HALL EFFECT AND THE MAGNETIC PROPERTIES OF SOME FERROMAGNETIC MATERIALS

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(Received September 20, 1930)

Abstract

The Hall effect and the magnetic properties have been measured simultaneously with considerable accuracy in K.S. magnet steel and in hardened high carbon steel. The Hall effect is found to be a single-valued straight-line function of the intensity of magnetization I, but is neither a single-valued nor a straight-line function of either the magnetic induction B or the magnetic field H. This is true on both the virgin curves and the broad hysteresis loops of these materials. The possibility of writing the Hall e.m.f. per unit current in a cm square as $\epsilon = R_0 H + R_1 I$ is considered. In the materials measured here R_0 is less than 0.005 R_1. The formula may hold for non-ferromagnetic materials, but it could not be tested so simply.

Introduction

Shortly after the discovery of the Hall effect it was found that ferromagnetic materials behave differently than paramagnetic and diamagnetic substances. While the transverse Hall e.m.f.'s in the latter substances are directly proportional to the magnetic field for all fields obtainable, in the ferromagnetic materials the direct proportionality holds only up to the region of maximum permeability.[1] Above that point the rate of increase of the Hall e.m.f. with the field decreases.[2]

Many attempts have been made to explain this phenomenon. Many of the early authors thought that in ferromagnetic substances the effect was proportional to the intensity of magnetization I rather than to the induction B, which is the quantity that is always measured. In fact, Kundt[3] in 1893 made experiments on iron, nickel, and cobalt which showed this to be the case. At low fields he could not possibly distinguish between B and I; but at higher fields, near the saturation point of the material, his results are certainly good enough to show that the Hall e.m.f. is more nearly proportional to the magnetic intensity I than to either the magnetic induction B or the magnetic field H.

This might have been expected since both the direction and magnitude of the Hall coefficient depends upon the materials in which it is measured. This seems to indicate that the Hall effect is a phenomenon more dependent upon the characteristics of the material than upon the applied magnetic field. However, we find that since that time most of the theoretical work

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[1] E. M. Pugh, Phys. Rev. **32,** 824 (1928).

[2] A. W. Smith, Phys. Rev. **30,** 1 (1910).

[3] A. Kundt, Wied. Ann. **49,** 257 (1893).

has been based upon the assumption that the effect is caused by the action of a uniform field on the electric current in the material.

Recently Smith and Sears[4] have found it possible to separate the Hall e.m.f. in permalloy into two components; one of which they consider due to I and the other to H.

The usual expression for the Hall e.m.f. E is

$$E = RHI/t$$

and this may be written

$$Et/I = \epsilon = RH \tag{1}$$

where ϵ represents the Hall e.m.f. per unit current in 1 cm square of the test piece, R the Hall coefficient depending upon the material and H the field strength in gauss. This equation holds for nearly all non-ferromagnetic materials, although, as the author[1] has previously pointed out, the H should be replaced by B, where

$$B = H + 4\pi I \tag{2}$$

since it is the magnetic induction which is actually measured. While it might be legitimate to neglect the second member of (2) in non-ferromagnetic materials, where it is small compared to the first, it certainly is not legitimate in ferromagnetic materials.

From the work of Smith and Sears, equation (1) for ferromagnetic materials should be written

$$\epsilon = R_0H + R_1I \tag{3}$$

where R_0 is a constant which may be nearly independent of the material and R_1 is a constant which may have any value, either positive or negative, depending upon the material.

As far as our experimental knowledge goes on non-ferromagnetic materials, equation (3) might also be correct for them, for in them the quantities B, H and I are all proportional to each other. Therefore, the Hall e.m.f. is a straight line when plotted against any one of the three.

If we are to develop a workable theory of these galvanomagnetic and thermomagnetic effects, of which the Hall effect is only one, we should first settle the question as to what part of these effects is due to H and what part is due to I. With this in mind, the following experiment was started to correlate as accurately as possible the Hall effect with the magnetic properties of ferromagnetic materials.

Since H is so small compared to I in most materials, it was decided to use some materials which were not easily magnetized so that H would be a larger proportion of B. It was also decided to use materials with wide hysteresis loops to see if the effect followed the same law on the loops as on the virgin curve of the material. The K.S. Magnet Steel which was kindly furnished by Professor Honda of the Imperial University at Tokyo, Japan was admirably suited for this purpose.

[4] Smith and Sears, Phys. Rev. **34,** 1466 (1929).

METHOD

The same general method was used as was previously employed by the author,[1] though with many changes and refinements to obtain greater accuracy. This method measures the Hall effect in bars instead of sheets. It eliminates the uncertainties caused by the close proximity of the surfaces of the sheets to the interior where the effect is presumably taking place. This may be especially important with ferromagnetic substances which become polarized in the field, and thus have free poles on their surfaces. The method also makes it possible to measure the magnetic quantities in the material at the same time as the Hall effect is being measured. This is important because the behavior of magnetic materials depends so much upon their previous magnetic history that we must know the magnetic quantities at the particular time when the Hall e.m.f. is measured.

The experimental arrangement is illustrated in Fig. 1. A rectangular bar of the material to be tested was fitted into the pole pieces of an electro-

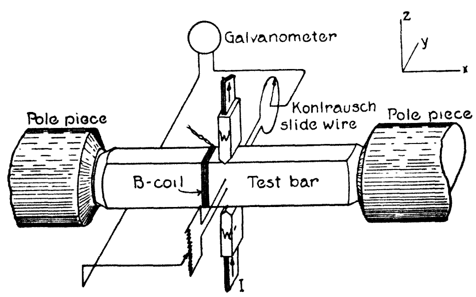

Fig. 1. Perspective diagram of experimental arrangement.

magnet, which made it possible to magnetize the bar longitudinally in the x-direction. A steady current of from 50 to 100 amps. from a large bank of storage batteries was passed through the bar in the z-direction by means of the heavy copper electrodes W and W'. The change in the Hall e.m.f. in the y-direction resulting from a change in the magnetization in the bar was measured by means of the Kohlrausch slide wire and high sensitivity galvanometer shown in Fig. 1.

Hall e.m.f. Any change in the magnetization in the bar causes a change in the Hall e.m.f., or a rotation of the equipotential surfaces in the bar about the lines of force, *i.e.* about the x-axis. Such a rotation changes the position of the contact on the Kohlrausch slide wire at which the galvanometer will not deflect. The change in position on the slide wire is then a measure of the change in the Hall e.m.f. The point on the slide wire representing zero Hall e.m.f. lies midway between the two points found by magnetically saturating the bar in opposite directions. Contact was made by two steel needles on each side of the bar. The needles were symmetrically spaced with respect to the center of the bar, 3 mm apart, and directly in line with the electrodes W and W'.

Magnetization

Both the virgin curves and the hysteresis loops were obtained by the step by step method using a ballistic galvanometer to measure the magnetic induction B. The Hall e.m.f. was measured between each of these steps. The search coil was wound on the bar as close to the center as the contacts would permit. The period of the ballistic galvanometer was made as large as possible, and the instrument was used greatly overdamped to avoid errors due to the fact that for each change in the e.m.f., a finite time is required for the magnetization to reach its final value. A number of tests were made to determine the extent of this error, and it was found to be entirely negligible with the hardened materials upon which the conclusions from this experiment are based. In soft annealed materials the errors were not negligible. For this reason, and because the low resistance of these materials made the method of measuring the Hall e.m.f. quite insensitive, little was done with such substances.

The electromagnet was wound with five coils which were connected in series. It was arranged that each of these coils could be reversed separately to obtain the necessary changes in e.m.f. In this way, two important errors were eliminated which would have been present had the steps been produced by changing the current in the coils. When the current is changed in a coil considerable time is required for it to reach its final value, for the temperature and resistance must change. This would cause grave errors in the step by step method of determining B. Also, in order to keep the temperature sufficiently constant, it was necessary that the heat loss in the coils be kept constant. The magnet and test bar were immersed in a constant temperature insulated oil bath.

The value of H can not be obtained directly. It was measured with a "saddle coil"[5] connected to the ballistic galvanometer. The "saddle coil" was made to fit over the test bar at the center. This method was chosen because it has been shown to measure the average value of H over the small section which it covers. It therefore approximated very closely the value of H where the Hall e.m.f. was measured. As a further check upon the absolute value of H in the test bars and upon the calibration of the "saddle coil," the value of H was measured with an independently calibrated Chattock potentiometer.[6]

Results

The results of this experiment on K.S. magnet steel are shown in Figs. 2, 3, 4, and 5. Fig. 2 shows the Hall e.m.f., E, plotted against H when the material was taken through a complete hysteresis loop. This E vs. H curve presents the same appearance as the usual B vs. H or I vs. H curves. In Fig. 3, the values of E plotted in Fig. 2 are replotted first against B and then against $(B-H)$. The E vs. B curve (points indicated by circles) is a very definite loop though quite narrow, while the E vs. $(B-H)$ (points indicated

[5] Dictionary of Applied Physics, Vol. II Electricity, p. 464, 1922 edition.

[6] Dictionary of Applied Physics, Vol. II Electricity, p. 467, 1922 edition.

by dots) curve is a straight line. It is seen from this that the Hall e.m.f. is not a single valued function of B, but it is a single valued function of $(B-H)$ which is proportional to the intensity of magnetization. This is quite significant because it shows that the same intensity of magnetization undoubtedly produces the same Hall e.m.f. whether it is a residual intensity or an intensity produced by the action of an external field. In Figs. 4 and 5 one of the many runs taken on the virgin curve of K.S. magnet steel has been plotted. In Fig. 4, E has been plotted first as a function of B (points indicated by dots) and second as a function of $(B-H)$ (points indicated by

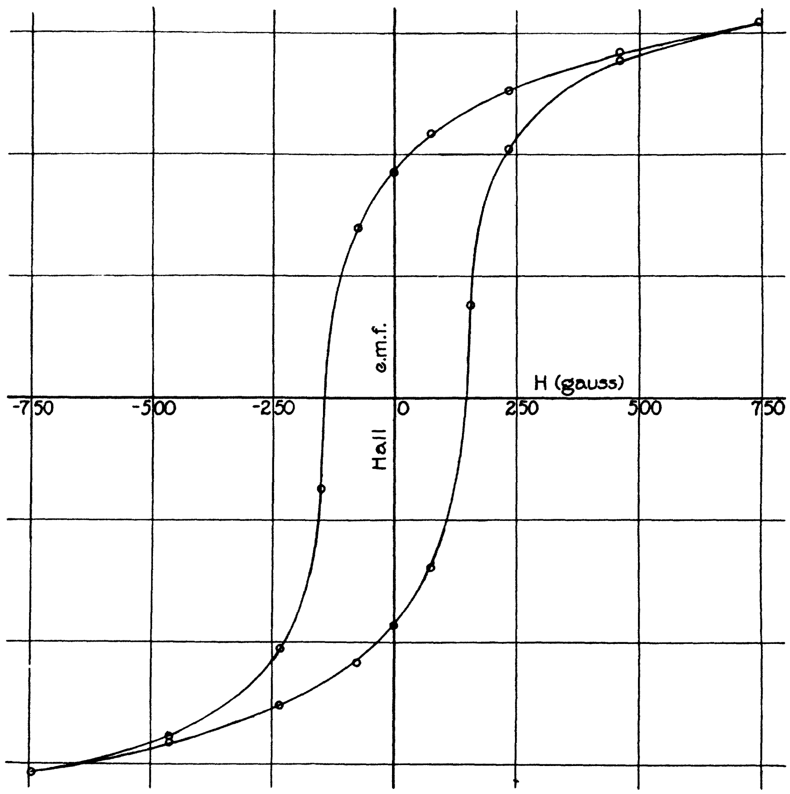

Fig. 2. Hall e.m.f. hysteresis loop for K.S. magnet steel.

circles) just as was done in Fig. 3. The accuracy of the experimental points could not be well shown in Figs. 3 and 4, so the deviations of the points in Fig. 4 from their respective straight lines have been shown in Fig. 5. The run plotted in Figs. 4 and 5 was chosen because it was taken under the most favorable conditions ever obtained. The slight deviation of the second point from the straight line in each of these figures is undoubtedly an error because it does not show up on other runs. Runs were also made on the first part of the virgin magnetization curve of K.S. magnet steel. Here H is small and consequently the difference between B and $(B-H)$ is very little, yet even in this case, when the Hall e.m.f. is plotted against B, a slight curvature can be detected which straightens out when plotted against $(B-H)$.

These results show that within the limits of error of this experiment on K.S. magnet steel *the Hall e.m.f. is a single valued, straight line function of I, while it is neither a single valued nor a straight line function of either B or H.*

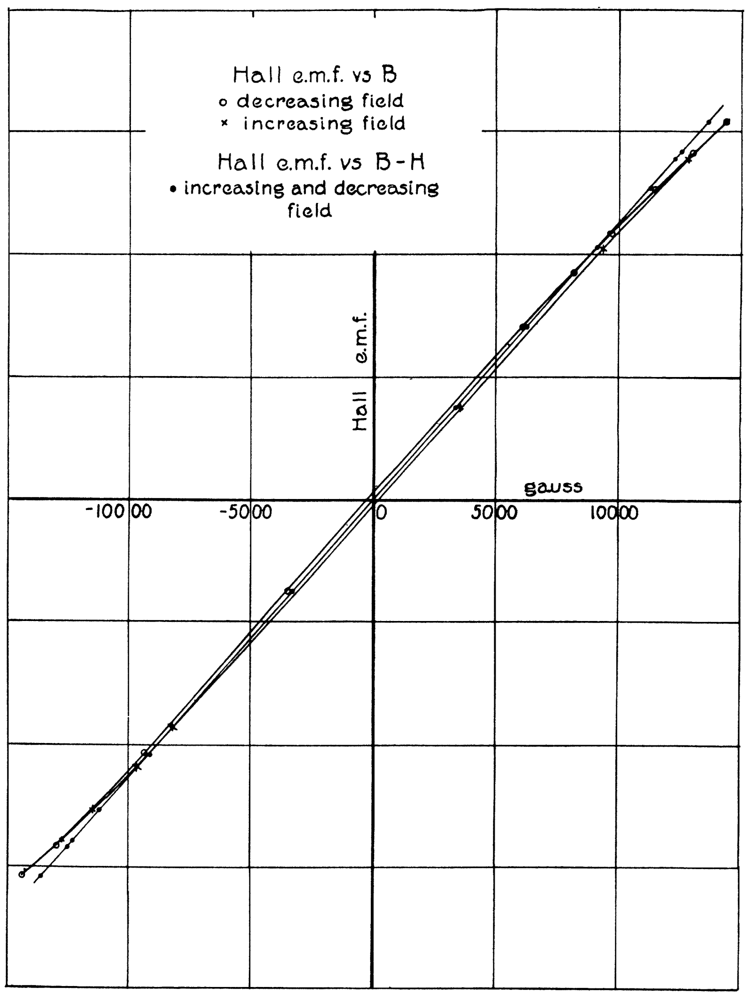

Fig. 3. Hall e.m.f. hysteresis loop for K.S. magnet steel plotted against B and $(B-H)$.

This is true whether the Hall e.mf. is measured on the virgin curve or on a hysteresis loop.

The same tests were performed with a hardened high carbon steel (1.1 percent carbon) and the results were just the same as with the K.S. magnet steel.

Just recently Stierstadt[7] has published an investigation of the change in

[7] O. Stierstadt, Phys. Zeits. 31, 561 (1930).

electrical conductivity of ferromagnetic materials in longitudinal magnetic fields in which he also takes the materials around their magnetic hysteresis loops. He states that this change in resistance is a function of B and not of I, but the quantity which he measures and calls magnetic induction is actually intensity of magnetization. It seems that in ferromagnetic materials the change in resistance and the Hall effect are both functions of the intensity of magnetization rather than of the magnetic induction. One wonders

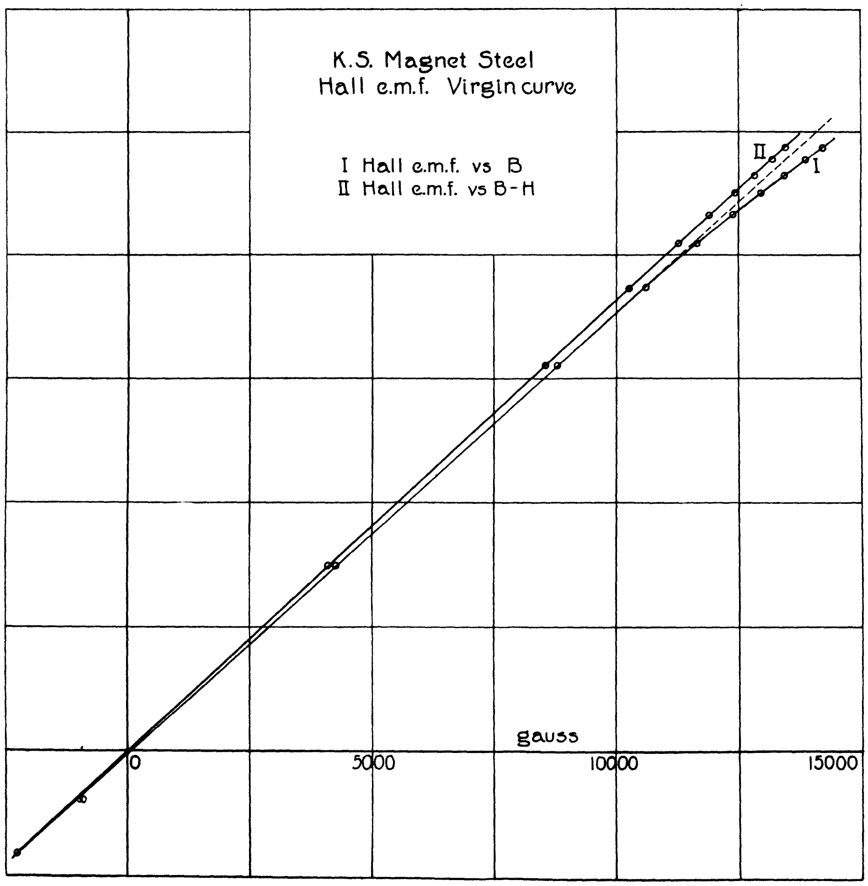

Fig. 4. Hall e.m.f. virgin curve for K.S. magnet steel plotted against B and $(B-H)$.

whether this might not be true of all the familiar galvanomagnetic and thermomagnetic effects in ferromagnetic substances.

Where the Hall effect is proportional to the intensity of magnetization it must mean that the part played by the uniform field is quite negligible. Is it not reasonable to suppose that if the uniform field plays a negligible part in the production of the Hall e.m.f. in some materials that it also plays a negligible part in others? Such an assumption would be difficult to test in paramagnetic and diamagnetic substances because in them the Hall e.m.f. gives a straight line whether it is plotted against B, H or I. It has been possible to make the test in these ferromagnetic materials only because I and B are not proportional to each other.

The recent work of Smith and Sears[4] in measuring the Hall effect on different permalloys differs from the results obtained here. They find that in all of their permalloys the Hall e.m.f. rises to a maximum and then decreases to negative values with increasing magnetic inductions. This may mean, as must be concluded from their explanation, that in equation (3) R_1 is positive while R_0 is negative and of the same order of magnitude as R_1. It may also mean that the annealed permalloys were not homogeneous but had segregated into two magnetically different components, which have Hall coefficients R_1 and R_1' of opposite sign. In which case the component having the positive coefficient reached saturation long before the one having the negative coefficient.

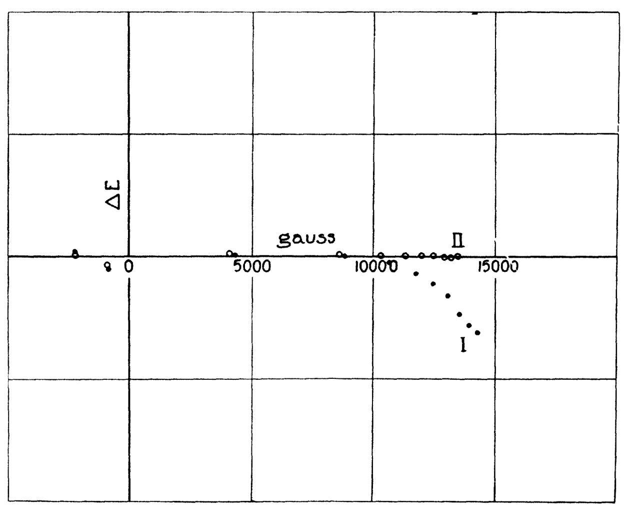

Fig. 5. Deviation from straight lines of the experimental points shown in Fig. 4.

Let us consider the first explanation. No experiment can prove that R_0 is zero. It can only be shown that R_0 is or is not negligible compared to R_1. The best test in this experiment shows that R_0 is less than 0.004 R_1. It is conceivable that while in some materials R_0 is negligible compared to R_1 this may not be true for others. However, if the absolute value of R_0 had been as large in the K.S. magnet steel as it was reported to be in the permalloy of Smith and Sears, it should have been possible to detect it here. Now let us consider the second explanation which assumes segregation of the permalloy into two components when it is annealed. Permalloy is an alloy of iron and nickel. Iron has a positive Hall coefficient and nickel has a negative one. If segregation occurs, we might expect the two components to have opposite signs for their Hall coefficients. Elman[8] has found that certain alloys of iron, nickel, and cobalt, when annealed for a long time, have

[8] G. W. Elman, Bell Tech. Journ. July 1929.

peculiar magnetic hysteresis loops which may be very nicely accounted for on the assumption that the alloy had segregated into two or more magnetic components which act individually. He could not detect the segregation in any other way. Since the permalloys of Smith and Sears were annealed they might have been segregated into two such components. If these components reached saturation at different times the reversal of the Hall e.m.f. with increasing induction would be perfectly possible. It should be stated that permalloys are very hard to investigate in this way on account of their extremely high initial permeability. Stray fields such as that of the earth or of the current flowing through the sheet itself may nearly saturate the permalloy when the applied field is apparently zero. This may help to account for the fact that the Hall e.m.f. vs. field curves of Smith and Sears depended so much upon the direction of the applied field.

The author wishes to thank Dr. A. Goetz for his assistance in planning and directing this research and Dr. W. V. Houston for his valuable suggestions.