Supplemental Material: Landau levels in strained optical lattices

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This supplement is organized as follows: (1) we summarize timescales reported in conventional synthetic gauge field experiments on a lattice; (2) we summarize the intensity profile that we used for Gaussian beams; (3) we present the details of our tight binding model and its relation to the light intensity in the lattice beams; (4) we tabulate optimal beam parameters for a range of sample sizes; (5) we show explicit details for the constraints on trap frequency presented in the main manuscript; (6) we define the local density of states; (7) we write out the transition rate for Bragg spectroscopy and (8) we present additional data on Bloch-Zener spectroscopy.

I. TIMESCALES IN EXPERIMENTS WITH OPTICAL LATTICES AND SYNTHETIC GAUGE FIELDS

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Method</th>
<th>Timescale [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aidelsburger et al. [1]</td>
<td>Raman</td>
<td>13</td>
</tr>
<tr>
<td>Atala et al. [2]</td>
<td>Raman</td>
<td>2</td>
</tr>
<tr>
<td>Aidelsburger et al. [3]</td>
<td>Raman</td>
<td>50</td>
</tr>
<tr>
<td>Jotzu et al. [4]</td>
<td>Shaking</td>
<td>10</td>
</tr>
<tr>
<td>Struck et al. [5]</td>
<td>Shaking</td>
<td>50</td>
</tr>
<tr>
<td>Kennedy et al. [6]</td>
<td>Raman</td>
<td>71</td>
</tr>
</tbody>
</table>

TABLE I. Timescales in conventional synthetic gauge field experiments on a lattice, typical timescale for static optical lattice experiments are several hundreds of milliseconds [7].

To compare conventional synthetic gauge field in lattice experiments to the proposed setup we would like to compare the heating rates in the two cases. Unfortunately (with the exception of Ref. [4]) heating rates are not typically reported. Hence, we use the longest reported experimental timescale as a proxy for how long the atoms remain cold within the lattice (see Table. I). We expect our proposed setup to extend the timescales to several hundreds of milliseconds, which is the typical timescale for experiments in static lattices [7].

II. GAUSSIAN BEAMS

In modeling the Gaussian lattice beams we use the following intensity profile:

\[ I_{\text{gaussian}}(r, z) = I_0 \left( 1 + \frac{z^2}{z_R^2} \right)^{-\frac{1}{2}} e^{-\frac{z^2}{w_0^2(1 + z^2/z_R^2)}} \] (1)

where \( z_R = \frac{\pi w_0^2}{\lambda} \), \( w_0 \) is the beam waist, \( r = |(\vec{r} - \vec{r}_0) \times \vec{v}_0| \) is the radial distance from the beam axis, \( z = |(\vec{r} - \vec{r}_0) \cdot \vec{v}_0| \) is the axial distance to the beam focal point \( \vec{r}_0 \), and \( \vec{v}_0 \) is a unit vector along the beam axis.

III. MODEL RELATING LASER INTENSITY TO THE TIGHT BINDING PARAMETERS

To relate the intensities of the three lattice beams \( I_1(\vec{r}), I_2(\vec{r}), \) and \( I_3(\vec{r}) \) to the tight binding parameters \( t_1(\vec{r}), t_2(\vec{r}), t_3(\vec{r}), \) and \( V(\vec{r}) \) at a given point in space \( \vec{r} \), we make two assumptions: (1) the beam intensities vary slowly in space on the length-scale of a unit cell and (2) the beam intensities are close to uniform \( I_1(\vec{r}) \approx I_2(\vec{r}) \approx I_3(\vec{r}) \).

For the case of the hopping matrix elements \( t_u(\vec{r}) \), the assumptions allow us to use the model

\[ t_u = a_0 I_{\text{avg}}^{3/4} e^{-\sqrt{I_{\text{avg}}(b_0 + b_1 \delta_u + b_2 \delta_u^2 + c_2 \sum_v \delta_v^2)}} \] (2)

where we have dropped the index \( \vec{r} \) for clarity, \( I_{\text{avg}} = \frac{1}{2} \sum_{\vec{r}=1} I_u, \delta_u = (I_u/I_{\text{avg}} - 1), \) and \( \{a_0, b_0, b_1, b_2, c_2\} \) are the fitting parameters. To obtain values for \( \{a_0, b_0, b_1, b_2, c_2\} \), we numerically computed \( t_u \)’s for a series of spatially uniform but anisotropic lattices and then fitted the resulting data set [see Table II]. The numerical computations were performed using the Wannier function method [8] with a sufficiently large basis to ensure convergence. As for the data set, we used various values of \( I_u \)’s ranging from 2.2\( E_R \) to 3.6\( E_R \) and keeping \( |\delta_u| < 0.3 \) which is the appropriate range of light intensities for the proposed setup with displaced \( 4E_R \) beams. We find good agreement between our model and the numerically computed hopping matrix elements as long as the beams have approximately the same intensity \( |\delta_u| < 0.3 \). In Fig. 1 we show this comparison along a particular slice through the data set, in which we set \( I_2 = I_3 = 3E_R \) while varying \( I_1 \). The maximal logarithmic error over our dataset was \( \left| \log \frac{t_u, \text{numerical}}{t_u, \text{fitted}} \right| = 0.016 \).
Using the two assumptions on $I_1(\vec{r})$, $I_2(\vec{r})$, and $I_3(\vec{r})$, we can model the onsite potential due to the optical lattice using the expression

$$V = d_1 I_{\text{avg}} + e_1 \sqrt{I_{\text{avg}}} + f_1 + \frac{g_1}{\sqrt{I_{\text{avg}}}} + h_1 I_{\text{avg}} \sum_v \delta_v,$$  

(3)

where $\{d_1, e_1, f_1, g_1, h_1\}$ are the fitting parameters. Fitting the same data set as the one we used for the hopping matrix elements, results in values for the fitting parameters listed in Table II. The maximal error over the dataset was $|V_{\text{numerical}} - V_{\text{fitted}}|/V_{\text{numerical}} = 0.004$.

### IV. OPTIMAL BEAM PARAMETERS

In order to achieve a uniform pseudo-magnetic field over the sample area we can tune two parameters – Gaussian beam waist $\omega$ and beam axis displacement $d$. We tabulate these parameters for a range of sample sizes in Table III. The parameters were chosen to ensure that the pseudo-magnetic field varies by less than 20% over the sample area. The corresponding gap between $n = 0$ and $n = 1$ Landau levels is listed for $^{87}\text{Rb}$ atoms in a 700 nm, $4E_R$ lattice with $t_{ij}/\hbar \approx 870 \text{Hz}$. While the pseudo-magnetic field strength is determined by the geometry [see main text], the Landau level gap scales with the hopping strength $E_1 - E_0 = \lambda t_{ij} \sqrt{2B/3}$.

<table>
<thead>
<tr>
<th>$R_0$ [µm]</th>
<th>$w_0$ [µm]</th>
<th>$d$ [µm]</th>
<th>B field [µm$^{-2}$]</th>
<th>LL gap [µHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6</td>
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<td>33</td>
<td>220</td>
<td>72</td>
<td>0.13</td>
<td>180</td>
</tr>
</tbody>
</table>

TABLE III. Optimal beam waist $w_0$ and displacement $d$ for different sample sizes $R_0$.

### V. CONSTRAINTS ON TRAP FREQUENCY

In order to observe the Landau levels in the presence of a trap potential, we want $\omega_{\text{eff}} = \sqrt{\omega_{\text{trap}}^2 - \omega_{\text{anti-trap}}^2}$ to be large enough to confine the atoms but not so large as to smear out the Landau levels. These two constraints provide us the upper and lower bound for $\omega_{\text{eff}}$.

For the lower bound, we begin with the lengthscale of the simple harmonic oscillator $\lambda_{\text{SHO}}$. First, we find the band mass to be:

$$m_{\text{band}} = \frac{9\hbar^2}{2t_0\lambda^2},$$  

(4)

where $t_0 = \langle t_{ij} \rangle$ represents the average value of the nearest neighbor hopping matrix element. Using the band mass, we can write down the continuum Hamiltonian for low energy states of our trapped system:

$$H = \frac{p^2}{2m_{\text{band}}} + \frac{1}{2}m_{\text{Rb}}\omega_{\text{eff}}^2 x^2$$

$$= \frac{p^2}{2m_{\text{band}}} + \frac{1}{2}m_{\text{band}} \left( \sqrt{\frac{m_{\text{Rb}}}{m_{\text{band}}} \omega_{\text{eff}}} \right)^2 x^2.$$  

(5)

Defining $\tilde{\omega} = \sqrt{\frac{m_{\text{Rb}}}{m_{\text{band}}} \omega_{\text{eff}}}$, we find that the radius of the lowest energy state of the harmonic the system is approximately

$$\lambda_{\text{SHO}} = \sqrt{\frac{\hbar}{m_{\text{band}} \tilde{\omega}}},$$  

(6)

In order to ensure that at least the ground state is trapped, we require that $\lambda_{\text{SHO}} \leq R_0$ where $R_0$ is the characteristic radius of our lattice system – i.e. the radius over which the pseudo magnetic field is uniform. Thus we obtain the lower bound for $\omega_{\text{eff}}$:

$$\omega_{\text{eff}}^2 \geq \frac{2t_0\lambda^2}{9m_{\text{Rb}}R_0^2}.$$  

(7)

For sample size $R_0 = 23.1\mu m$, $\omega_{\text{eff}} \geq 0.211 \times 2\pi \text{Hz}$.

To find the upper bound on $\omega_{\text{eff}}$, we first evaluate the Landau level energy

$$E_n = \hbar \nu \sqrt{2nB}$$

$$= \frac{\lambda t_{0}}{\sqrt{3}} \sqrt{2nB}. $$  

(8)
To avoid smearing the $n^{th}$ Landau level, we want 
\[ \frac{1}{2} m_{Rb} \omega_{n}^{2} \rho_{n} \leq E_{n+1} - E_{n} \], where $r_{n} = (2n + 1)/B$ is the typical radius of $n^{th}$ Landau level. Using the relation $B = 2.7/\lambda R_{0}$ obtained from Table I of the main text, we find

\[ \omega_{\text{eff}}^{2} \leq \frac{7.2t_{ij}}{m_{Rb}R_{0}^{1/2} \lambda^{1/2}} \left( \frac{\sqrt{|n + 1|} - \sqrt{|n|}}{2n + 1} \right). \tag{9} \]

For $R_{0} = 23.1 \mu m$, $\omega_{\text{eff}} \lesssim 91 \times 2 \pi Hz$ for $n = 0$ and $\omega_{\text{eff}} \lesssim 12 \times 2 \pi Hz$ for $n = \pm 1$.

\section{VI. LOCAL DENSITY OF STATES}

We define the local density of states $\rho(E, r)$ as

\[ \rho(E, r) = N \sum_{i, |E_{i} - E| < \delta} |\psi(\vec{r}_{i}, E_{i})|^{2} e^{-|\vec{r}_{i} - \vec{r}|^{2}/(2\sigma_{i}^{2})}, \tag{10} \]

as a function of position and energy, where $\{E_{i}\}$ and $\{\psi(\vec{r}_{i}, E_{i})\}$ are the single-particle eigenvalues and eigenfunctions of Eq. 3 in the main text, $N^{-1} = \delta \sum_{i} e^{-|\vec{r}_{i} - \vec{r}|^{2}/(2\sigma_{i}^{2})}$ is a normalization factor, and $\sigma_{0}$ specifies the range over which we measure the local density of states.

\section{VII. BRAGG SPECTROSCOPY}

For Bragg spectroscopy setup [9] described by the perturbation potential $V(\vec{r}, t) = V_{1} \cos(\vec{k} \cdot \vec{r}) \cos(\omega t)$, the transition rate is given by

\[ \sum_{i} |\psi_{i}(\vec{r})| \cos(\vec{k} \cdot \vec{r}) |\psi_{0}(\vec{r})|^{2} \delta(E_{i} - E_{0} - \omega). \tag{11} \]

Here, $\psi_{0}$ is the initial state with energy $E_{0}$ and $\psi_{i}$ is the excited states with energy $E_{i}$.

\section{VIII. BLOCH-ZENER SPECTROSCOPY}

Bloch-Zener spectroscopy [10] offers an alternative to Bragg spectroscopy that can detect the separation between the $n = 0$ and $n = \pm 1$ Landau levels without the complexity of the Bragg setup. We begin with a BEC in the ground state of the trap and then apply a tilt in the direction of one of the Dirac cones in order to induce Bloch oscillations [11]. The character of the Bloch oscillations strongly depends on the lattice tilt $\alpha$ [see Fig. 2]. The change in character is controlled by the Landau-Zener process across the largest gap in the system – the separation between $n = 0$ and $n = 1$ Landau levels – allowing us to measure this gap. If $(E_{1} - E_{0})/\alpha \lambda \gg 0.5 \ll 0.5$ the atoms remain on the lower branch [jump to the upper branch]. The change in character can be observed directly in the in-situ motion of the atom cloud via a change of the direction of the group velocity: If the tilt angle is small, atoms will be reflected back by the gap. On the other hand, if the tilt angle is large the atoms will jump across the gap to the upper band. The presence/absence of reflection can be detected by monitoring the motion of the center of mass of the atoms along the direction of the tilt, as depicted in Fig. 3. We identify the critical tilt by a plateau in the motion of the center of mass (half the atoms stay at the lower band and half go to the upper band) as depicted in Fig. 3b.

To verify the ability of Bloch-Zener spectroscopy to measure the gap, we calculate the critical tilt for various values of the pseudo-magnetic field and hence gap. We plot the relation between the gap and the critical tilt in Fig. 4. We observe a linear relation between the critical tilt and the largest energy gap $E_{1} - E_{0}$

\[ \frac{E_{1} - E_{0}}{\alpha \text{crit} \lambda} \approx 0.5. \tag{12} \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Bloch-Zener spectroscopy: spectral density of the atom cloud as a function of time for small tilt (top) and large tilt (bottom).}
\end{figure}
FIG. 3. Tilt spectroscopy: (a-c) Center of mass position of an atom cloud as a function of time for three different values of tilt $\alpha$. (d) Same data as (a-c) with both axis rescaled by $\alpha$.

FIG. 4. Relation between the critical tilt $\alpha_c$ (rescaled by the wavelength of optical lattice light $\lambda$) and the gap between the $n = 0$ and $n = 1$ Landau levels for various values of the pseudo-magnetic field. The line represents best fit to the linear law $E_1 - E_0 = c\lambda\alpha_c$ where $c$ is the constant of proportionality.

[11] In contrast to cyclotron motion experiment, we do not stop at the Dirac point but pass through it.