Deterministic Generation of Arbitrary Photonic States Assisted by Dissipation

A. González-Tudela,1 V. Paulisch,1 D. E. Chang,2 H. J. Kimble,3,4 and J. I. Cirac1

1Max-Planck-Institut für Quantenoptik Hans-Kopfermann-Strasse 1, 85748 Garching, Germany
2ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain
3Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California 91125, USA
4Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California 91125, USA

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A scheme to utilize atomlike emitters coupled to nanophotonic waveguides is proposed for the generation of many-body entangled states and for the reversible mapping of these states of matter to photonic states of an optical pulse in the waveguide. Our protocol makes use of decoherence-free subspaces (DFSs) for the atomic emitters with coherent evolution within the DFSs enforced by strong dissipative coupling to the waveguide. By switching from subradiant to superradiant states, entangled atomic states are mapped to photonic states with high fidelity. An implementation using ultracold atoms coupled to a photonic crystal waveguide is discussed.

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Recent work on optical emitters coupled to one-dimensional (1D) waveguides has opened new avenues to investigate light-matter interactions [1–20]. Particularly promising are the setups where atoms are strongly coupled to structured dielectrics [6–10], where large Purcell factors have been predicted [21,22]. Furthermore, collective effects can be enhanced by placing the atoms at particular positions [15,16,23–26]. The combination of atomlike emitters and nanophotonic waveguides may enable new regimes for the interaction of light and matter, leading to technologies that outperform current ones and qualitatively different physics. In this work we investigate the possibility of using atom nanophotonics interfaces to tailor arbitrary states for propagating photons on demand, which lies at the heart of many quantum information [30], metrology [31], and lithography [32] methods (see Ref. [33] for a review). We predict large fidelities even for relatively large numbers of photons, something which has been impossible to achieve with other platforms in the optical domain.

Our proposal uses $N+1$ three-level systems (with levels $\{\vert g \rangle, \vert s \rangle, \vert e \rangle \}$), where one of the optical transitions ($\vert g \rangle \leftrightarrow \vert e \rangle$) is strongly coupled to a 1D waveguide [see Figs. 1(a) and 1(b)]. We denote by $P_{1D}$ the Purcell factor corresponding to that transition, i.e., the ratio of the emission rate into the waveguide mode, $\Gamma_{1D}$, and the one for all other modes, $\Gamma^\ast$. The atoms must be separated by distances proportional to $\lambda_0 = 2\pi / q(\omega_0)$, where $q(\omega)$ is the wave number determined by the waveguide dispersion relation. Depending on their internal state, atoms may experience a collective decay into the waveguide, or become completely decoupled from it. The latter occurs if they are in a decoherence free subspace (DFS) [34–36]. Our protocol consists of two steps: in the first one, we generate certain states within the DFS, $\vert \Psi_P \rangle$, by driving the atoms with lasers and using the collective quantum Zeno effect [37–39] within the DFS with an infidelity $1 - F_1 \propto m / \sqrt{P_{1D}}$, where $m$ is the maximum number of photons we want to generate; in the second one, a laser pulse takes the atomic state out of the DFS so that atoms collectively emit into the waveguide, creating the desired state of a single propagating mode, $\vert \Psi_E \rangle$, with an infidelity $1 - F_2 \propto m^2 / (NP_{1D})$.

The atom-photon Hamiltonian of these systems is given by $H = H_{qb} + H_{field} + H_p$, with $H_{qb} = \sum_{n=1}^{N} \{ \omega_n \sigma_n^{a\dagger} \sigma_n^a + \omega_n \sigma_n^{s\dagger} \sigma_n^s \}$ and $H_{field} = \sum_q \omega_q a_q^\dagger a_q$, (using $\hbar = 1$), where $\omega_n$ is the two-level system energy, $\omega_q$ is the field dispersion relation of the 1D photonic modes, and $\sigma_n^{i\dagger} = \vert i \rangle_n \langle j \vert$. We consider only the coupling to a single polarization as justified for suitable dielectric waveguide modes [21], that is,

\[ \sigma_n^{a\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\Delta_n} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

FIG. 1 (color online). (a) Setup: $N$ atoms plus 1 ancilla atom coupled to a 1D photon bath. The ancilla must be individually addressed. (b) Atomic $\Lambda$ scheme: the transition $\vert g \rangle \leftrightarrow \vert e \rangle$ is coupled to the $a_q$ modes. A laser controls the transition $\vert s \rangle \leftrightarrow \vert e \rangle$ with amplitude, $\Omega_q$, and detuning $\Delta_q$. Another field, $\Omega_n$, controls $\vert s \rangle \leftrightarrow \vert g \rangle$, with frequency $\omega_n$. (c) Relevant states and steps for our protocol: (I) generation of superpositions of symmetric Dicke states, $\vert D_{mn} \rangle$, by using the excited dark states $\vert \Psi_E^{(m)} \rangle$. (II) Flipping $\vert s \rangle \rightarrow \vert e \rangle$ to generate the superradiant state $\vert S_{mn} \rangle$, which decays rapidly (III) to the desired photonic state.

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\[
H_1 = \sum_{\lambda \sigma} g_{\lambda \sigma} n_{\lambda \sigma} a_{\lambda \sigma}^\dagger e^{-i\theta_{\lambda \sigma}} + \text{H.c.},
\]
with \( g_{\lambda \sigma} \) the single-photon coupling constant to the mode of interest and where we have used the rotating wave approximation. When the 1D baths have a much faster relaxation time scale than the atomic system, the atoms are described by a density matrix, \( \rho \), which in the Born-Markov limit, is governed by a master equation \cite{23,24,40} of the form:
\[
d\rho/dt = \sum_{\lambda,m,n} \left( |\Gamma_{\lambda m}/2\rangle \langle \omega_m|e^{iz_{\lambda m}}|\omega_n\rangle \langle \omega_n|e^{-iz_{\lambda n}}|\omega_m\rangle \langle \omega_m| - \rho \right) + \text{H.c.}
\]
written in the interaction picture rotating with \( H_{\text{qf}} \). By appropriately choosing the atomic positions, e.g., \( z_n = n\lambda_0 = 2\pi/q(\omega_m) \), with \( n \in \mathbb{N} \), the coherent atom-atom interactions are eliminated \cite{41} and the effective interaction yields a pure Dicke model \cite{42} described by
\[
\mathcal{L}_D(\rho) = \frac{\Gamma_{\text{ID}}}{2} (S_{eg} \rho S_{eg}^\dagger - S_{eg}^\dagger S_{eg} \rho) + \text{H.c.},
\]
where we defined \( S_{ij} = \sum_{n=1}^{N-1} |\eta_{ij}|^2 \). One of the assets of the Dicke model is the emergence of subradiant and superradiant states. The excited states with \( |\Psi\rangle \) are subradiant with a decay rate proportional (at least) to the atom number \( N \), and are unique for each \( m \). On the other hand, the states satisfying \( S_{eg}|\Psi\rangle = 0 \) are dark states of the Liouvillian of Eq. (2), and therefore decoupled from collective dissipation. These dark states span the DFS that is highly degenerate for \( m > 1 \).

The atomic states that must be created in the first step of our protocol are very peculiar as (i) they must be prepared within the DFS to avoid dissipation, and (ii) they must be easily mapped to the appropriate superradiant states, to generate arbitrary superpositions of the photonic waveguide states. Our strategy consists of first, identifying states, denoted by \( |D_m\rangle \), in the subspace spanned by the ground levels \( g \) and \( s \), which can be mapped one-to-one to a basis \( |S_m\rangle \) of superradiant states using a simple laser pulse, and which in turn give rise to \( m \) photons in the waveguide via superradiance. Then, we use a more sophisticated scheme within the DFS to generate superpositions of \( |D_m\rangle \), which requires only \( m \) steps.

By introducing another metastable state, \( |s\rangle \), as depicted in Fig. 1(b), the candidates to map to superradiant states are the symmetric Dicke states \( |D_m\rangle \propto \text{sym} \{|s\rangle^{\otimes m} \otimes |g\rangle^{\otimes N-m}\} \), as they can be turned superradiant by switching \( |s\rangle \leftrightarrow |e\rangle \). Having identified the target, \( |D_m\rangle \), we need to find ways to build efficiently arbitrary superpositions. Previous studies have proposed implementing one or two-qubit universal gates within DFS \cite{34–38}, but the number of steps increases rapidly with \( N \), as well as using adiabatic passage methods \cite{43–46}, limited to small excitations number \( m \). Here, we use the collective character of the interaction to deterministically generate arbitrary \( N \)-qubit states for which the number of steps is independent of the number of atoms.

The scheme that we use is depicted in Fig. 1(a): we consider a system of \( N + 1 \) emitters, in which we aim to generate \( |D_m\rangle \) in the first \( N \) emitters using the ancilla as an auxiliary state. As the \( |D_m\rangle \)'s are invariant under the permutation of the first \( N \) atoms, we choose the control fields with the same symmetry and the same detuning \( \Delta_n = \Delta_x \) for all emitters:
\[
H_c = \frac{\Omega_0}{2} c_a^{N+1} + \text{H.c.},
\]
where we defined \( \Omega_0 = \Omega_{\text{anc}}^{N+1} \).

\[
H_{\text{las}} = \left( \frac{\Omega_0}{2} \sum_{n=1}^{N} \sigma_{se}^n + \frac{\Omega_{\text{anc}}}{2} \sigma_{se}^{N+1} + \text{H.c.} \right) + \Delta_x \sum_{n=1}^{N-1} \sigma_{ee}^n.
\]

This transition makes:...
Another advantage of our protocol is that it can be used to generate a superposition of photonic states in the 1D bath by dissipative means once we have |Ψ_D⟩. To make sure the ancilla atom can be neglected, we flip the ancilla state |g⟩_A → |s⟩_A and apply no fields to it. In order to map to the superradiant state of N atoms, we apply a fast resonant π pulse (Δ_e = 0 and Ω_e ≫ NΩ_{1D}) on the N emitters to switch all |s⟩_n → |e⟩_n, thus generating the superposition of |S_m⟩ ∝ sym{[|e⟩_m]⊗ |g⟩^{N-m}}. Because of their superradiant character, the |S_m⟩ decay completely to 1D-reservoir modes. Because H_f conserves the number of excitations, the superradiant state of m excitations decays to the Fock-state of m photons [47]:

$$|S_m⟩ \rightarrow |m⟩_{\{q\}} \equiv \sum_{\{q\}} A_{\{q\}}(t) m! a_{q_1}^+ ... a_{q_m}^+ |\text{vac}⟩,$$

(7)

where \(\{q\} = \{q_1, ..., q_m\}\) is the set of relevant momenta which run over the whole Brillouin Zone. The scattering amplitude A_{\{q\}}(t) is calculated using a generalized input-output formalism [47,52-54] and quantum regression theorem [55]:

$$A_{\{q\}}(t) = \sum_{r=1}^{m} i \sqrt{rN} e^{-i\omega_0 t} + r\Gamma_{1D}r_{fr}^{2} + P(\{q\})$$

(8)

for sufficiently large times \(t \gg 1/N_1\Gamma_{1D}\) when the atomic state has decayed completely and defining \(P(\{q\})\) as all the permutations of \(\{q\}\). The only dependence on \(t\) enters through \(e^{-i\sum_{r=1}^{m} \omega_0 t}\), which describes the center-of-mass motion of the wave packet when going to the real space. In the low excitation regime, one can either use the Holstein-Primakoff approximation [56] or directly substitute \(N_m \rightarrow N\) in the expression above, arriving at

$$A_{\{q\}}^{\text{HP}}(t) = \sum_{r=1}^{m} e^{-i\omega_0 t} i \sqrt{rN} e^{-i\omega_0 t} + r\Gamma_{1D}r_{fr}^{2} + P(\{q\})$$

(9)

that has a Lorentzian shape centered at \(\omega_a\) with bandwidth \(\Gamma_{1D}N_r/2\). Substituting A_{\{q\}}(t) → A_{\{q\}}^{\text{HP}}(t) into the definition of \(|m⟩_{\{q\}}\), yields a linear Fock state denoted by |m⟩_{\{q\}}^{\text{HP}}. In principle, the emission into the waveguide is bidirectional (±q), but combining both fields in phase, e.g., by placing a mirror at an appropriate distance, or by engineering the atom-photon coupling appropriately [3,4,14], it is possible to achieve emission in one direction only. Furthermore, by shaping the pulse, \(\Omega_e(t)\) (within a bandwidth \(\leq NT\Omega_{1D}\)) we generate any desired shape of the output photonic state, e.g., to create a time-symmetric photonic state [57] that ensures the reversibility of the process when mapping the photonic state to another sample. Moreover, because of the linearity of the calculation of A_{\{q\}} with respect to the input state [47], superpositions of atomic states decay to superpositions
\[ |\Psi_D \rangle \rightarrow \sum_{m=0}^{m_{\text{max}}} d_m |S_m \rangle \rightarrow |\Psi_B \rangle = \sum_{m=0}^{m_{\text{max}}} d_m |m_{\{q\}} \rangle, \] (10)

that will be generated in a single-mode wave packet, as required for most applications [33], as long as \( N \gg m \) because \( \langle m_{\{q\}} | m_{\{q\}}^{\text{HP}} \rangle \approx 1 - m^3/(20N^2) \) [47].

So far, we have only discussed the ideal protocol without considering, e.g., spontaneous emission into all other modes with rate \( \Gamma^e \). For the error in the preparation of the many-body entangled state, we derive an error rate \( \epsilon \) from perturbation theory, which, together with the time of the operation \( \tau \), gives an approximation of the fidelity, \( F_1 = \sqrt{\langle \Psi_D | \rho(\tau) |\Psi_D \rangle} \approx \tau - \epsilon \), with respect to the target state \( |\Psi_D \rangle \). The dominant errors assuming \( N \gg m \) and \( \Gamma_{1D} \gg \Delta_e \gg \Gamma^e \) come from [47] (i) the spontaneously emitted photons from \( |\Psi_e^{(m)} \rangle \) to decay channels other than the waveguide, which scales as \( \epsilon_f \approx \Gamma^e m |\Omega|^2/(4N\Delta_e^2) \), and (ii) from the photons emitted from the small populations of superradiant states. We estimate the rate of these errors by taking into account the second order corrections of the projected Hamiltonian which are finally given by \( \epsilon_f \approx \Gamma_{1D}^e (m |\Omega|^2/4\Delta_e^2 + (\Gamma_{1D}^e)^2) \). Summing up, the error for the \( m \text{th} \) step of the process, which takes \( \tau = \left( \frac{\pi}{\Omega^{(m)}} \right) \approx 2\pi N\Delta_e/(m |\Omega|^2) \) for full population transfer, is

\[
1 - F_1^{(m)} \approx \frac{\pi}{2} \left( \frac{\Gamma^e}{\Delta_e} - \frac{\Delta_e}{\Gamma_{1D}} \right),
\] (11)

that is optimized for \( \Delta_{e,\text{opt}} = \sqrt{\Gamma^e \Gamma_{1D}} \), which yields a scaling: \( 1 - F_1^{(m)} \propto 1/\sqrt{\Gamma_{1D}} \). To create a superposition \( |\Psi_D \rangle \), we require \( m_{\text{max}} \) steps. Thus, the total error of the first part of the protocol is \( 1 - F_2 \propto m_{\text{max}}/\sqrt{\Gamma_{1D}} \), that can be improved via postselection conditioned on detecting no photons in the waveguide [47].

To validate the scaling analysis, we study numerically the preparation of two relevant sets of states: (i) the \( |D_m \rangle \); (ii) the superpositions \( |\Phi_m \rangle = (1/\sqrt{2})(|D_0 \rangle + |D_m \rangle) \). Because of the imposed symmetry conditions the relevant Hilbert space depends only on the maximum number of excitations, \( m_{\text{max}} \), while the \( N \) only enters on the two-photon resonance condition, that fixes \( \Omega^{(m)} \) [47]. With this restriction, we use a non-Hermitian evolution governed by \( H_{\text{eff}} = H_{\text{las}} + H_c - i\Gamma_{1D} S_{eg} S_{ge}/2 - \Gamma^e S_{ee}/2 \). To generate the \( |D_m \rangle \), the pulse sequence consists of a complete transfer of populations in each step of Fig. 2(a), which is ensured by fixing the time of interaction, \( t \), to \( t\Omega (\Omega^{(m)}) = \pi \) for the microwave (Raman) transitions, whereas for the \( |\Phi_m \rangle \) the pulse sequence is calculated numerically. In Figs. 2(b) and 2(c), we show the numerical fidelities obtained when fixing the off-resonant transition to the optimal \( \Delta_{e,\text{opt}} \), confirming that our arguments give the correct scaling \( \propto 1/\sqrt{\Gamma_{1D}} \).

Finally, we estimate the fidelity of the photonic state considering the effect of \( \Gamma^e \). For a superradiant state with \( m \) excitations the error rate is \( m \Gamma^e \), while the average time to decay is \( 1/(N\Gamma_{1D}) \), which yields an error of \( 1 - F_2^{(m)} = 2m \Gamma^e/(N\Gamma_{1D}) \). When there are \( m_{\text{max}} \) excitations in the system, the total fidelity of the process is

\[
F_2 \approx 1 - \frac{m_{\text{max}} \Gamma^e}{N\Gamma_{1D}}.
\] (12)

The dissipative character of this mapping allows for the efficient generation of (arbitrary superpositions of) photonic states, e.g., Fock states, that typically are generated probabilistically [58–60] or via nonlinear interactions [39].

An appealing platform to implement these ideas is cold atoms trapped near photonic crystal waveguides [6–10], where \( \Gamma_{1D}/\Gamma_a = \xi n_g \sigma/(2A_m) \), where \( n_g = c/v_g \) is the group index, \( \sigma = 3\lambda_0^3/(2\pi) \) the radiative cross section, \( A_m \) the effective mode area, \( \Gamma_a \) the vacuum emission rate, and \( \xi \) a cavity enhancement factor. Current values for Cs atoms (\( \lambda_0 = 884 \text{ nm}, \Gamma_a/2\pi = 5.02 \text{ MHz} \)) and SiN alligator waveguides [6,10] have \( A_m \approx 0.2 \mu \text{m}^2, n_g \approx 10, \xi \approx 5, \) and \( Q \) factors of \( 10^6 \). Depending on the reduction of spontaneous emission, \( \Gamma^e = \alpha \Gamma_a \), these numbers lead to \( P_{1D} \approx 50/\alpha \). Intrinsically losses in the dielectric and reduced \( v_g \) set finite propagation lengths of waveguide modes, \( L_{\text{prop}}/\lambda_a \approx Q/(2\pi n_g) \), which is \( > 10^4 \) for state-of-art SiN values [6,10]. Retardation effects also set a maximum number of atoms for superradiant atom-photon mapping, e.g., assuming a separation \( \lambda_a/2, \) then \( N\Gamma_{1D} < 2t_{\text{sat}}/(\lambda_a) \), which for current structures leads to \( N \lesssim 500 \) atoms. Possible ways of avoiding retardation are to increase \( \Gamma_{1D} \) by increasing \( \xi \) (not \( n_g \) [10]); or by doing the atom-photon mapping off-resonantly, which decreases \( \Gamma_{1D} \) while keeping \( P_{1D} \) constant. Other potential problems that we neglected are (i) imperfect atomic separations limited ultimately by center of mass wave packets and atomic motion and (ii) group velocity dispersion that distorts the propagating wave packet. In Ref. [47], we estimate under which conditions they can be neglected; however, a thorough study should be done for each implementation to minimize the impact on the protocols described.

In conclusion, we have presented a protocol to generate deterministic superpositions of many-body entangled atomic states in the presence of strong dissipation. Remarkably, the errors in the preparation of complex superposition states increase only linearly with the number of excitations of the system and inversely with the square root of the Purcell factor. Furthermore, we have shown how to map these atomic states to photonic states with a very efficient scaling that depends linearly on the inverse collective Purcell Factor and how to engineer a time-symmetric wave packet that guarantees the reversibility of the mapping.

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[41] Notice that with $d = n\lambda_c/2$, similar physics can be observed if one adjusts the laser phases appropriately.
[47] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.115.163603 for more detailed discussion on (i) error analysis in the preparation of the atomic superpositions; (ii) atom-photon mapping; (iii) implementation considerations, which includes Refs. [48–51].