.015$ ($\log ft = 6.1$) with an interaction strength in the region of $\alpha_1' = -300$ MeV fm^3.

Whether or not the inadequacy in the description of these single-beta decays rates is carried over to the double-beta decay rate depends upon the relative contribution of the lowest intermediate state to the matrix elements $M_{GT}^{2\nu}$. As we have already indicated, in the case of ^{100}Mo this contribution is dominant. By contrast, the matrix element of the ^{128}Te decay is not significantly affected by the lowest intermediate state (except of course in the immediate vicinity of the zero crossing of $M_{GT}^{2\nu}$). It is, therefore, possible that despite the aforementioned difficulties, the QRPA is able to reproduce the double-beta decay rate and in fact we are able to obtain the experimental double-beta decay lifetime within the prescribed window of values for α_1'.

IV. EFFECT OF PARTICLE-PARTICLE FORCE ON THE "β^--LIKE" DECAY

The suppressive effect of particle-particle force on the β^+-like transitions, as discussed earlier, is well known. It has been used previously by a number of authors in attempts to explain the rates of β^+ decay in various nuclei [15–17]. On the other hand, the lifetimes of β^- decaying nuclei are usually calculated for $g_{pp}=0$, i.e., the effect of the particle-particle force is neglected [18,19]. We have seen above that this assumption is generally not correct, and that the inclusion of the particle-particle force often leads to an increase in rate of the β^- decays involving low-lying states. Below we discuss this result which is of interest on its own right, independently of the double-beta decay.

The action of the repulsive particle-hole force leads to the creation of the high-lying collective giant GT resonance and in doing so draws strength, both β^- and β^+, from the low-lying states. With the addition of an attractive particle-particle force some collectivity is returned to the lowest-lying state. Thus in the TDA we would expect an increase in both the β^- and β^+ transition strengths of the lowest-lying state as the particle-particle force is switched on. In the QRPA we must also consider the effects of the induced ground state correlations. It turns out that these have relatively little impact upon the "β^--like" transition strength which therefore continues to be reinforced by the particle-particle force. We illustrate this dependence in Fig. 4, which depicts the $[1_1^+ \to 0_i^+]$ transition amplitude in ^{100}Mo. In fact we can see that this enhancement is quite strong given that the contribution of the dominant $[p1g_{9/2}n1g_{7/2}]^{1^+}$ configuration, shown dashed (for which $t_{pn}^{-,1} = -0.65$ and $t_{pn}^{+,1} = +1.35$), is reduced to zero by the ground state correlations at $\alpha_1' = -440$ MeV fm^3.

The "β^+-like" transition, on the other hand, is influenced by the ground state correlations to such an extent that the Tamm-Dancoff behavior is overturned and the transition strength is suppressed by the particle-particle force.

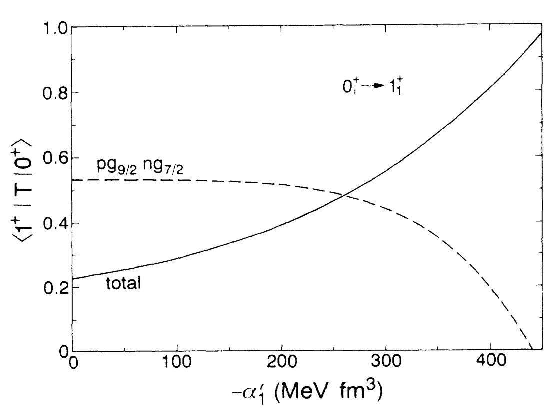

FIG. 4. The dependence of the "β^--like" $[1_1^+ \to 0_i^+]$ transition amplitude upon the particle-particle interaction strength. The dashed line indicates the contribution from the $[p1g_{9/2}n1g_{7/2}]^{1^+}$ pairing only configuration.

Thus, to conclude this section, while it is true that the particle-particle force has relatively little effect upon the excitation energy and "β^--like" strength of the giant GT resonance, it *can* significantly change the "β^--like" strength of the low-lying states. It is not therefore justified to simply ignore the particle-particle force in the calculation of "β^--like" transitions, and, in particular, in the calculation of β^- lifetimes.

Note added in proof. Recent reanalysis of the data shows that the half-life is somewhat shorter, about 10^{21} yr (F. Avignone, private communication).

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