were made to the six angles at each energy. The addition of the fourth term did not significantly improve the fit, and had only a slight effect on the other coefficients. The coefficients for fit (3) and the normalization-free ratios B/A and C/A are also presented in Table I.

Although the normalization of the data appears to differ considerably from that of other experiments,[4,5] the normalization-independent ratios (Fig. 2) are in good agreement over the region of interest. Since a violation of time-reversal invariance would be expected to manifest itself in the ratio C/A,[1] a meaningful comparison of Reactions (1) and (2) should be possible when data for the latter process become available.

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†Present address: Princeton-Pennsylvania Accelerator, Princeton, N. J.

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EXPERIMENTAL CONSISTENCY OF MULTI-REGGEISM IN A HIGH-ENERGY REACTION*

R. Lipes† and G. Zweig‡
California Institute of Technology, Pasadena, California 91109

and

W. Robertson
University of Wisconsin, Madison, Wisconsin 53706
(Received 9 December 1968)

The reactions $\pi^- + p \to \pi^- + X + p$, $X \to \pi^+ + \pi^-$, for incident π^- energy of 25 BeV, has been analyzed within the framework of the multi-Regge exchange model.

It has been conjectured independently by several authors[1] that the multiparticle production amplitude, in a certain well-defined kinematic region, is essentially the product of two-body Regge amplitudes. We shall refer to this as the multi-Regge exchange (MRE) hypothesis. The consistency of this assumption with experimental data has been investigated by several groups,[2,3] but always in kinematic regions where the validity of application is uncertain and additional assumptions are involved.[4] We have looked for MRE in the reactions

$$\pi^- + p \to \pi^- + X + p, \tag{1}$$

$$X \to \pi^+ + \pi^-, \tag{2}$$

at an incident π^- energy of 25 BeV, always working in a kinematic region where the theoretical assumptions leading to a MRE form for the amplitude are kept to a minimum. Simultaneous with the requirement that the p and π^- momentum transfers be small, we demand that all invariant masses (except that of the X) be large. This is the first time that this particular kinematic region has been investigated experimentally. The events were selected from an 80-in. bubble-chamber exposure at the Brookhaven National Laboratory by the Walker-Erwin group of the University of Wisconsin.

At a fixed incident energy, the amplitude for Reaction (1) will depend on four variables, if we treat X as a stable particle of definite mass. We

take these to be the two momentum transfers

$$t_\pi = (\pi_f - \pi_i)^2 < 0, \quad t_p = (p_f - p_i)^2 < 0,$$

and the two final-state invariant masses

$$s_{\pi X} = (\pi_f + X)^2, \quad s_{Xp} = (X + p_f)^2.$$

Here the particle symbol stands for the four-vector of that particle, while the subscripts "i" and "f" designate "initial" and "final" [Fig. 1(a)]. The MRE hypothesis states that when both final-state invariant masses are large, the amplitude for Reaction (1) in the region where both momentum transfers are small will receive contributions of the form[5]

$$A \sim \beta_{\pi\alpha_1\pi}(t_\pi)\zeta_1(t_\pi) s_{\pi X}^{\alpha_1(t_\pi)} \gamma_{\alpha_1 X \alpha_2}(t_\pi, \omega, t_p) s_{Xp}^{\alpha_2(t_p)} \zeta_2(t_p) \beta_{p\alpha_2 p}(t_p),$$

$$\zeta_k(t) = \frac{1 \pm \exp[-i\pi\alpha_k(t)]}{\Gamma[1+\alpha_k(t)]\sin\pi\alpha_k(t)}, \quad k = 1, 2. \tag{3}$$

where $\alpha_1(t_\pi)$ and $\alpha_2(t_p)$ are two exchanged trajectories which will depend on our choice for X. The trajectory $\alpha_1(t_\pi)$ couples to the initial and final pions with strength $\beta_{\pi\alpha_1\pi}(t_\pi)$; $\alpha_1(t_\pi)$ and $\alpha_2(t_p)$ couple with X through the residue $\gamma_{\alpha_1 X \alpha_2}(t_\pi, \omega, t_p)$, while $\alpha_2(t_p)$ couples to the external protons via $\beta_{p\alpha_2 p}(t_p)$. The variable ω occurring in $\gamma_{\alpha_1 X \alpha_2}(t_\pi, \omega, t_p)$ is the angle, as measured in the X rest frame, between the normals to the plane determined by the pions and the plane determined by the protons. The crucial point here is that the trajectories $\alpha_1(t_\pi)$ and $\alpha_2(t_p)$, as well as the residues $\beta_{\pi\alpha_1\pi}(t_\pi)$ and $\beta_{p\alpha_2 p}(t_p)$, may be determined from appropriate two-body reactions. The only unknown occurring in the amplitude is $\gamma_{\alpha_1 X \alpha_2}(t_\pi, \omega, t_p)$.

The primary object of this Letter is to demonstrate that the data are consistent with MRE if $\alpha_1(0) \approx \frac{1}{2}$, $\alpha_2(0) \approx 1$. This indicates that if the isospin I of the X is 1, ρ and Pomeranchukon (P) exchange are most important; while if the X has $I = 0$, the P' and P may be the dominant contributors. The data clearly exclude double-Pomeranchukon exchange as the dominant production mechanism for Reaction (1). If the $(I=0)/(I=1)$ X production ratio is small, then the $I = 0$ X may still be produced primarily via double-P exchange.

There is, in general, an ambiguity as to whether a particular π^- should be grouped with the π^+ to form an X, or whether it should be called π_f^-. We take π_f^- to be, by definition, that π^- which makes the smallest momentum transfer with the

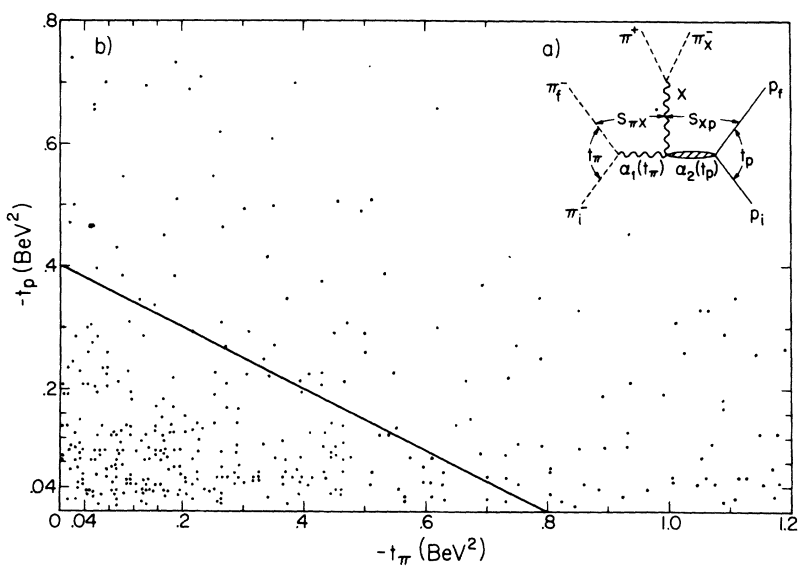

FIG. 1. (a) Kinematic diagram. (b) t_π–t_p scatter plot. The line $|t_p + 2t_\pi| = 0.8$ is used in making a momentum-transfer cut of the data. To the scale shown, the kinematic boundaries are given by the lines $t_p = 0$ and $t_\pi = 0$.

incident π^-, i.e.,

$$|(\pi_f^- - \pi_i^-)^2| = |t_\pi| < |(\pi_X^- - \pi_i^-)^2|,$$

where π_X^- designates the π^- that is included in X.

We have indicated that Eq. (3) is expected to hold when both final-state invariant masses are large, while both momentum transfers are small. More precisely, we shall restrict ourselves to events where $s_{\pi X}$ and s_{Xp} lie outside the final-state two-body resonance region, i.e.,

$$s_{\pi X} \geq 2 \text{ BeV}^2 \ (40\%), \quad s_{Xp} \geq 4 \text{ BeV}^2 \ (25\%),$$

$$s_{\pi p} = (\pi_f + p_f)^2 \geq 4 \text{ BeV}^2 \ (2\%). \qquad \text{(a)}$$

The percentage indicates the fraction of events that are removed, at each stage, as a result of the cut employed; we begin with ~2000 four-prong, four-constraint events with identifiable proton. Our results are insensitive to the exact location of these, and subsequent, cuts. The momentum-transfer constraints will be described shortly. Other investigators[3] have not required that both final-state invariant masses be large. Justification for this must rest on some, as yet ill-defined, generalization of Dolen-Horn-Schmid "duality" to multiparticle amplitudes.[4] Since this "duality" principle frequently does not work in two-body reactions when we include only one or two trajectories,[6] we feel that if we want to demonstrate the validity of Eq. (3), we had best work in kinematic regions which are free of "duality" uncertainties.

When the X mass is large (above the $\pi^+\pi_X^-$ resonance region), the dynamics presumably are described by triple-Regge exchange. Since the number of events here is small, we ignore these for simplicity and confine ourselves to the double-Regge-exchange region by requiring

$$m_X^2 = (\pi^+ + \pi_X^-)^2 \leq 2 \text{ BeV}^2 \ (8\%), \qquad \text{(b)}$$

except for $X = g(1.650 \text{ BeV})$.

We have, in addition, removed those events where one of the π's from the X resonates with either the final proton or π^-, i.e.,[7]

$$(\pi^+ + p_f)^2 \neq m_\Delta^2 \ (18\%),$$

$$(\pi^+ + \pi_f^-)^2 \neq m_\rho^2, m_f^2 \ (31\%). \qquad \text{(c)}$$

In order to see if our data are consistent with MRE, we first look for an accumulation of events [satisfying (a)-(c)] when the momentum transfers t_π and t_p are both simultaneously small. This is one of the most striking features contained in Eq. (3) (exponentials in momentum transfer arise both from the β's and the s^α factors), and will determine if a multiperipheral signal is present in the data when the final-state invariant masses $s_{\pi X}$ and s_{Xp} are both large. The result is shown on a t_π-t_p plot in Fig. 1(b). Note that there is a large excess of events when both t's are small even though phase space vanishes at the boundaries of this plot. We now isolate this multiperipheral signal by restricting ourselves to the small momentum-transfer events contained within the region

$$|t_\pi + 2t_p| \leq 0.8 \text{ BeV}^2 \ (40\%), \qquad \text{(d)}$$

and examine them to see if they are consistent with the expected detailed MRE structure. We have not treated t_p and t_π symmetrically in (d) because the peaking in t_p is sharper than the peaking in t_π. This asymmetry is our first indication that double-P exchange is not dominant. Double-P exchange would require all distributions for Reaction (1) to be approximately symmetric under the interchange $t_\pi \leftrightarrow t_p$ [recall that πp and pp elastic scattering have similar diffraction peak slopes implying, via factorization, that $\beta_{\pi\alpha_P\pi}(t) \approx \beta_{p\alpha_P p}(t)$]. Note that earlier analysis[2,3] did not incorporate this type of momentum-transfer cut (d). We have found it useful in sharpening the MRE signal.

The cross section for the 250 events which remain after the application of conditions (a)-(d) is 95 ± 10 μb.

To proceed further in the analysis, we assume that γ is independent of ω and may be parametrized[8] by $\gamma \propto \exp(g_1 t_\pi + g_2 t_p)$. The consistency of this assumption will be checked later. Similarly, we parametrize the β's by $\beta_{\pi\alpha_1\pi}(t_\pi) \propto \exp(b_1 t_\pi)$, $\beta_{p\alpha_2 p}(t_p) \propto \exp(b_2 t_p)$. The signature factors $\xi_k(t)$ are slowly varying in t and will be set equal to constants. Altogether, with a linear approximation for the trajectories, the MRE amplitude may be put into the form

$$A \propto \exp(c_1 t_\pi) s_{\pi X}^{\alpha_1(0) + \alpha_1'(0) t_\pi} s_{Xp}^{\alpha_2(0) + \alpha_2'(0) t_p} \exp(c_2 t_p); \quad c_i = b_i + g_i, \quad i = 1, 2. \qquad (4)$$

To obtain some feeling for the relative size of $\alpha_1(t)$ and $\alpha_2(t)$, we examine the $s_{\pi X}$ and s_{Xp} distributions [Figs. 2(c) and 2(d)]. The differences in scale are striking. Approximately half the events have $s_{Xp} > 15$ BeV^2; there are no events with $s_{\pi X} > 15$ BeV^2. This asymmetry automatically excludes dominant double-P exchange and implies $\alpha_1(t) < \alpha_2(t)$ in the small-t region under investigation.

In this Letter we will not attempt a determination of $\alpha_1'(0)$ and $\alpha_2'(0)$; anticipating the fits we will obtain (i.e., $1=\rho$ or P', $2=P$) we shall take these as inputs to be 1/BeV^2 and 0, respectively.

The following iterative scheme is used to compare the amplitude with the data. We first guess that $\alpha_1(0) \approx \frac{1}{2}$, $\alpha_2(0) \approx 1$, and $c_2 \approx 5$, being guided by our knowledge of two-body reactions and our expectation that $1=\rho$ or P', $2=P$ (we know $b_2 \approx 2.5$ and we might expect a comparable value for g_2). We then integrate over $s_{\pi X}$, s_{Xp}, and t_p, fitting c_1 by comparing the resulting t_π distribution with experiment. The results are shown in Fig. 2(a) and favor $c_1 \approx 1$. Since this distribution is more sensitive to c_1 than to any of the other parameters, iteration will be possible. The next step is to take $\alpha_1(0) \approx \frac{1}{2}$, $\alpha_2(0) \approx 1$, and $c_1 \approx 1$, and to fit c_2 with a t_p plot as shown in Fig. 2(b). Note that $c_2 \approx 5$ works quite well although the first t_p bin appears somewhat underpopulated. Fixing $c_1 \approx 1$, $c_2 \approx 5$, we now fit $\alpha_1(0)$ and $\alpha_2(0)$ from the $s_{\pi X}$ and s_{Xp} distributions [Figs. 2(c) and 2(d)]. Note that although a plot in one of the s's will be rather insensitive to c_1 and c_2, it will depend rather critically on both $\alpha_1(0)$ and $\alpha_2(0)$. For example, in the s_{Xp} distribution, high values of s_{Xp} depend most sensitively on $\alpha_2(0)$, as expected, but low values depend both on $\alpha_1(0)$ and $\alpha_2(0)$. A consistent fit to the qualitative features of both s distributions is found with $\alpha_1(0) \approx \frac{1}{2}$, $\alpha_2(0) \approx 1$, in agreement with the predictions of MRE if $1=\rho$ or P', $2=P$ (recall that the ρ and P' trajectories have comparable intercepts at $t=0$).

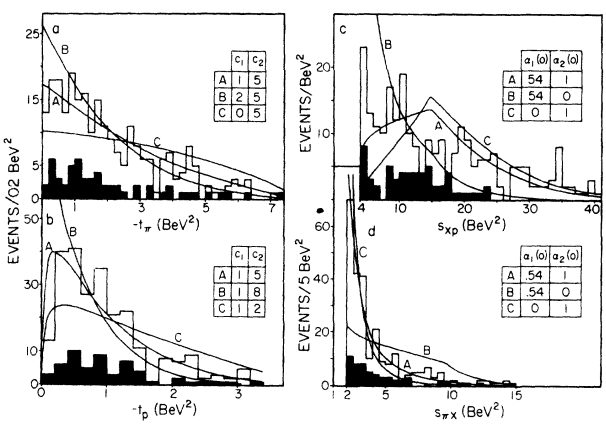

FIG. 2. (a)–(d) t_π, t_p, s_{Xp}, and $s_{\pi X}$ distributions, respectively. Note scale changes. The shaded events in this and succeeding graphs correspond to 0.7 BeV $< M_X <$ 0.83 BeV, where M_X is the mass of the X. Only about $\frac{1}{3}$ of these events are actual ρ's. To simplify the calculations, the theoretical curves were computed assuming an average X mass equal to 0.765 BeV.

Although our s_{Xp} fit [curve A, Fig. 2(c)] works quite well for large values of s_{Xp}, there are definite discrepancies at low s_{Xp}. These can be corrected, without altering the goodness of fit to the other three distributions, by adding in a small contribution with $\alpha_1(0) \approx 1$, $\alpha_2(0) \approx \frac{1}{2}$ and allowing it to interfere with the main term in the amplitude.[9]

Identifying $2=P$, taking $c_P \approx 5$ from our analysis, and using $b_P \approx 2.5$ from two-body reactions, we have $g_P \approx 2.5$. Since b_1 is essentially unknown, we are unable to estimate g_1.

We now check our assumption of the independence of γ on ω by computing the expected ω distribution and comparing it with experiment [Fig. 3(b)]. Note the fit is satisfactory; no variation of γ with ω is needed.

To understand what comprises X, we have plotted the invariant X mass for events which satisfy all constraints except Fig. 3(b). Note that while some ρ is present (~20 events), no strong f (<15 events) or g (<3 events) signal is observed.[10] Since the g lies on the same trajectory as the ρ, the difference between g and ρ production may be attributed solely to differences in γ and phase space factors. Phase space favors g over ρ by a

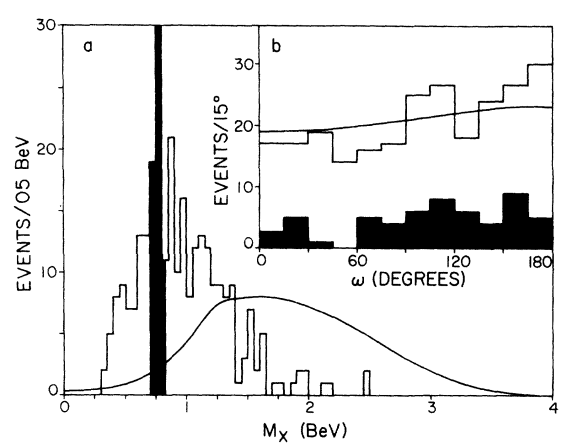

FIG. 3. (a) X invariant-mass plot. The solid line is obtained from Eq. (3) assuming $\gamma_{\alpha_1 X \alpha_2}$ is independent of X. (b) ω angular distribution. The solid curve follows from Eq. (3) assuming $\gamma_{\alpha_1 X \alpha_2}$ is independent of ω.

factor of 4 [solid line, Fig. 3(a)]. Consequently, the middle residue γ must fall dramatically as we move along the X trajectory from ρ to g. We have estimated theoretically the f/ρ production cross-section ratio to be 25 assuming $\alpha_1 = \alpha_P$, $\alpha_2 = \alpha_P$ for f, $\alpha_1 = \alpha_\rho$, $\alpha_2 = \alpha_P$ for ρ, and $\gamma_{\alpha_P f \alpha_P} = \gamma_{\alpha_\rho \rho \alpha_P}$. Since the experimental f/ρ production ratio is ≤ 1, we have additional evidence that double-Pomeranchukon exchange is either severely suppressed or absent.

In summary, we have found that (1) multiperipheral events exist even when final-state invariant masses are large. (2) The multiperipheral events are consistent with a MRE structure. We find $\alpha_P(0) \approx 1$, $\alpha_\rho(0) \approx \frac{1}{2}$, and/or $\alpha_{P'}(0) \approx \frac{1}{2}$, in good agreement with determinations of these parameters from two-body reactions. Double-Pomeranchukon exchange is not dominant. (3) The internal vertex γ is independent of the angle ω. (4) Multiperipheral f production is suppressed by a factor of at least 25 over what one might expect from double-Pomeranchukon exchange. (5) Multiperipheral g production is small indicating that residues considered as functions of external mass are strongly damped with increasing mass. (6) The cross section for the MRE events is 95 ± 10 μb. We have used the MRE model, normalized to this cross section, to predict cross sections for this same reaction at other energies. We find that the cross section peaks at around 10 BeV with a maximum value of ~165 μb. However, at this low energy, ambiguities in grouping the final-state particles may become serious.

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[7]More precisely, we require $(\pi^+ + p_f)^2 \geq (1.350\ \text{BeV})^2$, $(\pi^+ + \pi_f^-)^2 \neq (0.765 \pm 0.065\ \text{BeV})^2$, $(1.25 \pm 0.05\ \text{BeV})^2$.

[8]We would, of course, expect that g_1 and g_2 are both functions of X. We assume here that this dependence is "smooth" so that g_1 and g_2 may be taken to represent average values of these exponential factors.

[9]The relative phase of the main term and the added correction cannot be distinguished from the relative absolute magnitudes of these two terms since the square of the correction term is negligible. Therefore we have essentially one degree of freedom in correcting our fits to the distributions.

[10]The ρ signal continues to exist and dominate the f even when the $s_{\pi X}$ cut is increased from 2 to 3 BeV^2.