Collins asymmetries in inclusive charged $KK$ and $K\pi$ pairs produced in $e^+e^-$ annihilation


(The BABAR Collaboration)

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We present measurements of Collins asymmetries in the inclusive process $e^+e^- \to h_1h_2X$, $h_1h_2 = KK, K\pi, \pi\pi$, at the center-of-mass energy of 10.6 GeV, using a data sample of 468 fb$^{-1}$ collected by the BABAR experiment at the PEP-II B factory at SLAC National Accelerator Center. Considering hadrons in opposite thrust hemispheres of hadronic events, we observe clear azimuthal asymmetries in the ratio of unlike- to like-sign, and unlike- to all charged $h_1h_2$ pairs, which increase with hadron energies. The $K\pi$ asymmetries are similar to those measured for the $\pi\pi$ pairs, whereas those measured for high-energy $KK$ pairs are, in general, larger.

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The Collins effect [1] relates the transverse spin component of a fragmenting quark to the azimuthal distribution of final state hadrons about its flight direction. The chiral-odd, transverse momentum-dependent Collins fragmentation function (FF) provides a unique probe of quantum chromodynamics (QCD), such as factorization in semi-inclusive deep inelastic scattering experiments (SIDIS) [6–9]. These are sensitive to the product of a fragmenting quark to the azimuthal distribution of final state hadrons about its flight direction.

Experimental, we do not know $\hat{k}$ or $S_q$ for a given $e^+e^- \to q\bar{q}$ event, but the quark and anti-quark must be produced back-to-back in the $e^+e^-$ c.m. frame, with their spins aligned with each other and polarized along the $e^+$ or $e^-$ direction. The event thrust axis $\hat{n}$ [20,21] approximates the $q\bar{q}$ axis, so an azimuthal correlation between two hadrons in opposite thrust hemispheres reflects the product of the two Collins functions.

In this Letter, we report the measurement of the Collins effect (or Collins asymmetry) for inclusive production of hadron pairs in the process $e^+e^- \to q\bar{q} \to h_1h_2X$, where $h_{1,2} = K^\pm$ or $\pi^\pm$, $q$ stands for light quarks $u$ or $d$ or $s$, and $X$ for any combination of additional hadrons.

The probability that a transversely polarized quark $q^\uparrow$ with momentum direction $\hat{k}$ and spin $S_q$ fragments into a spinless hadron $h$ with momentum $P_h$, is defined in terms of unpolarized $D^q_1$ and Collins $H^q_1$ fragmentation functions [19]:

$$D^q_1(z, P_{hT}) = D^q_1(z, F^2_{hT}) + H^q_1(z, P_{hT}^2) \frac{(k \times P_{hT}) \cdot S_q}{z M_h},$$

where $M_h$, $P_{hT}$, and $z = 2E_h/\sqrt{s}$ are the hadron mass, momentum transverse to $\hat{k}$, and fractional energy, respectively, with $E_h$ its total energy and $\sqrt{s}$ the $e^+e^-$ center-of-mass (c.m.) energy. The term including $H^q_1$ introduces an azimuthal modulation around the direction of the fragmenting quark, called Collins asymmetry.

Figure 1 shows the thrust reference frame (RF12) [10]. If not otherwise specified, all kinematic variables are defined in the $e^+e^-$ c.m. frame. The Collins effect results in a cosine modulation of the azimuthal angle $\phi_{12} = \phi_1 + \phi_2$ of the di-hadron yields. Expressing the yield as a function of $\phi_{12}$ [10] (after the integration over $P_{hT}$), and dividing by the average bin content, we obtain the normalized rate

$$R_{12}(\phi_{12}) = 1 + \frac{\sin^2 \theta_{1h}}{1 + \cos^2 \theta_{1h}} \cos \phi_{12} \frac{H^1_1(z_1)H^1_1(z_2)}{D_1^1(z_1)D_1^1(z_2)},$$

where $\theta_{1h}$ is defined in Fig. 11. $z_1(2)$ is the fractional energy.
of the first (second) hadron, and the bar denotes the FF for the \bar{q}.

Other reference frames \cite{10,11} have been proposed to overcome the finite resolution in the determination of the thrust axis. The RF0 frame \cite{22} uses the momentum of one hadron as a reference axis, and defines a single angle \phi_0 between the plane containing the two hadron momenta and the plane defined by the beam and the reference axis. The corresponding normalized yield in the \( e^+e^- \) c.m. system is

\[
R_0(2\phi_0) = 1 + \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos 2\phi_0 \cdot \frac{\mathcal{F}[H^1_0(z_1)H^0_1(z_2)]}{\mathcal{F}[D_1(z_1)D_1(z_2)]},
\]

where \( \theta_2 \) is the angle between the hadron used as reference and the beam axis, and \( \mathcal{F} \) is used to denote the convolution integral

\[
\mathcal{F}[DD] \equiv \sum_q \epsilon_q^2 \int d^2k_T d^2p_T \delta^2(p_T + k_T - q_T)
\]

\[
D^q(z_1,z_2) D^q(z_2,z_2) p_T^2,
\]

with \( k_T, p_T, \) and \( q_T \) the transverse momentum of the fragmenting quark, antiquark, and virtual photon from \( e^+e^- \) annihilation, respectively, in the frame where the two hadrons are collinear.

For this analysis we use a data sample of 468 fb \(^{-1} \) \cite{23} collected at the c.m. energy \( \sqrt{s} \approx 10.6 \) GeV with the \textit{BaBar} detector \cite{23} at the SLAC National Accelerator Laboratory. We use tracks reconstructed in the silicon vertex detector and in the drift chamber (DCH) and identified as pions or kaons in the DCH and in the Cherenkov ring imaging detector (DIRC). Detailed Monte Carlo (MC) simulation is used to study detector effects and to estimate contribution from various background sources. Hadronic events are generated using the \textit{Jetset} \cite{26} package and undergo a full detector simulation based on \textit{GeANT4} \cite{27}.

We make a tight selection of hadronic events in order to minimize biases due to detector acceptance and hard initial-state photon radiation (ISR) or final-state gluon (qg) radiation, since they can introduce fake azimuthal modulations. Requiring at least three charged tracks consistent with the \( e^+e^- \) primary vertex and a total visible energy of the event in the laboratory frame \( E_{\text{tot}} > 11 \) GeV, we reject \( e^+e^- \rightarrow \tau^+\tau^- \) and two-photon backgrounds, as well as ISR (qg) events with the photon (one jet) along the beam line. About 10% of ISR photons are within our detector acceptance, and we reject events with a photon candidate with energy above 2 GeV. We require an event thrust value \( T > 0.8 \) to suppress qg and B\( \bar{B} \) events, and \( |\cos \theta_{th}| < 0.6 \) so that most tracks are within the detector acceptance.

We assign randomly the positive direction of the thrust axis, and divide each event into two hemispheres by the plane perpendicular to it. To ensure tracks are assigned to the correct hemispheres, we require them to be within a 45° angle of the thrust axis and to have \( z > 0.15 \). A “tight” identification algorithm is used to identify kaons (pions), which is about 80% (90%) efficient and has misidentification rates below 10% (5%). We select those pions and kaons that lie within the DIRC acceptance region with a polar angle in laboratory frame \( 0.45 \text{rad} < \theta_{\text{lab}} < 2.46 \text{rad} \). To minimize backgrounds, such as \( e^+e^- \rightarrow \mu^+\mu^-\gamma \) followed by photon conversion, we require \( z < 0.9 \).

We construct all the possible pairs of selected tracks reconstructed in opposite thrust hemispheres, and we calculate the corresponding azimuthal angles \( \phi_1, \phi_2, \) and \( \phi_0 \) in the respective reference frames. In this way, we identify three different samples of hadron pairs: \( K\bar{K}, K\pi, \) and \( \pi\pi \). To reduce low-energy gluon radiation and the contribution due to wrong hemispheres assignment, we require \( Q_i < 3.5 \) GeV/c, where \( Q_i \) is the transverse momentum of the virtual photon from \( e^+e^- \) annihilation in the frame where the two hadrons are collinear.

The analysis is performed in intervals of hadron fractional energies with the following boundaries: 0.15, 0.2, 0.3, 0.5, 0.9, for a total of 16 two-dimensional \((z_1,z_2)\) intervals.

For each of the three samples, we evaluate the normalized yield distributions \( R_{12} \) and \( R_0 \) for unlike (U), like (L), and any charge combination (C) of hadron pairs as a function of \( \phi_1 + \phi_2 \) and \( 2\phi_0 \), as shown in the left plot of Fig. 2. These combinations of charged hadrons contain different contribution of favored and disfavored FFs, where a favored (disfavored) process refers to the pro-
duction of a hadron for which one (none) of the valence quarks is of the same kind as the fragmenting quark. In particular, by selecting \( KK \) pairs, we are able to study the favored contribution \( H_s^{\text{fav}} \) of the strange quark, not accessible when considering \( \pi\pi \) pairs only.

The normalized distributions can be parametrized with a cosine function: \( R^i_{\alpha} = b^i_{\alpha} + a^i_{\alpha} \cos \beta^i_{\alpha} \), where \( \alpha = 0, 12 \) indicates the reference frames, \( i = U, L, C \) the charge combination of hadron pairs, and \( \beta_{12(0)} = \phi_{12(0)} \).

The \( R^i_{\alpha} \) distributions are strongly affected by instrumental effects. In order to reduce the impact of the detector acceptance, we construct two double ratios (DR) of normalized distributions, \( R^U_{\alpha}/R^L_{\alpha} \) and \( R^U_{\alpha}/R^C_{\alpha} \). The two ratios give access to the same physical quantities as the independent \( R^i_{\alpha} \), that is the favored and disfavored FFs, but in different combinations. We report the results for both kind of DRs, which are strongly correlated since they are obtained by using the same data set. These are shown in the right plot of Fig. 2 for \( KK \) pairs in RF12. For small asymmetry values, the double ratios are still parametrized by a function that is linear in the cosine of the corresponding combination of azimuthal angles:

\[
R^{ij}_{\alpha} = \frac{R^i_{\alpha}}{R^j_{\alpha}} \simeq B^{ij}_{\alpha} + A^{ij}_{\alpha} \cdot \cos \beta^i_{\alpha},
\]

with \( B \) and \( A \) free parameters, and \( i, j = U, L, C \). The constant term \( B \) must be consistent with unity, while \( A \) contains the information about the favored and disfavored Collins FFs.

We fit the binned \( R^{ij}_{\alpha} \) distributions independently for \( KK, K\pi, \) and \( \pi\pi \) hadron pairs. Using the MC sample, we evaluate the \( K/\pi \) (mis)identification probabilities for the 16 \((z_1, z_2)\) intervals in each of the three samples. For example, the probability \( f_{KK}^{KK} \) that a true \( KK \) pair is reconstructed as \( KK \) pair is about 90% on average, slightly decreasing at higher momenta, while the probability \( f_{KK}^{KK} \) that a true \( K\pi \) pair is identified as \( KK \) is about 10%, and \( f_{\pi\pi}^{KK} \) is negligible.

The presence of background processes could introduce azimuthal modulations not related to the Collins effect, and modifies the measured asymmetry as follows:

\[
A_{KK}^{\text{meas}} = f_{KK} \cdot \left( \sum_{nm} f_{KK}^{nm} \cdot A_{nm} \right) + \sum_i F_i^{KK} \left( \sum_{nm} f_{KK}^{nm} \cdot A_{nm} \right),
\]

with \( nm = KK, K\pi, \pi\pi, \) and \( i = e^+, e^-, \tau^+, \tau^- \). In Eq. 5, \( A_{nm} \) are the true Collins asymmetries produced from the fragmentation of light quarks in the three samples, \( A_{nm}^i \) is the \( i \)-th background asymmetry contribution, and \( f_{KK}^{nm} \) are the fractions of kaon pairs coming from \( uds \) and background events, calculated from the respective MC samples. By construction, \( \sum_i F_i + F_{uds} = 1 \). A similar expression holds for \( K/\pi \) and \( \pi/\pi \) samples.

Previous studies show that \( f_{\pi\pi}^{KK} \) is very small, and \( A_{\pi\pi} \) are unknown, we determine \( A_{\pi\pi}^{\text{meas}} = \sum_i F_{\pi\pi}^{nm} \) in Eq. 5 from samples enhanced in \( e^+ e^- \) with a decay \( D^{*\pm} \rightarrow D^0 \pi^{\pm} \), with the \( D^0 \) candidate reconstructed in the following four Cabibbo-favored decay modes: \( K^- \pi^+, K^-\pi^+\pi^- \), \( K^0\pi^+\pi^- \), and \( K^0\pi^+\pi^- \). We solve the system of equations for \( A_{KK}^{\text{meas}}, A_{K\pi}^{\text{meas}}, A_{\pi\pi}^{\text{meas}} \), for the standard and charm-enhanced samples, and we extract simultaneously the Collins asymmetries \( A_{KK}, A_{K\pi}, \) and \( A_{\pi\pi} \), corrected for the contributions of the background and \( K/\pi \) (mis)identification.

We test the DR method on the MC sample. Spin effects are not simulated in MC, and so the DR distributions should be uniform. However, when fitting the distributions for reconstructed \( KK \) pairs with Eq. 5, we measure a cosine term in the full sample of 0.004 ± 0.001 and 0.007 ± 0.001 in the RF12 and RF0 frames, respectively, indicating a bias. Smaller values are obtained for \( K\pi \) and \( \pi\pi \) pairs. Studies performed on the MC samples, both at generation level and after full simulation, demonstrate that the main source of this bias is due to the emission of ISR. We subtract the bias, which is everywhere smaller than the asymmetries measured in the data sample in the RF12 and RF0 frames, respectively.
instead of the $q\bar{q}$ axis, while they are consistent with the simulated ones in RF0, where only particle identification and tracking reconstruction effects could introduce possible dilution. Since we measure the same dilution for $KK$, $K\pi$, and $\pi\pi$ samples, the asymmetry is corrected by rescaling $A_{KK}$, $A_{K\pi}$, and $A_{\pi\pi}$ using the same correction factor, which ranges from 1.3 to 2.3 increasing with $z$. No corrections are needed for the asymmetries measured in RF0. The errors on the correction factors are assigned as systematic errors.

All systematic effects, if not otherwise specified, are evaluated for each bin of $z$. The main contribution comes from the MC bias. We compare the bias results from the nominal selection, with those obtained by requiring different cuts on $E_{\text{tot}}$, and/or by changing the detector acceptance region for the hadrons. The largest variation of the bias is combined in quadrature with the MC statistical error and taken as systematic uncertainty. The effects due to the particle identification are evaluated using tighter and looser selection criteria. The largest deviations with respect to the nominal selection are taken as systematic uncertainties: the average relative uncertainties are around 10%, 7%, and 5% for the $KK$, $K\pi$, and $\pi\pi$ pairs. Fitting the azimuthal distributions using different bin sizes, we determine systematic uncertainties, which are not larger than 5%, 1.9%, and 1% for the three samples. The systematic uncertainty due to the $E_{\text{tot}}$ cut is obtained by comparing the measured asymmetries with those obtained with the looser selection $E_{\text{tot}} > 10$ GeV. The average systematic contribution is around 10% for the three samples in both reference frames. We use different fitting functions with additional higher harmonic terms. No significant changes in the value of the cosine moments with respect to the standard fits are found. As a cross-check of the double ratio method we fit the difference of $R^2$ distributions, and we compare the two results. The difference between the two procedures is negligible for $K\pi$ and $\pi\pi$ pairs, while it reaches 1% and 3% for kaon pairs in RF12 and RF0, respectively. All the other systematic contributions are negligible.

The Collins asymmetries measured for the 16 two-dimensional $(z_1, z_2)$ bins, for reconstructed $KK$, $K\pi$, and $\pi\pi$ hadron pairs, are shown in Fig. 3 for RF12 and RF0, and are summarized in tables reported in the Supplemental Material [28]. The asymmetries are corrected for the background contributions and $K/\pi$ contamination following Eq. 5, the MC bias is subtracted, and the corrections due to the dilution effects are applied. The total systematic uncertainties are obtained by adding in quadrature the individual contributions, and are represented by the bands around the data points.

An increasing asymmetry with increasing hadron energies is visible for the $U/L$ double ratio in both reference frames. The largest effects, but with less precision, are observed for $KK$ pairs, for which $A_{UL}^{12}$ is consistent with zero at low $z$, and reaches 22% in the last $z$ bin, while somewhat smaller values are seen for $\pi\pi$ and $K\pi$ pairs. In particular, at low $(z_1, z_2)$ bins $A_{UL}^{12}$ for $\pi\pi$ pairs is nonzero, in agreement with the behavior observed in [10]. The small differences between the two data sets are due to the different kinematic region selected after the cut on $\cos\theta_\text{th}$. The $A_{UL}$ asymmetry is smaller than $A_{UL}^{12}$ in all cases, and, for the $KK$ pairs, the rise of the asymmetry with the hadron energies is not evident.

In summary, we have studied for the first time in $e^+e^-$ annihilation the Collins asymmetry for inclusive production of $KK$ and $K\pi$ pairs as a function of $(z_1, z_2)$ in two distinct reference frames. We measure the azimuthal modulation of the double ratios $U/L$ and $U/C$, which are sensitive to the favored and disfavored Collins FFs for light quarks. We simultaneously extract also the Collins asymmetries for $\pi\pi$ pairs, which are found to be in agreement with those obtained in previous studies [10, 12]. The results reported in this Letter and those obtained from SIDIS experiments can be used in a global analysis to extract the favored contribution of the strange quark, and to improve the knowledge on the $u$ and $d$ fragmentation processes [13–15].

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FIG. 3. (color online). Comparison of U/L (top) and U/C (bottom) Collins asymmetries in RF12 (left) and RF0 (right) for $KK$, $K\pi$, and $\pi\pi$ pairs. The statistical and systematic uncertainties are represented by the bars and the bands around the points, respectively. The 16 $(z_1, z_2)$ bins are shown on the x-axis: in each interval between the dashed lines, $z_1$ is chosen in the following ranges: $[0, 0.15]$, $[0.2, 0.3]$, $[0.3, 0.5]$, and $[0.5, 0.9]$, while within each interval the points correspond to the four bins in $z_2$.

[28] See (URL to be inserted by the publisher) for Supplemental Material containing tables and figures.
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<td>0.382 / 0.792</td>
<td>10.97 / 1.31 / 0.63</td>
<td>2.41 / 0.65 / 0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3 / 0.5</td>
<td>0.383 / 0.5 / 0.9</td>
<td>0.610 / 0.784</td>
<td>9.84 / 1.16 / 0.92</td>
<td>1.71 / 0.52 / 0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 / 0.9</td>
<td>0.609 / 0.15 / 0.2</td>
<td>0.175 / 0.785</td>
<td>6.15 / 1.09 / 0.98</td>
<td>1.78 / 0.74 / 0.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 / 0.9</td>
<td>0.608 / 0.2 / 0.3</td>
<td>0.248 / 0.783</td>
<td>11.75 / 1.03 / 0.91</td>
<td>2.81 / 0.60 / 0.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 / 0.9</td>
<td>0.610 / 0.3 / 0.5</td>
<td>0.383 / 0.784</td>
<td>7.40 / 1.13 / 0.91</td>
<td>1.11 / 0.52 / 0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 / 0.9</td>
<td>0.615 / 0.5 / 0.9</td>
<td>0.615 / 0.776</td>
<td>22.36 / 2.09 / 1.69</td>
<td>2.63 / 0.61 / 0.62</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE I. Light quark ($uds$) Collins asymmetries obtained by fitting the U/L and U/C double ratios as a function of ($z_1$, $z_2$) for kaon pairs. In the first two columns, the $z$ bins and their respective mean values for the kaon in one hemisphere are reported; in the following two columns, the same variables for the second kaon are shown; in the fifth column the mean value of $\sin^2 \theta_{kz}/(1 + \cos^2 \theta_{kz})$ is summarized, calculated in the RF12 (upper table) or RF0 (lower table) frames; in the last two columns the asymmetry results are summarized. The quoted errors are statistical and systematic, respectively. The mean values of the quantities reported in the table are calculated by summing the corresponding values for each KK pair and dividing by the number of KK pairs that fall into each ($z_1$, $z_2$) interval. Note that the $A_{UL}^{UC}$ and $A_{UC}^{UC}$ results are strongly correlated since they are obtained by using the same data set.
### TABLE II. Light quark (uds) Collins asymmetries obtained by fitting the U/L and U/C double ratios as a function of \((z_1, z_2)\) for \(K\pi\) hadron pairs. In the first two columns, the \(z\) bins and their respective mean values for the hadron \((K\pi\) or \(\pi\)) in one hemisphere are reported; in the following two columns, the same variables for the second hadron \((K\) or \(\pi\)) are shown; in the fifth column the mean value of \(\sin^2 \theta_{h(2)}/(1 + \cos^2 \theta_{h(2)})\) is summarized, calculated in the RF12 (upper table) or RF0 (lower table) frames; in the last two columns the asymmetry results are summarized. The quoted errors are statistical and systematic, respectively. The mean values of the quantities reported in the table are calculated by summing the corresponding values for each \(K\pi\) pair and dividing by the number of \(K\pi\) pairs that fall into each \((z_1, z_2)\) interval. Note that the \(A^{UL}\) and \(A^{UC}\) results are strongly correlated since they are obtained by using the same data set.

<table>
<thead>
<tr>
<th>(z_1)</th>
<th>(z_2)</th>
<th>(\langle z_1 \rangle)</th>
<th>(\langle z_2 \rangle)</th>
<th>(\sin^2 \theta_{h(2)}/(1 + \cos^2 \theta_{h(2)}))</th>
<th>(A^{UL} (10^{-4}))</th>
<th>(A^{UC} (10^{-4}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15, 0.2</td>
<td>0.174</td>
<td>0.174</td>
<td>0.174</td>
<td>0.794</td>
<td>0.19 ± 0.77 ± 0.89</td>
<td>0.29 ± 0.69 ± 0.25</td>
</tr>
<tr>
<td>0.15, 0.2</td>
<td>0.174</td>
<td>0.174</td>
<td>0.174</td>
<td>0.792</td>
<td>0.17 ± 0.62 ± 0.36</td>
<td>1.49 ± 0.55 ± 0.23</td>
</tr>
<tr>
<td>0.15, 0.2</td>
<td>0.174</td>
<td>0.305</td>
<td>0.305</td>
<td>0.791</td>
<td>0.34 ± 0.53 ± 0.39</td>
<td>1.93 ± 0.48 ± 0.24</td>
</tr>
<tr>
<td>0.15, 0.2</td>
<td>0.174</td>
<td>0.509</td>
<td>0.509</td>
<td>0.784</td>
<td>0.73 ± 0.64 ± 0.56</td>
<td>4.12 ± 0.56 ± 0.35</td>
</tr>
<tr>
<td>0.2, 0.3</td>
<td>0.245</td>
<td>0.15, 0.2</td>
<td>0.174</td>
<td>0.791</td>
<td>0.20 ± 0.63 ± 0.36</td>
<td>1.10 ± 0.56 ± 0.23</td>
</tr>
<tr>
<td>0.2, 0.3</td>
<td>0.245</td>
<td>0.203</td>
<td>0.203</td>
<td>0.790</td>
<td>0.36 ± 0.50 ± 0.38</td>
<td>2.02 ± 0.45 ± 0.23</td>
</tr>
<tr>
<td>0.2, 0.3</td>
<td>0.245</td>
<td>0.305</td>
<td>0.305</td>
<td>0.789</td>
<td>4.94 ± 0.47 ± 0.39</td>
<td>2.79 ± 0.42 ± 0.24</td>
</tr>
<tr>
<td>0.2, 0.3</td>
<td>0.245</td>
<td>0.509</td>
<td>0.611</td>
<td>0.782</td>
<td>7.56 ± 0.61 ± 0.52</td>
<td>4.13 ± 0.52 ± 0.32</td>
</tr>
<tr>
<td>0.3, 0.5</td>
<td>0.380</td>
<td>0.15, 0.2</td>
<td>0.174</td>
<td>0.791</td>
<td>3.76 ± 0.55 ± 0.39</td>
<td>2.04 ± 0.50 ± 0.25</td>
</tr>
<tr>
<td>0.3, 0.5</td>
<td>0.380</td>
<td>0.203</td>
<td>0.245</td>
<td>0.789</td>
<td>4.27 ± 0.42 ± 0.39</td>
<td>2.04 ± 0.42 ± 0.24</td>
</tr>
<tr>
<td>0.3, 0.5</td>
<td>0.379</td>
<td>0.305</td>
<td>0.305</td>
<td>0.788</td>
<td>5.14 ± 0.58 ± 0.41</td>
<td>2.82 ± 0.47 ± 0.25</td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.612</td>
<td>0.15, 0.2</td>
<td>0.174</td>
<td>0.784</td>
<td>4.75 ± 0.65 ± 0.55</td>
<td>2.38 ± 0.56 ± 0.35</td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.612</td>
<td>0.203</td>
<td>0.245</td>
<td>0.782</td>
<td>8.65 ± 0.63 ± 0.52</td>
<td>4.57 ± 0.52 ± 0.32</td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.612</td>
<td>0.305</td>
<td>0.379</td>
<td>0.781</td>
<td>9.74 ± 0.92 ± 0.57</td>
<td>5.01 ± 0.63 ± 0.34</td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.615</td>
<td>0.509</td>
<td>0.615</td>
<td>0.777</td>
<td>12.83 ± 1.54 ± 0.81</td>
<td>5.66 ± 0.85 ± 0.46</td>
</tr>
</tbody>
</table>
### Table III

Light quark (uds) Collins asymmetries obtained by fitting the U/L and U/C double ratios as a function of \((z_1, z_2)\) for pion pairs. In the first two columns, the \(z\) bins and their respective mean values for the pion in one hemisphere are reported; in the following two columns, the same variables for the second pion are shown; in the fifth column the mean value of \(\sin^2 \theta_{h(2)/}\langle 1 + \cos^2 \theta_{h(2)} \rangle\) is summarized, calculated in the RF12 (upper table) or RF0 (lower table) frames; in the last two columns the asymmetry results are summarized. The quoted errors are statistical and systematic, respectively. The mean values of the quantities reported in the table are calculated by summing the corresponding values for each \(\pi\pi\) pair and dividing by the number of \(\pi\pi\) pairs that fall into each \((z_1, z_2)\) interval. Note that the \(A^{UL}\) and \(A^{UC}\) results are strongly correlated since they are obtained by using the same data set.

<table>
<thead>
<tr>
<th>(z_1)</th>
<th>(\langle z_1 \rangle)</th>
<th>(z_2)</th>
<th>(\langle z_2 \rangle)</th>
<th>(\frac{\sin^2 \theta_{h(1)} \langle 1 + \cos^2 \theta_{h(1)} \rangle}{1 + \cos^2 \theta_{h(1)}})</th>
<th>(A^{UL}_2 (10^{-4}))</th>
<th>(A^{UL}_2 (10^{-4}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15, 0.2</td>
<td>0.174 [0.15, 0.2]</td>
<td>0.174</td>
<td>0.791</td>
<td>2.64 ± 0.37 ± 0.39</td>
<td>1.25 ± 0.26 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>0.15, 0.2</td>
<td>0.174 [0.2, 0.3]</td>
<td>0.244</td>
<td>0.789</td>
<td>3.72 ± 0.29 ± 0.40</td>
<td>1.74 ± 0.21 ± 0.19</td>
<td></td>
</tr>
<tr>
<td>0.15, 0.2</td>
<td>0.174 [0.3, 0.5]</td>
<td>0.378</td>
<td>0.786</td>
<td>4.06 ± 0.24 ± 0.43</td>
<td>1.87 ± 0.19 ± 0.21</td>
<td></td>
</tr>
<tr>
<td>0.15, 0.2</td>
<td>0.174 [0.5, 0.9]</td>
<td>0.617</td>
<td>0.781</td>
<td>6.26 ± 0.34 ± 0.57</td>
<td>2.80 ± 0.23 ± 0.29</td>
<td></td>
</tr>
<tr>
<td>0.2, 0.3</td>
<td>0.244 [0.15, 0.2]</td>
<td>0.174</td>
<td>0.789</td>
<td>3.76 ± 0.29 ± 0.40</td>
<td>1.76 ± 0.21 ± 0.19</td>
<td></td>
</tr>
<tr>
<td>0.2, 0.3</td>
<td>0.244 [0.2, 0.3]</td>
<td>0.244</td>
<td>0.788</td>
<td>4.69 ± 0.21 ± 0.41</td>
<td>2.17 ± 0.17 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>0.2, 0.3</td>
<td>0.244 [0.3, 0.5]</td>
<td>0.377</td>
<td>0.785</td>
<td>4.99 ± 0.21 ± 0.44</td>
<td>2.24 ± 0.16 ± 0.21</td>
<td></td>
</tr>
<tr>
<td>0.2, 0.3</td>
<td>0.244 [0.5, 0.9]</td>
<td>0.617</td>
<td>0.780</td>
<td>8.27 ± 0.36 ± 0.58</td>
<td>3.57 ± 0.22 ± 0.28</td>
<td></td>
</tr>
<tr>
<td>0.3, 0.5</td>
<td>0.378 [0.15, 0.2]</td>
<td>0.174</td>
<td>0.786</td>
<td>4.53 ± 0.25 ± 0.43</td>
<td>2.08 ± 0.19 ± 0.21</td>
<td></td>
</tr>
<tr>
<td>0.3, 0.5</td>
<td>0.377 [0.2, 0.3]</td>
<td>0.244</td>
<td>0.785</td>
<td>4.73 ± 0.21 ± 0.44</td>
<td>2.12 ± 0.16 ± 0.21</td>
<td></td>
</tr>
<tr>
<td>0.3, 0.5</td>
<td>0.377 [0.3, 0.5]</td>
<td>0.377</td>
<td>0.782</td>
<td>6.23 ± 0.33 ± 0.48</td>
<td>2.70 ± 0.19 ± 0.23</td>
<td></td>
</tr>
<tr>
<td>0.3, 0.5</td>
<td>0.378 [0.5, 0.9]</td>
<td>0.619</td>
<td>0.777</td>
<td>9.47 ± 0.59 ± 0.62</td>
<td>3.85 ± 0.29 ± 0.30</td>
<td></td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.617 [0.15, 0.2]</td>
<td>0.174</td>
<td>0.781</td>
<td>6.58 ± 0.37 ± 0.58</td>
<td>2.94 ± 0.24 ± 0.29</td>
<td></td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.617 [0.2, 0.3]</td>
<td>0.244</td>
<td>0.780</td>
<td>7.45 ± 0.35 ± 0.58</td>
<td>3.21 ± 0.22 ± 0.28</td>
<td></td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.619 [0.3, 0.5]</td>
<td>0.378</td>
<td>0.777</td>
<td>8.77 ± 0.59 ± 0.62</td>
<td>3.55 ± 0.29 ± 0.30</td>
<td></td>
</tr>
<tr>
<td>0.5, 0.9</td>
<td>0.622 [0.5, 0.9]</td>
<td>0.622</td>
<td>0.772</td>
<td>18.46 ± 1.31 ± 0.98</td>
<td>6.93 ± 0.54 ± 0.43</td>
<td></td>
</tr>
</tbody>
</table>

The table presents the mean values for the pion in one hemisphere and the corresponding mean values for the second pion, along with the Collins asymmetry results calculated in the RF12 (upper table) or RF0 (lower table) frames. The quoted errors are statistical and systematic, respectively.
FIG. 4. (color online). Asymmetries measured in the MC sample in RF12 (top) and RF0 (bottom) for \(KK\), \(K\pi\), and \(\pi\pi\) pairs. The upper plots show the U/L double ratio, while the lower plots the U/C double ratio. The 16 \((z_1, z_2)\) bins are shown on the x-axis: in each interval between the dashed lines, \(z_1\) is chosen in the following ranges: \([0.15, 0.2]\), \([0.2, 0.3]\), \([0.3, 0.5]\), and \([0.5, 0.9]\), while within each interval the points correspond to the four bins in \(z_2\). We subtract these biases from the background-corrected asymmetry, and the statistical errors (represented by the bars around the points) are included into the systematic uncertainties.
FIG. 5. (color online). Correction factors for the dilution of the asymmetry due to the difference between the thrust and the $q\bar{q}$ axis. The 16 $(z_1, z_2)$ bins are shown on the x-axis: in each interval between the dashed lines, $z_1$ is chosen in the following ranges: [0.15, 0.2], [0.2, 0.3], [0.3, 0.5], and [0.5, 0.9], while within each interval the points correspond to the four bins in $z_2$. The open markers, triangles and circles, show the corrections applied to the U/L and U/C double ratios in the RF12 frame, respectively, after the background correction and MC biases subtraction. The correction factors for in RF0, full markers, are consistent with unity for both double ratios, and so no corrections are applied. Since we find the same dilutions for $K K$, $K \pi$, and $\pi \pi$, we apply the same correction factors to the three samples.