### Supplemental Material

**Computation of $f_p$.** In BD the overdamped Langevin equation (Eq. 2 in the letter) is integrated with respect to time. Since the interactions in hard-sphere suspensions are singular, particle overlap is unavoidable regardless of the step size $\Delta t$. In the 'potential free' algorithm, the overlap-free condition is maintained in a predictor-corrector fashion: in the predictor step, the dynamics are evolved without the hard-sphere potential, and in the corrector step, the algorithm checks the particle overlap that violates the hard-sphere potential and moves the overlapping particles back to contact pairwise along the line that connects the particle centers until the suspension is overlap-free. If particle $i$ at $r_i$ overlaps particle $j$ at $r_j$, i.e., $|r_{ij}| > (a_i + a_j)$, where $r_{ij} = r_i - r_j$ and $a_i$ and $a_j$ are the particle radii, the particles are moved along $r_{ij}$ according to

$$\Delta r_i = a_j \Delta r_{ij} \quad \text{and} \quad \Delta r_j = -a_i \Delta r_{ij}$$

where $\Delta r_{ij} = r_{ij} [[r_{ij}]^{-1} - (a_i + a_j)^{-1}]$. The interparticle force on particle $i$ is computed according to Stokes law,

$$f_{p,i} = 6\pi \eta_0 a_i \Delta r_i / \Delta t,$$

and here the Newton’s third law is satisfied $f_{p,i} + f_{p,j} = 0$.

**System size dependence.** Fig. 1 presents the system size dependence on the suspension shear viscosity $\eta_s$, the long-time self-diffusivity $d_{\infty}$, and the maximum of the dynamic susceptibility $\max(\chi_4)$ as functions of $Pe_\sigma$ with different system sizes $N$. The imposed pressure is $\Pi = 5$. The dynamic susceptibility $\chi_4$ for an $N$-particle system is defined as [1],

$$\chi_4(\mathbf{k}, t) = N\left[\langle F_s(\mathbf{k}, t)^2 \rangle - \langle F_s(\mathbf{k}, t) \rangle^2 \right],$$

where $F_s(\mathbf{k}, t) = N^{-1} \sum e^{i\mathbf{k} \cdot \mathbf{x}_i(t) - \mathbf{k} \cdot \mathbf{x}_i(0)}$ is the self-intermediate scattering function, $\mathbf{k}$ is the wave vector, $\nu = \sqrt{-1}$, and $\mathbf{x}_i(t)$ is the particle position at time $t$. In Fig. 1c the wave vector $\mathbf{k}$ is in the vorticity direction and $|\mathbf{k}|a = 3.72$, near the first peak of the static structure factor. When $Pe_\sigma > 5$, $\eta_s / \eta_0$, $d_{\infty} / d_0$, and $\max(\chi_4)$ show little $N$-dependence for flowing suspensions. Near the flow-arrest transition, there are quantitative differences at different $N$. For example, with increasing $N$, the $Pe_\sigma$ corresponding to the flow-arrest point shifts slightly towards lower $Pe_\sigma$, and the suspensions can achieve higher $\max(\chi_4)$ and lower $d_{\infty}$. However, the qualitative behaviors in Fig. 1 remain consistent. Using a modest system size of $N = 200$ captures the physics behind the flow-arrest transition with only small quantitative differences in the $N \rightarrow \infty$ limit, and allows us to explore a wide range of parameters in a reasonable amount of time.

**Initial condition dependence.** To investigate the effect of initial conditions on the BD simulation results, we performed simulations at $N = 200$ for a few $(\Pi, Pe_\sigma)$ pairs near the flow-arrest transitions. Each $(\Pi, Pe_\sigma)$ pair corresponds to 50 independent runs with distinct initial configurations generated from the Lubachevsky-Stillinger algorithm [2], which is also used in the letter. Each run lasts $5 \times 10^5$ dimensionless time units with a step size $10^{-3}$. Further decreasing the step size to the value used in the letter $(10^{-4})$ does not alter the results. To quantify the initial condition dependence, we randomly partitioned the results to independent groups of $N_{\text{samp}} = 1$, 2, 3, 5, 10, and 29 runs and compute the group average of the volume fraction $\phi$ and the scaled strain rate $\dot{\gamma}^2 / \dot{\gamma}_0$ (after the initial transient). Fig. 2 presents the results at $\Pi = 5$ (open symbols) and 50 (filled symbols), representing the behaviors at low and high imposed pressures, as functions of the group size $N_{\text{samp}}$. The average from the entire 50 runs are shown dashed lines in the corresponding color. For reference, the flow-arrest transition takes

![Figure 1](https://via.placeholder.com/150)

**Figure 1.** (Color online) The system size dependence on (a): the suspension shear viscosity $\eta_s / \eta_0$, (b): the long-time self-diffusivity $d_{\infty} / d_0$, and (c): the maximum of the dynamic susceptibility $\max(\chi_4)$ as functions of $Pe_\sigma$ for constant stress and pressure simulations at $\Pi a^3/k_B T = 5$. The filled (open) symbols represent the flowing (arrested) suspension states.
Figure 2. (Color online) The initial condition dependence on (a): the average volume fraction $\phi$ and (b): the average strain rate $\dot{\gamma}a^2/d_0$, with $d_0 = k_BT/(6\pi \eta_0 a)$, as functions of the number of independent simulations in the group $N_{\text{samp}}$. The simulations are performed at $\Pi a^3/k_BT = 5$ (open symbols) and 50 (filled symbols). The stress Péclet number $Pe_\sigma$ are annotated in on the graph. The dashed lines show the overall average of all 50 independent runs.

Flow visualization. To provide direct visualization of the arrested and the flowing suspensions, we also present videos $\text{SM1}_{\text{pe}1.0}$ and $\text{SM2}_{\text{pe}10.0}$, respectively. Both videos have a system size $N = 6000$, and the imposed pressure $\bar{\Pi} = 5$. For $\text{SM1}_{\text{pe}1.0}$, $Pe_\sigma = 1$ and for $\text{SM2}_{\text{pe}10.0}$, $Pe_\sigma = 10$. At the beginning of each video, the suspensions are colored as blue and white stripes to highlight the particle movement.

Fig. 2 shows the group size $N_{\text{samp}}$ does not affect the average $\phi$ and $\dot{\gamma}$ when $Pe_\sigma$ is higher than the yield Péclet number. Here, the suspension flows as liquid and the thermal and mechanical fluctuations erase any influences of the initial conditions. However, close to the flow-arrest transitions, i.e., $(\bar{\Pi}, Pe_\sigma) = (50, 145)$ and $(5, 5)$, the results are more sensitive to $N_{\text{samp}}$: both $\phi$ and $\dot{\gamma}$ fluctuates around the mean value without a definite trend. This fluctuation is also reflected in the large error bars in Fig. 3 in the letter. Fig. 2 further demonstrates that $N_{\text{samp}} \geq 3$ adequately reflects the system behavior and justifies the computational protocol in the letter. It also validates that our results and conclusions are independent of the initial conditions.